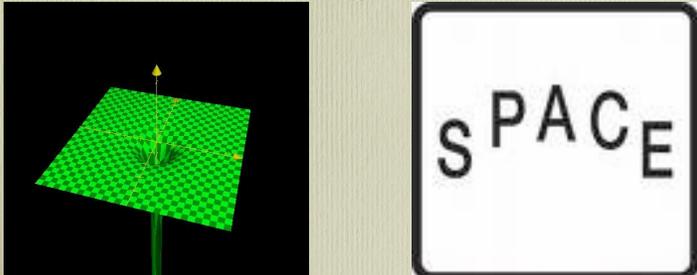


Going Straight in a Bent Space: How Matter bends Time

Peter Watson



- Einstein's next question as
- Why do all masses fall at same rate?

All normal forces (e.g. electrical, friction, elastic...) don't produce same acceleration in all bodies.

$$F = m_i a$$

The inertial mass  $m_i$  measures how hard things are to accelerate (2nd. law)

But the gravitational mass ( $m_G$ ) measures gravitational force or weight

$$F = m_G g$$

but we know everything falls at the same rate (well, in a vacuum) so  $a = g$  only if the "inertial mass" = "gravitational mass".

so

$$m_i \equiv m_G$$

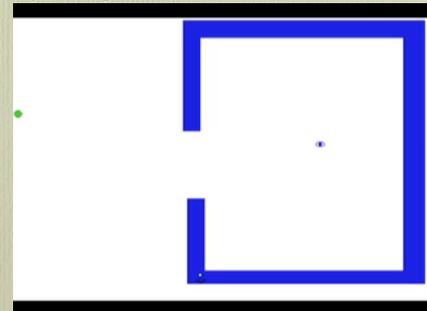
Are we really sure the  $m$ 's are the same? This concerned Newton.

Can demonstrate this is true to 1 part in a trillion ( $10^{12}$ ) (Eötvös experiment)

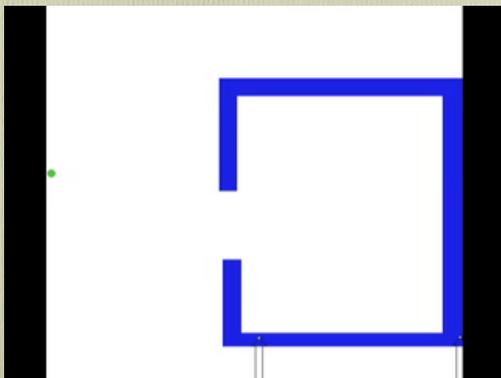
Special relativity said you cannot do an experiment to decide if you are moving.

General says that you cannot do an experiment to distinguish between a gravitational field and an acceleration (!!!!!!!!!!!)

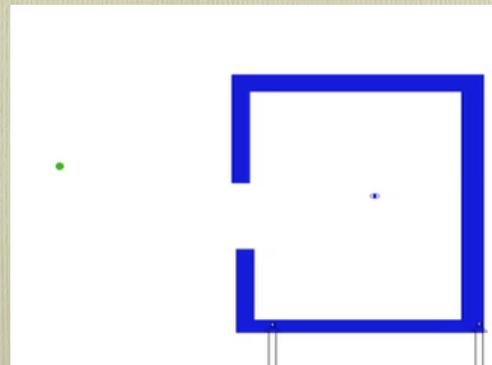
Suppose you are in a stationary elevator, and a bullet is shot horizontally through a window, it will fall due to gravity..



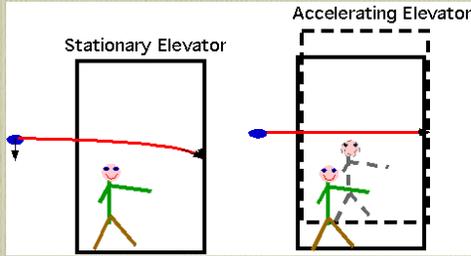
- Suppose you are in an accelerating elevator, and a bullet is shot horizontally, it will travel in a straight line (but the elevator will move up)



- so it will appear to fall inside the elevator.



## You cannot distinguish them

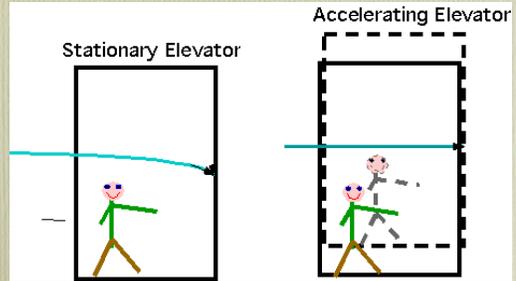


- This is known as the equivalence principle

Suppose you are in an accelerating elevator, and a beam of light is shot horizontally, it will appear to fall..

