Beyond the Standard Model at Colliders

(part 2 of 2)

Heather Logan

Carleton University

TRIUMF Summer Institute 2009
Outline

== Lecture 1 ==

1. Why Beyond the Standard Model
2. Resonances

== Lecture 2 ==

3. Decay chains to a dark matter particle
4. Summary
What about the dark matter?

Pink – hot gas via x-ray emission
Blue – mass density as reconstructed from gravitational lensing
What about the dark matter?

Particle dark matter: what do we know?
- Needs to be neutral.
- Needs to be stable.
- Limits on interaction cross section from direct detection searches.
- Thermal production ↔ EW-strength coupling, 0.1–1 TeV mass.

Note: without thermal production, all bets are off.
- Axions: super-light particles, produced coherently in a “cold” state, search via resonant conversion to photons in a microwave cavity.
- WimpZillas: way too heavy to produce in colliders, number density too low to detect.
- SuperWimps: coupling extremely weak; produced in decay of some other relic particle. Collider: search for parent particle?
Dark matter: direct experimental evidence that we need something new. Not guaranteed to be a new weak-scale particle. Many BSM models provide a dark matter candidate. (Weakly-Interacting Massive Particle = WIMP)
- SUSY
- Universal extra dimensions
- Little Higgs with T-parity

WIMP needs to be stable → some conserved quantum number.
- Lightest particle carrying the conserved quantum number is forced to be stable.

- SUSY: R-parity, a $Z_2$ parity wanted for proton stability.

- Universal extra dimensions: KK-parity, also an imposed $Z_2$

- Little Higgs with T-parity: an imposed $Z_2$ parity motivated to improve EWP consistency.

- Twin Higgs, inert doublet model, singlet scalar dark matter, etc etc... pretty much any model with a dark matter candidate.
$Z_2$ parities: particles have quantum number either $+1$ or $-1$ under the parity:

\[ \phi \to +\phi \text{ (even)} \quad \psi \to -\psi \text{ (odd)} \]

A Lagrangian invariant under the $Z_2$ can only contain terms with even powers of odd-charged fields. This means that interaction vertices must involve only even numbers of odd-charged fields.

– Starting from a $Z_2$-even initial state, $Z_2$-odd particles can be produced only in pairs. [SUSY particles must be pair produced.]

– A $Z_2$-odd particle must decay to an odd number of $Z_2$-odd particles plus any number of $Z_2$ even particles. [SUSY particles decay via a decay chain to the lightest SUSY particle (LSP), which is stable.]

– Two $Z_2$-odd particles can annihilate into a final state involving only $Z_2$-even particles. [Two LSPs in the galactic halo can annihilate to SM particles.]
These $Z_2$ parities give a good WIMP dark matter candidate, which is obviously nice.

But they also greatly improve the consistency of the model with electroweak precision measurements (and flavour constraints), without interfering with the solution to the hierarchy problem.

This second feature was first clearly articulated with the introduction of the Little Higgs with T-parity (2005).

Long story short:
(1) If the new states are odd under a $Z_2$, they cannot be exchanged at tree-level, and contributions to EW or flavour observables can only appear at 1-loop $\rightarrow$ much suppressed.
(2) The cancellation of the $\Lambda^2$-divergent Higgs mass radiative corrections already involves loops of new particles, so new particles being odd under a $Z_2$ does not interfere with this.
Let’s look at some models:

- SUSY
- Little Higgs with T-parity
- Universal extra dimensions

I’ll also talk about some collider techniques for studying events with pairs of decay chains to a dark matter particle.
- Masses
- Spins
Supersymmetry (SUSY)

The “super symmetry” itself is an extension of the Poincare algebra discovered in the early ’70s.

The new generators are spinor objects $Q_\alpha, \bar{Q}_\beta$ which talk to the Poincare group [translations, rotations, boosts] via:

$$\{Q_\alpha, \bar{Q}_\beta\} = 2(\sigma^\mu)_{\alpha\dot{\beta}}P_\mu$$

- A SUSY generator acting on a scalar produces a fermion.
- A SUSY generator acting on a fermion produces either a scalar or a vector (depending on how the spinor indices are contracted).
- A SUSY generator acting on a vector produces a fermion.

