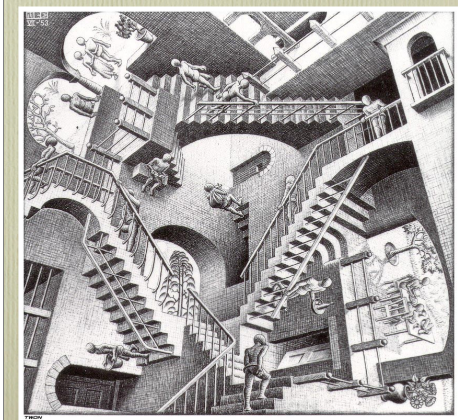
## Going Straight in a Bent Space: I



Peter Watson

M. C. Escher:
"Relativity"

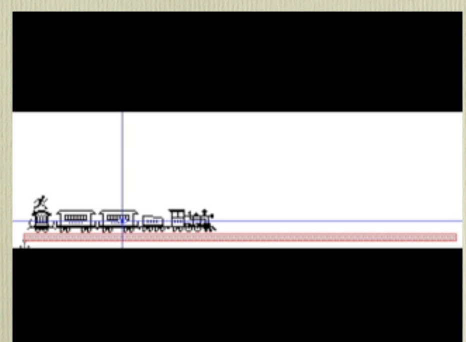
## Statutory Warning

- This lecture is for mature audiences only
- Extreme violence may be caused to your pre-conceptions

## Relative Motion

- Suppose a train is travelling at 5 m/s and a bandit is running towards the front at 2 m/s, relative to the train.
- How fast is he moving relative to the ground?

## How fast is he moving relative to the train?



## How fast is the ground moving relative to him?

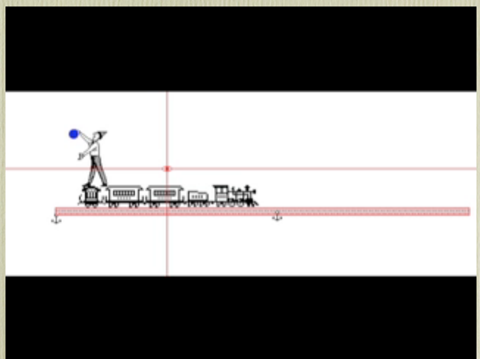
## Frames of Reference

The proper name for "point of view" is "frame of reference": a frame of reference which is not accelerating is an "inertial frame"

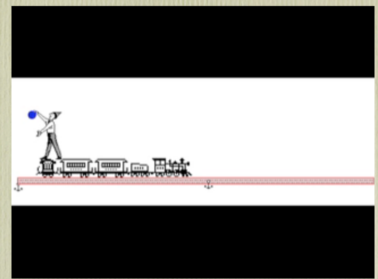
- This is Galilean Relativity: All inertial frames are equivalent
- We can consider doing an experiment in two different frames:
    1. Earth Frame: if we measure a distance(velocity) in this frame, we will call it x(v)
    2. Train Frame: if we measure a distance(velocity) in this frame, we will call it x'(v')

## e.g. just dropping a ball

- In the train frame



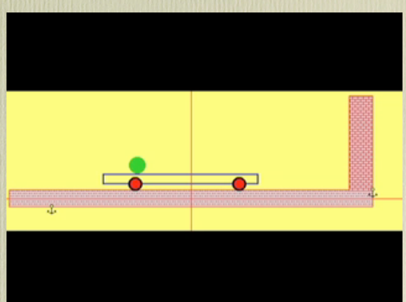
- In the earth frame



- Results of any experiment can be described in any frame: no frame is preferred.
- Put differently: you cannot do an experiment to decide if you are moving, since one man's motion is another man's station!.

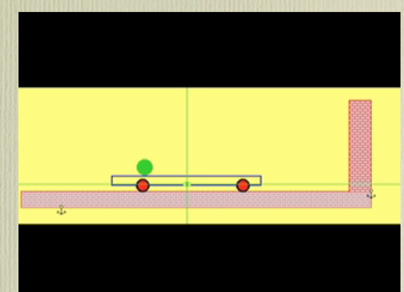
# Inertial Frames

- An inertial frame is one that does not accelerate
- Stationary objects stay stationary

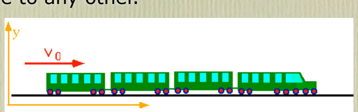


# Non-inertial Frames

- An non-inertial frame does accelerate
- Stationary objects can accelerate without forces

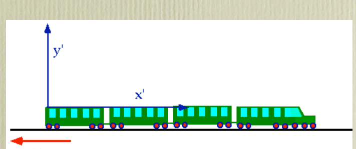


- Can transform the results of an experiment in any one frame to any other.