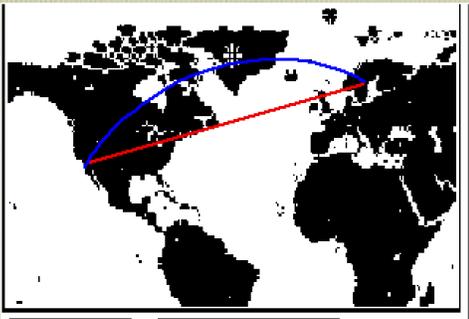
Suppose you are in a stationary elevator in a gravitational field, and a beam of light is shot horizontally, it will fall..

You cannot distinguish the two. Light gets affected by gravity?



## General relativity:

- What is a straight line?
- A Socratic dialog.....



Which is the straight line?

- **A Body continues at rest or in a state of uniform motion unless acted on by a force.**

Uniform motion means in a straight line.

.....But we are in a curved space .....

Need a new word: **Geodesic**

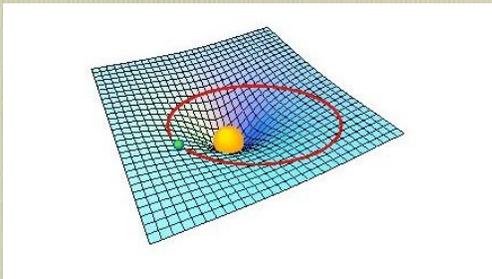
A geodesic in Euclidean space  $\equiv$  straight line  $\equiv$  shortest path

Can either say:

1. There is a force called gravity which acts on all energies (and hence attracts light)
2. There is no such thing as gravity, it's just that masses distort space-time in their neighbourhood

Either way, don't jump off tall buildings: you can be just as dead in a curved space!

Massive bodies follow timelike geodesics so planets are actually moving in "straight" lines in a curved space...

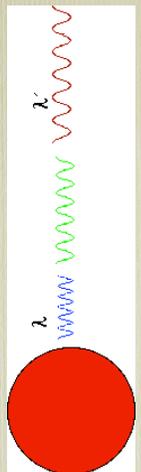


- "Lenses extend unwish through curving wherewhon till unwish returns on its unself" [e.e.cummings](#)

## Time and Gravity

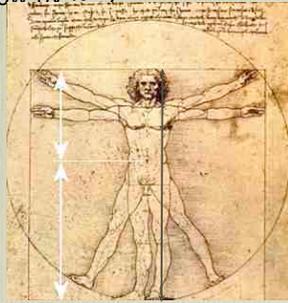
A ball thrown up near the earth's surface will lose energy.

- How about light?
- Back the the elevator: Light emitted from floor hits ceiling after a time  $t = h/c$ .
- During this time, lab (elevator) has accelerated to a speed  $u = gt$ ,
- so the light gets stretched out
- But light is a clock ....
- Implies that clocks run slow in gravitational fields



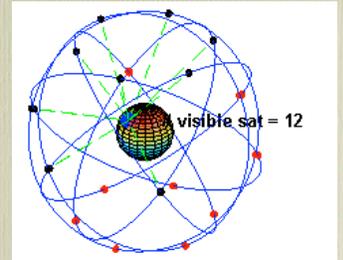
# Gravitational Red-shift

- This is another consequence of the equivalence principle:
- confirmed in numerous experiments over the last 40 years, starting with Pound-Rebka
- Means clocks at Earth's surface run slow by  $\sim 7$  ns per second
- difference in time over height  $h$  is
  - $\delta t = gh/c^2$
  - so  $10^{-16}$  secs/m
  - Can just get this with next generation clocks!

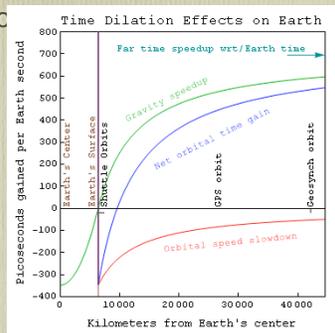


# GPS

- needs to be corrected for relativity
- 3 separate effects:
- Sagnac effect: earth rotates, so is not an inertial frame, so events are not simultaneous: can eliminate by using satellites to E and W



- Special relativity: satellite clock is moving relative to earth, so slows down  $\sim 10^{-10}$  or  $7 \mu\text{s/day}$
- GR: satellite clock is in free fall, so speeds up  $\sim 5 \times 10^{-10}$  or  $46 \mu\text{s/day}$
- Would give an 11.7 km error after one day!