Fermions and bosons can thus be grouped into supermultiplets that transform within themselves under the supersymmetry.
Supersymmetry and the hierarchy problem

Fermion masses don’t have a hierarchy problem. E.g., fermion self-energy diagram with a gauge boson loop gives

\[ \delta m_f \sim \frac{g^2}{16\pi^2} m_f \ln \left( \frac{\Lambda^2}{m_f^2} \right) \]

Note that \( \delta m_f \propto m_f \). This is a manifestation of chiral symmetry:
- In the limit \( m_f = 0 \) the system has an extra symmetry: the left- and right-handed components of the fermion are separate objects.
- In this limit, radiative corrections cannot give \( m_f \neq 0 \) – fermion mass is protected by chiral symmetry.

Scalars have no such symmetry protection (in a non-SUSY theory).

But Supersymmetry relates a scalar to a partner fermion:
- it links the scalar mass to the fermion mass!
  (In unbroken SUSY, members of a supermultiplet are degenerate)
So the scalar mass is also protected by chiral symmetry – the \( \Lambda^2 \) divergences all cancel and only \( \ln(\Lambda^2/m^2) \) divergences are left.
The Minimal Supersymmetric Standard Model (MSSM)

The MSSM is defined by adding the minimal set of new particles for a working supersymmetric theory that contains the SM.

Particle content:

Each fermion gets a boson (scalar) partner:

\[ e_L, e_R \leftrightarrow \tilde{e}_L, \tilde{e}_R \quad \text{“selectrons”} \]
\[ t_L, t_R \leftrightarrow \tilde{t}_L, \tilde{t}_R \quad \text{“top squarks” (or “stops”)} \]

and similarly for the rest of the quarks and leptons

The number of degrees of freedom match:

chiral fermion has 2 d.o.f $\leftrightarrow$ complex (charged) scalar has 2 d.o.f.

Each gauge boson gets a fermionic partner:

\[ W^\pm \leftrightarrow \tilde{W}^\pm \quad \text{“winos”} \]
\[ Z, \gamma \leftrightarrow \tilde{Z}, \tilde{\gamma} \quad \text{“zino”, “photino”} \]
\[ \text{(or } W^0, B \leftrightarrow \tilde{W}^0, \tilde{B} \text{ “neutral wino”, “bino”)} \]

Again the number of degrees of freedom match:

Transverse gauge boson has 2 d.o.f. (polarizations) $\leftrightarrow$ chiral fermion
Supersymmetric Lagrangian

In a supersymmetric theory, the Lagrangian must be invariant under supersymmetry transformations.

This turns out to be a really strict requirement. For ease of Lagrangian-building, all terms are lumped into generating functions (called the superpotential and Kahler potential) with prescribed rules for generating the various terms in the supersymmetric Lagrangian.

Allowed Lagrangian terms:
- Gauge interactions (which also fix Higgs, squark, and slepton self-interaction terms)
- Fermion-Higgs Yukawa interactions (which also show up in squark and slepton interactions)
- A Higgsino mass term called the $\mu$ parameter
- and some problematic fermion-fermion-sfermion Yukawa couplings.
“Problematic”? 

These problematic Yukawa couplings couple $QL\tilde{D}^c$ (violates lepton number) and $U^cD^c\tilde{D}^c$ (violates baryon number). These two couplings together allow very fast proton decay:

$$uu \rightarrow e^+\bar{d} \text{ via } t\text{-channel down-type squark} \Rightarrow p \rightarrow e^+\pi^0$$

Very very bad! Need to forbid at least one of these two couplings.

R-parity gets rid of them both: $R = (-1)^{2S+3B+L}$

$S$ = spin, $B$ = baryon number, $L$ = lepton number.

Upshot: familiar SM particles are R-parity even; SUSY partners are R-parity odd.

Conserved R-parity $\rightarrow$ lightest R-odd particle (LSP) is stable $\rightarrow$ dark matter candidate!