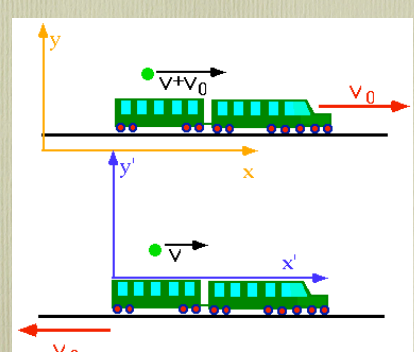



- Velocity in earth frame = Velocity of train frame + Velocity in train frame

- $v = v_0 + v'$




## and can compare them both



Have gone through this in (sordid) detail since it is wrong!

- We have assumed:

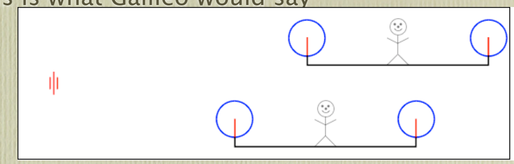
1. Laws of Physics are the same in all inertial frames,
2. Time is the same in all frames

- 2. is a hidden assumption, that was never written down.
- The correct statement (Einstein) is

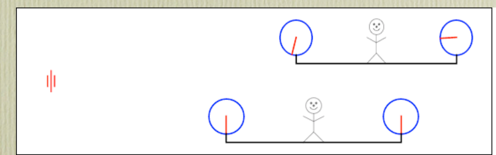
1. Laws of Physics are the same in all inertial frames,
2. **The speed of light is the same in all frames**

2. This means that (since speed = distance/time) distance and/or time must change when we go from one frame to another.

- This is what Galileo would say



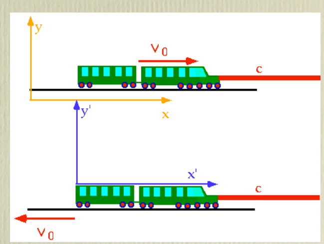
- And this is what Einstein would say



Means clocks must measure different times

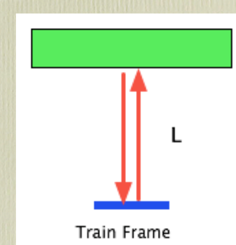
Suppose we fire a beam of light from the front of a train.

- From the point of view of the earth we would expect
- $c=v_0+c'$
- in fact
- $c=c'$

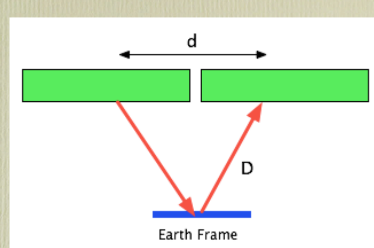



# Time Dilation

- To find out how the time changes from one frame to another, consider bouncing a light off a mirror as the train goes past.
- $$t=\frac{L}{c}$$




- In the earth frame, the light has to travel further, since the train has moved.




We can solve this
- giving

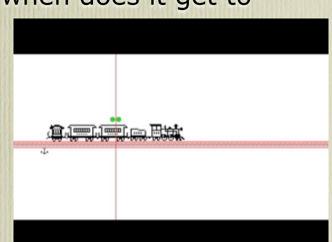
$$t'=t\sqrt{1-\frac{v^2}{c^2}}$$

- so $t'<t$

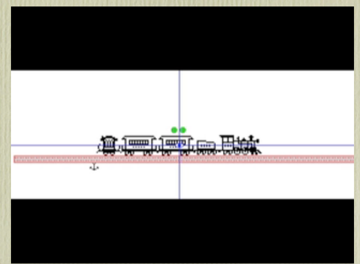
i.e. moving clocks run slow

# Simultaneity

- Since time is not the same in two frames, events which are simultaneous in one frame are not in another
- e.g suppose a flash of light is emitted at the centre of a train: when does it get to the end?
- in the earth frame