• But suppose gravity was **REALLY** strong.....

• But suppose gravity was **REALLY** strong.....

# Black Holes

- A particle will escape from the earth if it has positive energy

At the earth's surface,  $v \sim 11$  km/s

However we can interpret this differently: what radius would the earth have for a given escape velocity? In particular, if the escape velocity is the speed of light  $c$ , nothing can escape

$$R = \frac{2GM}{c^2}$$

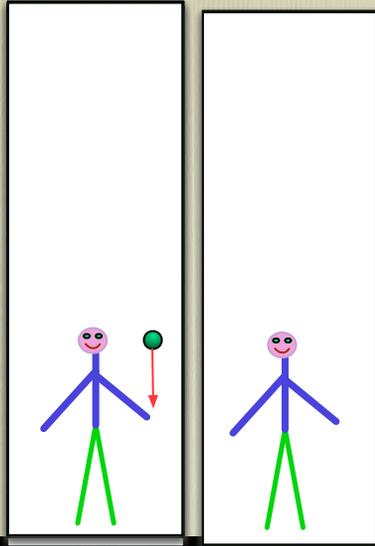
- This is the Schwarzschild radius (loosely the black-hole radius) for any mass.
- What is this for the earth?
- $\sim 8$  mm
- **Statutory Warning:** This is a fudge: you cannot treat light as a massive particle, nor can you handle a very strong gravitational field as if it were a weak one..... (there are actually two factors of 2 error which cancel out.....weren't we lucky!)

# Black Holes

- Invented by .....?
- Einstein
- Hawking?
- Well, actually, **John Michell**, rector of Thornhill Church in Yorkshire
- geologist? philosopher? astronomer? Seismologist?
- Polymath.
- presented his ideas to the Royal Society in London in 1783.



- A particle will escape from the earth if it has positive energy
- At the earth's surface, "escape velocity" is 11 km/s

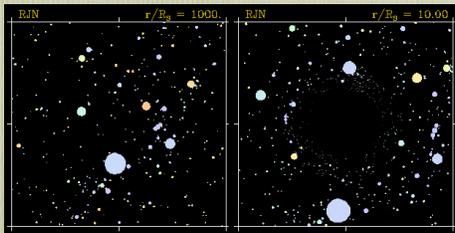


- If we throw something up from the earth, it will fall back
- Throw it
- But if the
- nothing
- If the ea
- hole



Peter Watson

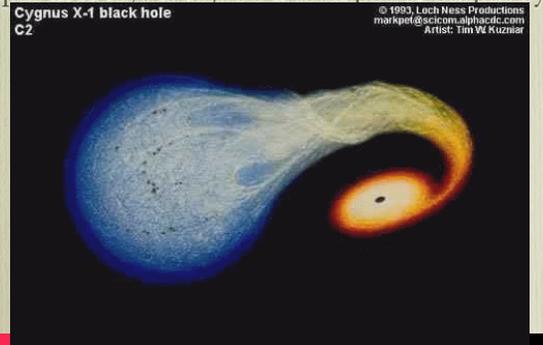
- One way to see a black hole: it's black!
- If we are really lucky...(or unlucky) as a gap in the sky



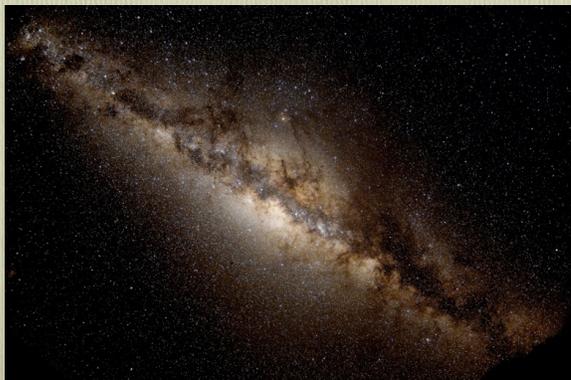
Too Close to a Black Hole  
Credit & Copyright: Robert Nemiroff (MTU)-

Peter Watson

- Stuff falling in will become very hot and produce X-rays
- Cygnus X-1: visible star ~20 mass of sun
- Invisible object  $M \sim 9M_{\odot}$
- Power output in X-rays is 10,000 x total power output by sun!