Heather Logan (Carleton U.)  
BSM at Colliders (2)  
TSI '09
### Summary: the particle content of the MSSM

<table>
<thead>
<tr>
<th>Names</th>
<th>Spin</th>
<th>$P_R$</th>
<th>Gauge Eigenstates</th>
<th>Mass Eigenstates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Higgs bosons</td>
<td>0</td>
<td>+1</td>
<td>$H_u^0 \ H_d^0 \ H_u^+ \ H_d^-$</td>
<td>$h^0 \ H^0 \ A^0 \ H^\pm$</td>
</tr>
<tr>
<td>squarks</td>
<td>0</td>
<td>−1</td>
<td>$\tilde{u}_L \ \tilde{u}_R \ \tilde{d}_L \ \tilde{d}_R$</td>
<td>“ ”</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\tilde{s}_L \ \tilde{s}_R \ \tilde{c}_L \ \tilde{c}_R$</td>
<td>“ ”</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\tilde{t}_L \ \tilde{t}_R \ \tilde{b}_L \ \tilde{b}_R$</td>
<td>$\tilde{t}_1 \ \tilde{t}_2 \ \tilde{b}_1 \ \tilde{b}_2$</td>
</tr>
<tr>
<td>sleptons</td>
<td>0</td>
<td>−1</td>
<td>$\tilde{e}_L \ \tilde{e}_R \ \tilde{\nu}_e$</td>
<td>“ ”</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\tilde{\mu}_L \ \tilde{\mu}<em>R \ \tilde{\nu}</em>\mu$</td>
<td>“ ”</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\tilde{\tau}_L \ \tilde{\tau}<em>R \ \tilde{\nu}</em>\tau$</td>
<td>$\tilde{\tau}_1 \ \tilde{\tau}<em>2 \ \tilde{\nu}</em>\tau$</td>
</tr>
<tr>
<td>neutralinos</td>
<td>1/2</td>
<td>−1</td>
<td>$\tilde{B}^0 \ \tilde{W}^0 \ \tilde{H}_u^0 \ \tilde{H}_d^0$</td>
<td>$\tilde{N}_1 \ \tilde{N}_2 \ \tilde{N}_3 \ \tilde{N}_4$</td>
</tr>
<tr>
<td>charginos</td>
<td>1/2</td>
<td>−1</td>
<td>$\tilde{W}^\pm \ \tilde{H}_u^+ \ \tilde{H}_d^-$</td>
<td>$\tilde{C}_1^\pm \ \tilde{C}_2^\pm$</td>
</tr>
<tr>
<td>gluino</td>
<td>1/2</td>
<td>−1</td>
<td>$\tilde{g}$</td>
<td>“ ”</td>
</tr>
<tr>
<td>gravitino/</td>
<td>3/2</td>
<td>−1</td>
<td>$\tilde{G}$</td>
<td>“ ”</td>
</tr>
</tbody>
</table>
goldstino      |

... plus the usual SM quarks, leptons, and gauge bosons.

If Supersymmetry were an exact symmetry, the SUSY particles would be degenerate with their SM partners. Clearly they are not $\rightarrow$ SUSY must be broken. Most general set of SUSY-breaking terms $\rightarrow > 100$ new parameters [specific SUSY-breaking-mediation models $\rightarrow \mathcal{O}(5 – 10)$ new params]
Most of the SUSY phenomenology is controlled by the (unknown) SUSY-breaking parameters.

A schematic sample SUSY spectrum:
(This may or may not have anything to do with reality)

Some features:
- $\tilde{N}_1$ is the LSP
- $\tilde{t}_1$ and $\tilde{b}_1$ are the lightest squarks
- $\tilde{\tau}_1$ is the lightest charged slepton
- Coloured particles are heavier than uncoloured particles

from Martin, hep-ph/9709356
Where do these features come from?

SUSY particle masses are (presumably) set at a high scale by some SUSY-breaking mechanism.

Masses “run” down by Renormalization Group equations.

E.g., “Constrained MSSM” (CMSSM) model (a.k.a. mSUGRA):

from Martin, hep-ph/9709356
Running of the gauge couplings (the other reason people love SUSY)

Dashed lines: SM   Solid lines: MSSM
(Bands are the uncertainties in the low-energy values.)

The MSSM at 1 TeV gives gauge coupling unification!
SUSY particle decays and collider phenomenology

The general features of SUSY particle decays are controlled by:

R-parity conservation [introduced to avoid fast proton decay]
Lightest R-odd particle (LSP) is stable
Decay chains of R-odd (SUSY) particles must end in LSP
LSP as dark matter: require LSP to be neutral and uncoloured
  → escapes from detector → missing energy

Mass spectrum [controlled by SUSY breaking and RGEs]
Heavier particles decay through a cascade of lighter particles
  → High multiplicity of objects in SUSY events – multijets, multileptons
NLSP affects event content:
  – light stau → events with taus
  – light sbottom → events with $b$-jets

Couplings
In general, couplings are just the supersymmetrized version of SM couplings.
Superparticle production at hadron colliders

SUSY particles are always produced in pairs (because of R-parity).

