1 Parameters

In what follows, use the following parameters unless otherwise indicated

$$
\begin{aligned}
&H_0 = 70kms^{-1}Mpc^{-1}\\
&1eV = 1.602 \times 10^{-19}J\\
&G = 6.67 \times 10^{-11}m^3kg^{-1}s^{-2}\\
&c = 2.998 \times 10^8 ms^{-1}\\
&\hbar = 1.055 \times 10^{-34}Js = 6.582 \times 10^{-16}eVs\\
&k = 1.381 \times 10^{-23}JK^{-1}\\
&m_ec^2 = .511MeV\\
&m_pc^2 = 938.272MeV\\
&m_nc^2 = 939.566MeV\\
&1AU = 1.496 \times 10^{11}m\\
&1pc = 3.086 \times 10^{16}m\\
&T_{CMBR} = 2.736K\\
&L_{sun} = 3.846 \times 10^{26}W\\
&M_{sun} = 1.989 \times 10^{30}kg\\
&m_{hydrogen} = 1.67 \times 10^{-27}kg
\end{aligned}
$$

2 Qualitative Questions

Give short answers to 6 of the following questions.

1. Black holes would seem to be a good candidate for dark matter. Why does this seem to be incorrect?
2. What is meant by the angular distance in cosmology? How do we expect it to vary at very large red-shifts?
3. What is meant by the Planck time or Planck energy? What is their relevance to cosmology?
4. Neutrinos are said to have "frozen out" at approximately 1 second after the Big Bang. What is meant by this? Why did it occur for a much earlier time than photon freeze-out?
5. Hubble originally measured his constant to be around $550 kms^{-1} Mpc^{-1}$. What (approximately) would this imply for the age of the universe? Why is it unacceptable?
6. One of the alternatives to dark energy is to drop the Copernican principle: namely the assertion that the earth is not at a privileged position in the universe. What is meant by this and why might it work?
7. Suppose the neutron were totally stable: this could be the case if (e.g.) the electron had a mass of 2 MeV, with all other parameters staying the same. How would you expect this to affect nucleosynthesis?
8. The CMBR shows a large dipole effect, with the "hottest" area in the constellation Leo. How is this interpreted?
9. How does gravitational lensing give us information on the dark matter distribution in galaxy clusters?

3 Problems

Attempt 4 of the following problems.

1. Suppose the universe has a critical density of matter entirely in the form of hydrogen atoms. How many would there be in a cubic kilometre of space? Assuming the H_0 given above, what is the change in the 1 km^3 volume over one year? Suppose the steady state model of the universe were correct (so that none of the parameters change), how many hydrogen atoms would need to be created in this volume to maintain the density constant?

2. Someone proposes a model of the universe which is dominated by a (very weird) matter with equation of state parameter w = -2/3. Solve the fluid equation to show that the energy density

$$\varepsilon(t) = \frac{\varepsilon_0}{a(t)}$$

Assuming the universe is flat, use the Friedman equation to find ε_0 in terms of H. Show that the scale parameter

$$a(t) \propto t^2$$

Do you believe in it?

3. Suppose we have a metric distance formula given by

$$ds^2 = e^{-2\kappa r} dr^2 + r^2 d\varphi^2$$

What is the curvature of the space? What is the proper distance from the origin to the point r? What do your results become as $r \to 0$?

4. A spiral galaxy has a rotation curve given by

$$\begin{aligned} v(r) &= v_0 \tfrac{r}{r_0}, r < r_0 \\ &= v_0, r > r_0 \end{aligned}$$

Show this can be understood if it consists of a spherical core of constant density and a halo with

$$\begin{aligned} \rho_0 &= \tfrac{3v_0^2}{4\pi G r_0^2}, (r < r_0) \\ \rho(r) &= \tfrac{3v_0^2}{4\pi G r^2}, (r > r_0) \end{aligned}$$

Hence find the total mass out to 50 kPc, if $r_0 = 2$ kPc and $v_0 = 150$ km/s.

5. A cluster of galaxies has a spherical shape with a luminous mass of $10^{13} M_o$ and a radius of 3 MpC. Finds its density and hence its collapse time: compare your result to the Hubble time. If the dark matter density is measured to be $\rho_{DM} \approx 100\rho_{lum}$ what is the collapse time? Comment on your results.