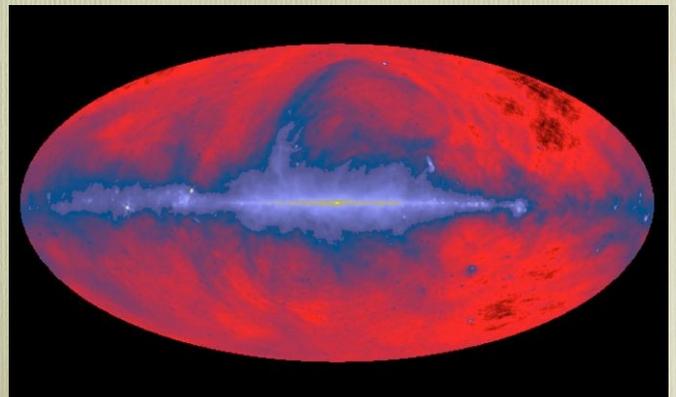
But there are much bigger black holes around



- This is the Milky way, showing the whole sky

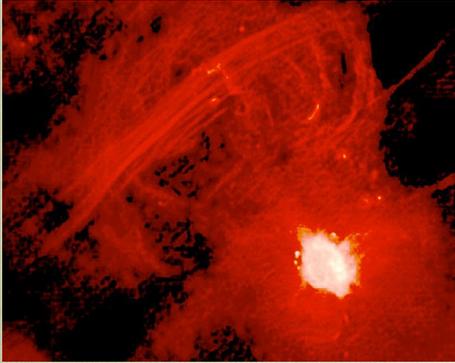
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- If we look at it with radio waves, see very intense source at centre



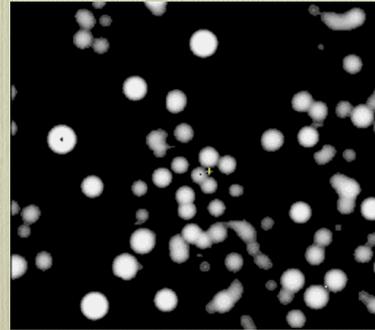
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- which gets brighter as we zoom in



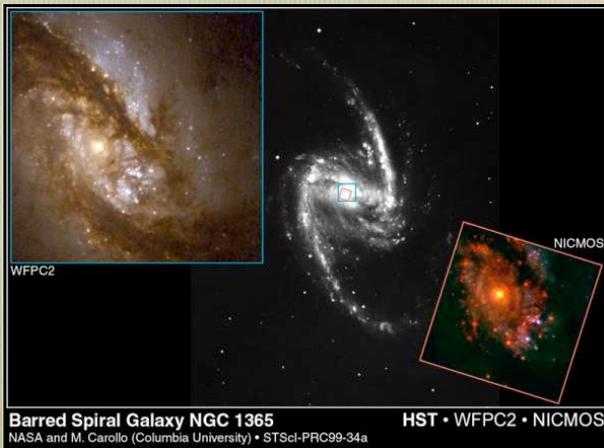
Peter Watson

- The stars there are swirling round something 10 million times as heavy as the sun



Peter Watson

- All galaxies seem to have a huge black hole at the centre



Barred Spiral Galaxy NGC 1365 HST • WFPC2 • NICMOS  
NASA and M. Carollo (Columbia University) • STScI-PRC99-34a



Peter Watson

- And this is maybe where it is happening now:
- Two galaxies have collided and the black holes seem to be coalescing



3C75 X-rays from Chandra

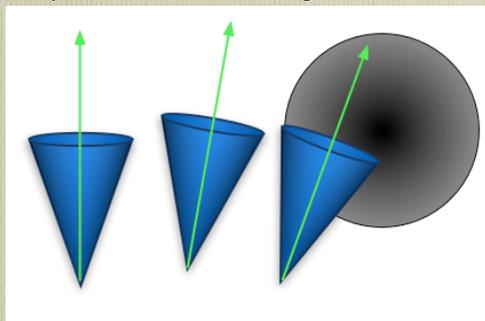


Peter Watson

## What happens to time near a black hole

- Gravity modifies the light cone
- close to a black hole, all your futures include falling into it!

A consequence: time stops at the edge of a black-hole for an external observer.