Production via QCD generally dominates, even though squarks and gluinos are typically heavy:

- **Gluino pairs**
- **Squark pairs**
- **Squark + gluino**

LHC reach depends on mass spectrum. Reach for gluinos & squarks is typically out to about 2 TeV.

*Heather Logan (Carleton U.)*  
*BSM at Colliders (2)*  
*TSI '09*
Superparticle decays

**Gluino decays:** always to quark + squark.

If $M_{\tilde{g}} < M_{\tilde{q}}$, then gluino will decay via an off-shell squark:

3-body decays, $\tilde{g} \to q\tilde{q}^* \to q\tilde{q}\tilde{N}_i$ or $q\tilde{q}'\tilde{C}_i$

**Squark decays:** decay to quark + gluino (strong coupling) if kinematically allowed. Otherwise quark + neutralino or quark + chargino or (for 3rd gen.) quark + Higgsino.

Decay branching fractions controlled by quark and -ino compositions.

**Slepton decays:** decay to lepton + neutralino or lepton + chargino.

**Neutralino and chargino decays:** to lepton + slepton or quark + squark, or to gauge or Higgs boson + lighter neutral-/charg-ino

Typically get **decay chains**, which always end with the LSP.

For example:  

\[
\begin{array}{c}
\tilde{g} \\
\tilde{q}_L \\
\tilde{N}_2 \\
\tilde{f} \\
\tilde{N}_1
\end{array}
\]
Generic signatures of SUSY at hadron colliders:

**Missing transverse energy**
From two escaping LSPs

**Large jet multiplicity**
Produce heavier SUSY particles via QCD; long decay chains

**Large \( \sum E_T \) in event**
Decay of heavy particles produces energetic jets, leptons
Relatively spherical distribution in detector

**Like-sign leptons or \( b \)-jets**
Gluino is Majorana – decays equally likely to \( \tilde{q} \) or \( \tilde{q}^* \)
Decay chain gives leptons – like-sign if \( \tilde{q}\tilde{q} \) or \( \tilde{q}^*\tilde{q}^* \)

Many more specific signatures have been studied in detail.
Signatures depend strongly on mass spectrum.

*Heather Logan (Carleton U.)*

*BSM at Colliders (2)*

*TSI '09*
After discovery, want to measure SUSY masses and couplings.

A new challenge:
Each SUSY event contains two invisible massive particles. Can’t reconstruct SUSY masses directly Can’t even measure transverse mass like for $W \rightarrow \ell \nu$

Need to use more sophisticated techniques:
take advantage of decay chains.
- Kinematic endpoints
- Four-momentum conservation relations
SUSY kinematic at the LHC

Difficult:
- $\sqrt{s}$ not known; varies event-by-event
- Boost of CM along beam direction not known

But: LHC can produce heavy sparticles: long decay chains, many kinematic variables to play with.
Since we don’t know the boost of individual events, need to use kinematic invariants, like invariant masses.

Consider the decay chain $\tilde{N}_2 \rightarrow \tilde{\ell}^+_R \ell^+_R \rightarrow \tilde{N}_1 \ell^+ \ell^-$
(First need to select events that contain a $\tilde{N}_2$ and identify the $\ell^+ \ell^-$ coming from the $\tilde{N}_2$ decay.)
Invariant observable: invariant mass of $\ell^+ \ell^-$: $M_{\ell\ell}$

How is this related to the SUSY masses?
Consider the decay chain $\tilde{N}_2 \rightarrow \ell_R^\pm \ell^\mp \rightarrow \tilde{N}_1 \ell^+ \ell^-$

Momentum and energy conservation in each decay:

$$p_{\tilde{N}_2} = p_{\ell_1} + p_{\tilde{\ell}} \quad \quad p_{\tilde{\ell}} = p_{\ell_2} + p_{\tilde{N}_1}$$

Combine and rearrange:

$$M_{\ell\ell}^2 = (p_{\ell_1} + p_{\ell_2})^2 = (p_{\tilde{N}_2} - p_{\tilde{N}_1})^2 = m_{\tilde{N}_2}^2 + m_{\tilde{N}_1}^2 - 2p_{\tilde{N}_2} \cdot p_{\tilde{N}_1}$$