- but in the train frame



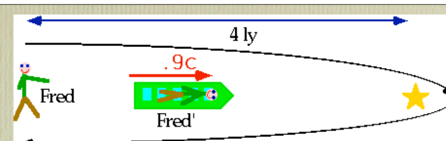
## Note there are a lot of other consequences

- Length contraction (moving objects appear to be shorter)
- Increase of mass (objects get heavier the faster they go, so cannot go faster than light)
- and
- $E = mc^2$

## The Twin Paradox

- How much does this slowing down of time matter?
- e.g. Suppose you are in an OC Transpo bus ($v_0 = 10ms^{-1}$):
- how slow will your watch appear to run compared to your clock at home?
- T = 1 hour at home
- Corresponds to 1 hour - 1 picosecond on the bus
- Note that this correction term is tiny for all cases we are familiar with (which is just as well!)

## Twin Paradox



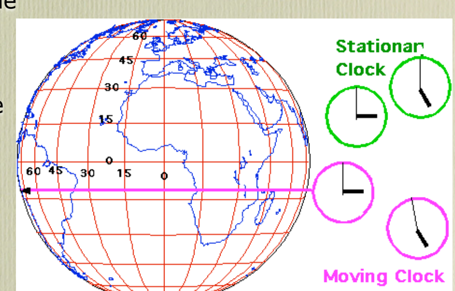

- The star α-Centauri is 4 light-years distant from earth.
- Fred and Fred' are both 20.
- Fred' leaves for α-Centauri at .9 c.
- How old is Fred when Fred' gets back?
- 28.89 yrs
- How old is Fred'?
- 23.87 yrs

Your reaction to all this should be:

- **"This is really stupid. What really happens?"**
- Answer: In physics you cannot ask
- "What really happens?"
- The best one can do is ask
- "What can I measure?"
- Reality is a dangerous concept

## So can we measure it?

- Hafele-Keating experiment done in 1980:
- atomic clock flown round the world (first-class!) and compared to time of atomic clock "at rest".
- Time lost by moving clocks ~ 190 ns



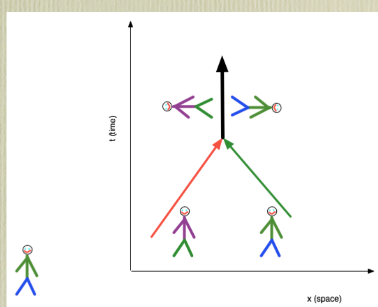
Vladimir: That passed the time
Estragon: It would have passed in any case.
Vladimir: Yes, but not so quickly.
**Beckett: waiting for Godot.**

## Time as a fourth dimension

The changes to space and time that Einstein found show that they are aspects of the same thing: space-time.

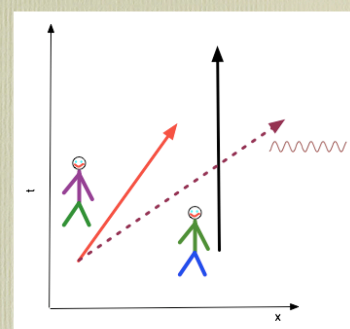
- Galileo-Newton Space is 3-D, time is an independent quantity
- Einstein-Minkowski, Space-time is 4-D, and motion mixes space and time in different ways

- Einstein's concept of time can be expressed graphically by "worldlines" in a space-time diagram.
- Reduced to 1 (2) space and 1 time dimension, can describe interactions as events: e.g 2 men walk into each and fall over.



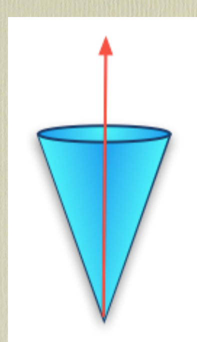

Following constraints must be satisfied by world-lines:

Must be oriented from past to future "flow of time".
- Static object remains at same x, but time still moves.
- Moving objects cannot have a slope > c (otherwise object would be moving faster than light.)
- Events occur when worldlines intersect



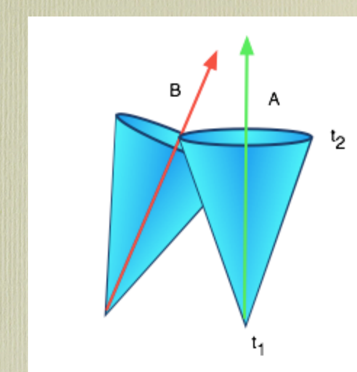

# Light Cone

- Possible light paths are rep. by "Light Cone".
- Cannot escape the light cone (it includes all possible futures for you!)