Note that we can still just escape the BH if we move fast enough

Alice never could quite make out, in thinking it over afterwards, how it was that they began: all she remembers is, that they were running hand in hand, and the Queen went so fast that it was all she could do to keep up with her: and still the Queen kept crying 'Faster! Faster!' but Alice felt she COULD NOT go faster, though she had not breath left to say so..... ..

Alice looked round her in great surprise. 'Why, I do believe we've been under this tree the whole time! Everything's just as it was!

"Of course it is," said the Queen, 'what would you have it?"

"Well, in OUR country," said Alice, still panting a little, 'you'd generally get to somewhere else—if you ran very fast for a long time, as we've been doing.'

'A slow sort of country!' said the Queen. 'Now, HERE, you see, it takes all the running YOU can do, to keep in the same place. If you want to get somewhere else, you must run at least twice as fast as that!'



## Geometry of Curved spaces

Note we have carefully avoided saying what we mean by a curved space

- Bending of light by gravity allows two (or more) geodesics: i.e. many time-lines connecting same points
- The 2-D curved surface of the Earth is embedded in a 3-D space. Hence if a massive body curves space, it implies extra dimensions.

In fact we can carry out tests to decide if we live in a "normal" 3-D space (Euclidean)

e.g. parallel lines may be impossible (they get further apart or closer together!)



## Geometry of Curved spaces

Note we have carefully avoided saying what we mean by a curved space

- Bending of light by gravity allows two (or more) geodesics: i.e. many time-lines connecting same points
- If you take the example of the 2-D curved surface of the Earth, this is embedded in a 3-D space. Hence if a massive body curves space, it almost implies extra dimensions.

In fact we can carry out tests to decide if we live in a "normal" 3-D space (Euclidean) e.g. parallel lines may be impossible (they get further apart or closer together!)

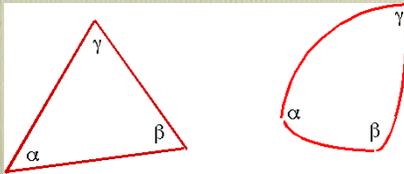


## Geometry of Curved spaces

- angles of a triangle add up to  $180^\circ$

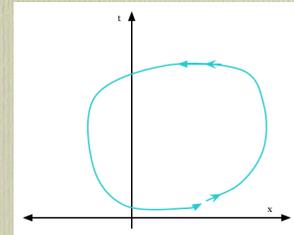
$$\alpha + \beta + \gamma = 180$$

These are experiments that we can almost do.  
(Gauss tried the 2nd!).

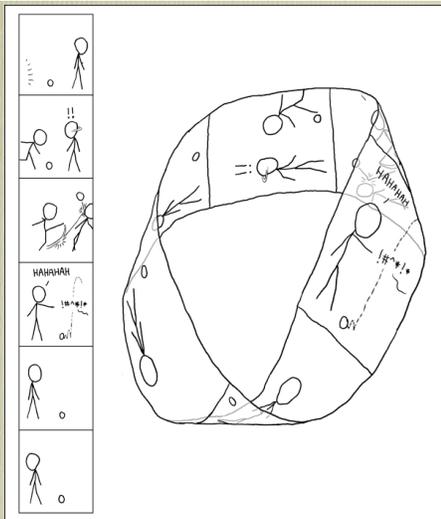


## So can we build a time-machine?

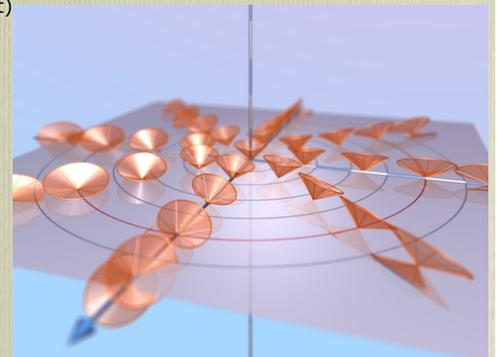
- Now we know the question to ask;
- Can we arrange for world-lines to be closed?



- xkcd.com



- Godel invented a model universe consistent with GR with closed time-lines. Not like ours:
- it has a centre (ours has no centre)
- it is not homogenous (ours is)
- It rotates (ours doesn't)





## Finally

- A somewhat subtle point
- Originally we had “universal time”
- Not crazy to think that time can change, but then it should be tied to measurement (e.g. clock, photon, biology .....)
- It is now connected to a “frame of reference”, disconnected from any measurement
- Lets do something simpler! Can we at least predict things?!