What is this? Let’s work in the $\tilde{N}_2$ rest frame (can do that because we’re calculating kinematic invariants!)

$$\rightarrow \quad p_{\tilde{N}_2} \cdot p_{\tilde{N}_1} = m_{\tilde{N}_2} E_{\tilde{N}_1}$$

where $E_{\tilde{N}_1}$ is energy in the $\tilde{N}_2$ rest frame, so

$$M_{\ell\ell}^2 = m_{\tilde{N}_2}^2 + m_{\tilde{N}_1}^2 - 2m_{\tilde{N}_2} E_{\tilde{N}_1}$$

Now we need to find the kinematic endpoint(s) of $E_{\tilde{N}_1}$ in the $\tilde{N}_2$ rest frame in terms of the SUSY masses.

**Strategy:**

Relate the energies to masses and the $\tilde{\ell}$ decay angle $\theta$
Relate the energies to masses and the $\tilde{\ell}$ decay angle $\theta$ in $\tilde{N}_2$ rest frame.

Look at $\tilde{N}_2$ decay: $m_{\tilde{N}_2} = E_{\ell_1} + E_{\tilde{\ell}}, \quad \vec{p}_{\ell_1} = -\vec{p}_{\tilde{\ell}}$

Solve using four-momentum conservation (with $m_\ell \simeq 0$):

\[
E_{\ell_1} = \frac{1}{2m_{\tilde{N}_2}} \left( m_{\tilde{N}_2}^2 - m_{\tilde{\ell}}^2 \right) \quad |\vec{p}_{\ell_1}| = E_{\ell_1}
\]

\[
E_{\tilde{\ell}} = \frac{1}{2m_{\tilde{N}_2}} \left( m_{\tilde{N}_2}^2 + m_{\ell}^2 \right) \quad |\vec{p}_{\tilde{\ell}}| = |\vec{p}_{\ell_1}| = E_{\ell_1}
\]
Now let’s do the $\tilde{\ell}$ decay in the $\tilde{\ell}$ rest frame (denoted by a star – will need to boost back to the $\tilde{N}_2$ rest frame at the end!)

4-momentum conservation: $m_{\tilde{\ell}} = E_{\tilde{\ell}2}^* + E_{\tilde{N}_1}^*$, \[ \vec{p}_{\tilde{\ell}1}^* = -\vec{p}_{\tilde{N}_1}^* \]

$$E_{\tilde{\ell}2}^* = \frac{1}{2m_{\tilde{\ell}}} \left( m_{\tilde{\ell}}^2 - m_{\tilde{N}_1}^2 \right) \quad |\vec{p}_{\tilde{\ell}2}^*| = E_{\tilde{\ell}2}$$

$$E_{\tilde{N}_1}^* = \frac{1}{2m_{\tilde{\ell}}} \left( m_{\tilde{\ell}}^2 + m_{\tilde{N}_1}^2 \right) \quad |\vec{p}_{\tilde{N}_1}^*| = |\vec{p}_{\tilde{\ell}2}^*| = E_{\tilde{\ell}2}$$

Have $E_{\tilde{N}_1}^*$ in the $\tilde{\ell}$ rest frame; need to boost to $\tilde{N}_2$ rest frame.

Work out the kinematic boost from the $\tilde{\ell}$ energy and momentum:

$$\gamma = \frac{E_{\tilde{\ell}}}{m_{\tilde{\ell}}} = \frac{m_{\tilde{N}_2}^2 + m_{\tilde{\ell}}^2}{2m_{\tilde{\ell}} m_{\tilde{N}_2}}, \quad \gamma \beta = \frac{|\vec{p}_{\tilde{\ell}}^*|}{m_{\tilde{\ell}}} = \frac{m_{\tilde{\ell}}^2 - m_{\tilde{N}_2}^2}{2m_{\tilde{\ell}} m_{\tilde{N}_2}}$$