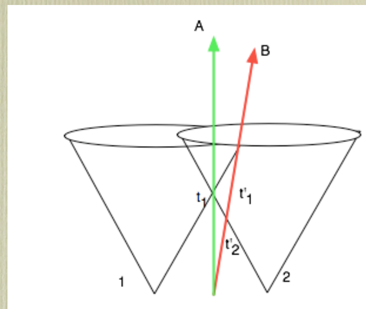


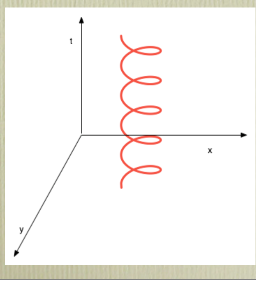
"Light Cone" represents possible paths of signals:

- i.e. interaction need not be direct, but can send (e.g.) phone message.
- Shows person A sending signal at time $t_1$ which is received by B at time $t_2$
- Note times are measured in A's frame



- Can see the violation of simultaneity:
- e.g. two flashes of light are seen as simultaneous by observer A but not by B

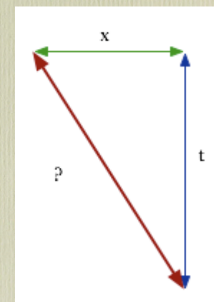



Some world-lines are more complex: e.g. a planet with 2 space dimensions

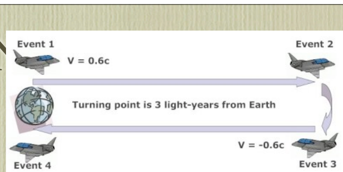
Statutory Warning: We have represented time as a 4th dimension: this does not mean it **is** the fourth dimension.

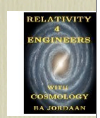
- e.g suppose we have an event now and one in the future at time t and position x: the distance is **not**
- $s^2=x^2+t^2$
- (in fact we can't even add space and time).



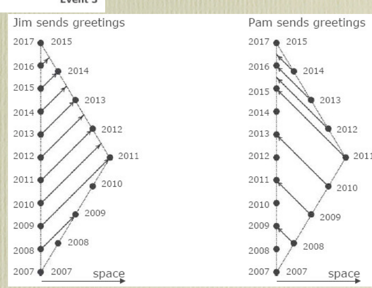

- We can use (note the minus sign)
- $s^2=x^2-c^2t^2$
- but even this needs careful interpretation.

N... etation




- Twins Jim(stay-at-home) and Pam (traveller at 60% of c)
- Exchange Xmas greetings
- Note must describe times very carefully




e.g. Suppose we send a flash of light:

How does time move in the frame of the light?

- It doesn't: there is no time in this frame.
- Can we describe this in English?
- Imagine you are a photon
- You can't

How about a novel?

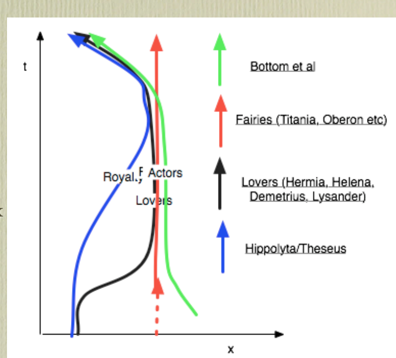
*My Life as a Photon*

*By*

*Bit Delight*

However this 4-D picture is useful. We can analyze the time in any creative work in the same way:

- e.g. Midsummer Nights Dream.
- Note this is a gross over-simplification:
- e.g the lovers + fairies + Bottom have very complex crossing world-lines
- I'll put a girdle round the earth in forty minutes (Puck)




The assertion: prior to 1900 the space-time diagram for any work satisfied the standard conditions: Aristotle's three unities become "space-time causality is preserved" or "special relativity is satisfied".

- Einstein's next question was
- Why do all masses fall at same rate?
- All normal forces (e.g. electrical, friction, elastic...) don't produce same acceleration in all bodies.

$$F = m_I a$$

- The inertial mass $m_I$) measures how hard things are to accelerate (2nd. law)