Now do the boost:

$$E_{\tilde{N}_1} = \gamma \left( E_{\tilde{N}_1}^* + \beta |\vec{p}_{\tilde{N}_1}^*| \cos \theta^* \right)$$

where $\theta^*$ is the angle between the $\tilde{\ell}$ decay direction and the $\tilde{\ell}$ boost (in the $\tilde{\ell}$ rest frame)
Plug in $\gamma$ and $\gamma\beta$:

$$E_{\tilde{N}_1} = \frac{1}{4m_{\tilde{N}_2} m_\ell^2} \left[ \left( m_{\tilde{N}_2}^2 + m_\ell^2 \right) \left( m_\ell^2 + m_{\tilde{N}_1}^2 \right) + \left( m_{\tilde{N}_2}^2 - m_\ell^2 \right) \left( m_\ell^2 - m_{\tilde{N}_1}^2 \right) \cos \theta^* \right]$$

Remember our original formula for the $\ell\ell$ invariant mass:

$$M_{\ell\ell}^2 = m_{\tilde{N}_2}^2 + m_{\tilde{N}_1}^2 - 2m_{\tilde{N}_2} E_{\tilde{N}_1}$$

Kinematic endpoint: the maximum of $M_{\ell\ell}$ corresponds to the minimum of $E_{\tilde{N}_1}$, which occurs for $\cos \theta^* = -1$:

$$E_{\tilde{N}_1} \bigg|_{\text{min}} = \frac{1}{2m_{\tilde{N}_2} m_\ell^2} \left( m_\ell^4 + m_{\tilde{N}_2}^2 m_{\tilde{N}_1}^2 \right)$$

Plugging in to $M_{\ell\ell}^2$ formula and simplifying gives

$$M_{\ell\ell} \bigg|_{\text{max}} = \left[ \frac{\left( m_{\tilde{N}_2}^2 - m_\ell^2 \right) \left( m_\ell^2 - m_{\tilde{N}_1}^2 \right)}{m_\ell^2} \right]^{1/2}.$$
One endpoint measurement constrains a combination of three SUSY masses.

\[ M_{\ell\ell}^{\text{max}} = \left( \frac{m_{\tilde{N}_2}^2 - m_{\ell}^2}{m_{\ell}^2} \right) \left( m_{\ell}^2 - m_{\tilde{N}_1}^2 \right) \]^{1/2}

from Paige, hep-ph/0211017
LHC can do more if we look at longer decay chains: 
→ more kinematic invariants to play with.

Add a squark to the top of our decay chain:
\[ \tilde{q} \rightarrow \tilde{N}_2 q \rightarrow \tilde{\ell}^\pm \ell^\mp q \rightarrow \tilde{N}_1 \ell^+ \ell^- q \]

Invariant mass of \( q \) and the first lepton emitted (\( \ell_1 \)) has an endpoint analogous to the \( \ell \ell \) endpoint:

\[
M_{q\ell_1}^{\text{max}} = \left[ \frac{\left( m_\tilde{q}^2 - m_{\tilde{N}_2}^2 \right) \left( m_{\tilde{N}_2}^2 - m_{\ell}^2 \right)}{m_{\tilde{N}_2}^2} \right]^{1/2}
\]

How to distinguish \( \ell_1 \) from \( \ell_2 \)?
→ \( \ell_1 \) likely to have higher energy.

With \( M_{q\ell_1}^{\text{max}} \) and \( M_{\ell\ell}^{\text{max}} \) we have 2 measurements and 4 unknowns.
Not doing better than before... yet.

from Paige, hep-ph/0211017
Decay chain has an extra kinematic invariant:

**Invariant mass of** $q\ell^+\ell^-$. 

$$M_{q\ell\ell}|_{\text{max}} = \left[ \frac{(m_{\tilde{q}}^2 - m_{\tilde{N}_2}^2)}{m_{\tilde{N}_2}^2} \right]^{1/2} \left[ \frac{(m_{\tilde{N}_2}^2 - m_{\tilde{N}_1}^2)}{m_{\tilde{N}_2}^2} \right]^{1/2}$$

3 measurements and 4 unknowns. 

Doing better!

from Paige, hep-ph/0211017

---

Heather Logan (Carleton U.)  

BSM at Colliders (2)  

TSI '09
There are also lower kinematic edges:

After applying a cut $M_{\ell\ell} > M_{\ell\ell}^{\text{max}}/\sqrt{2}$, get a complicated formula for a lower kinematic endpoint for $M_{q\ell\ell}$.

Can also consider the decay chain $\tilde{q} \rightarrow \tilde{N}_2 q \rightarrow \tilde{N}_1 h q$ with $h \rightarrow b\bar{b}$

[The Higgs mass can be measured elsewhere]

Then $M_{hq}$ has a threshold (lower kinematic edge)

Get enough measurables to extract all the masses!
Uncertainties from blurring of the kinematic endpoints by backgrounds, wrong jet/lepton combinations, also gluon radiation off the jet at NLO.
**Kinematic endpoints:** Statistics are not super; we’re only making use of the events right near the endpoints. Can we use the events from the middles of the distributions to do better? Some avenues of research:

**Kinematic shapes:**
Fit to the whole shape of the invariant mass distributions, not just the endpoint. Helps to deal with background.

Gjelsten, Miller, & Osland, hep-ph/0410303, 0501033

**Exact kinematic relations:**
Completely solve the kinematics of each SUSY cascade decay. Need longer decay chain: at least 5 sparticles

E.g.: \[ \tilde{g} \rightarrow q\tilde{q} \rightarrow qq\tilde{N}_2 \rightarrow q\ell\ell \rightarrow qq\ell\ell\tilde{N}_1 \]

Kawagoe, Nojiri, & Polesello, PRD 71, 035008 (2005)
**Exact kinematic relations** Kawagoe, Nojiri, & Polesello, PRD 71, 035008 (2005)

Completely solve the kinematics of each SUSY cascade decay.
- Selected events must be from one particular decay chain
- SUSY particles in the decay chain must be on mass shell

Each event gives you the 4-momenta of all the decay products except $\tilde{N}_1$.

Have to consider a longer decay chain: $\tilde{g} \to q\bar{q} \to qq\tilde{N}_2 \to qq\ell\bar{\ell} \to qq\ell\ell\tilde{N}_1$. 5 sparticles involved → 5 mass-shell conditions:

\[
\begin{align*}
m_{\tilde{N}_1}^2 &= p_{\tilde{N}_1}^2 \\
m_{\ell}^2 &= (p_{\tilde{N}_1} + p_{\ell_1})^2 \\
m_{\tilde{N}_2}^2 &= (p_{\tilde{N}_1} + p_{\ell_1} + p_{\ell_2})^2 \\
m_{q}^2 &= (p_{\tilde{N}_1} + p_{\ell_1} + p_{\ell_2} + p_{q_1})^2 \\
m_{g}^2 &= (p_{\tilde{N}_1} + p_{\ell_1} + p_{\ell_2} + p_{q_1} + p_{q_2})^2
\end{align*}
\]

Each $qq\ell\ell\tilde{N}_1$ event contains 4 unmeasured degrees of freedom, the 4 components of the $\tilde{N}_1$ 4-momentum.

→ Each event picks out a 4-dimensional hypersurface in a 5-dimensional mass parameter space.

Overlap multiple events in this hyperspace → find a discrete set of solutions from overlap of different hypersurfaces.
Solve shorter chains by using both sides of the event.
6 constraint equations from one event:

\[
\begin{align*}
(M^2_Z &=) \ (p_1 + p_3 + p_5 + p_7)^2 = (p_2 + p_4 + p_6 + p_8)^2, \\
(M^2_Y &=) \ (p_1 + p_3 + p_5)^2 = (p_2 + p_4 + p_6)^2, \\
(M^2_X &=) \ (p_1 + p_3)^2 = (p_2 + p_4)^2, \\
(M^2_N &=) \ p_1^2 = p_2^2.
\end{align*}
\]

\[
p_1^x + p_2^x = p_{miss}^x, \quad p_1^y + p_2^y = p_{miss}^y.
\]

8 unknown components of missing (invisible) particle 4-momenta \((p_1 \text{ and } p_2)\)

Still 2 unknowns: cannot solve.
Add a second event: 8 more unknowns ($q_1$ and $q_2$) but 10 more equations:

$$q_1^2 = (q_1 + q_3)^2 = (q_1 + q_3 + q_5)^2 = (q_1 + q_3 + q_5 + q_7)^2 = (q_2 + q_4)^2 = (q_2 + q_4 + q_6)^2 = (q_2 + q_4 + q_6 + q_8)^2 = p_2^2,$$

$$p_2^2 = (p_2 + p_4)^2 = (p_2 + p_4 + p_6)^2 = (p_2 + p_4 + p_6 + p_8)^2,$$

$$q_1^x + q_2^x = q_{miss}^x, \quad q_1^y + q_2^y = q_{miss}^y.$$

Can invert for the masses directly!
SPS1a: Ideal from 100 events (no combinatorics or resolution)  

300 fb$^{-1}$ after ATLFAST, combinatorics, some cuts to reduce wrong combinations

Cheng et al, PRL 100, 252001 (2008)

Can reconstruct genuine mass peaks!  
Relies on all decays being 2-body decays.

SUSY mass reconstruction techniques are looking good.  
But what about other models with dark matter candidates?

Heather Logan (Carleton U.)  

BSM at Colliders (2)  

TSI ’09
So far so good with SUSY... until:

PHYSICAL REVIEW D 66, 056006 (2002)

Bosonic supersymmetry? Getting fooled at the CERN LHC

Hsin-Chia Cheng
Enrico Fermi Institute, The University of Chicago, Chicago, Illinois 60637

Konstantin T. Matchev
Department of Physics, University of Florida, Gainesville, Florida 32611
and TH Division, CERN, Geneva 23, CH–1211, Switzerland

Martin Schmaltz
Department of Physics, Boston University, Boston, Massachusetts 02215
(Received 3 June 2002; published 23 September 2002)

We define a minimal model with universal extra dimensions, and begin to study its phenomenology. The collider signals of the first Kaluza-Klein (KK) level are surprisingly similar to those of a supersymmetric model with a nearly degenerate superpartner spectrum. The lightest KK particle (LKP) is neutral and stable because of KK parity. KK excitations cascade decay to the LKP yielding missing energy signatures with relatively soft jets and leptons. Level 2 KK modes may also be probed via their KK number violating decays to standard model particles. In either case we provide initial estimates for the discovery potential of the Fermilab Tevatron and the CERN Large Hadron Collider.

Universal extra dimensions introduced as a “straw-man” model to compare to SUSY.
Universal extra dimensions:
Flat 5th dimension with periodic boundary conditions
Get particle-in-a-box KK excitations: \( M^{(n)} = n/R \) (5-dim)

Generic 5th dimension: tree-level exchange of gauge boson KK modes \( \rightarrow \) electroweak precision constraints give \( 1/R \gtrsim 6 \text{ TeV} \).

Fermion KK modes: letting fermions into the 5th dimension complicates things.
A chiral 5-dim fermion contains both a left- and right-handed 4-dim fermion!
Need to get rid of the extra components of the zero-modes, so SM fermions stay chiral.

Deal with this by “orbifolding”: impose a reflection symmetry down the middle of the 5th dimension.
- Projects out the bad 5-dim fermion components.
- Preserves a \( Z_2 \) remnant of 5-dim momentum conservation:
KK parity \( = (-1)^n \) (\( n \) is KK number).
UED with KK parity:

Level-1 KK modes are odd under KK parity: have to be pair produced.

Electroweak precision constraints much weaker: 1-loop, not tree level: limits on KK quark masses $\sim$ few hundred GeV from direct searches.

Lightest KK mode is stable due to conserved KK parity:
- Dark matter candidate
- Decay chains to stable particle

Engineered to look a lot like SUSY...
UED phenomenology

KK mode masses get radiative corrections from loops of SM particles. Get splitting in spectrum:

This spectrum is for a common boundary mass [like $m_0$ in CMSSM]
Coloured particles get largest radiative corrections: get shifted upwards.
Lightest odd-parity particle (LKP) is stable: dark matter candidate; missing energy in decay chains.
LKP is naturally $\gamma^{(1)}$ for common boundary terms.
Because of KK parity, get cascade decay chains:

Spectrum tends to be more degenerate than SUSY, but collider signals are similar. Jets, leptons, missing $p_T$.

Couplings related to corresponding SM couplings, just like SUSY. KK-odd particles must be pair-produced.

Major difference is particle spins!

SUSY: partners have opposite spin.

UED: partners have same spin.

from Cheng, Matchev, & Schmaltz, hep-ph/0205314
Another model: **Little Higgs with T-parity**

Hubisz, Meade, Noble, & Perelstein, hep-ph/0506042

Strongest electroweak precision constraints on Little Higgs models come from tree-level exchange of new gauge bosons between fermions.

The new-physics scale $f$ is fairly tightly constrained: $M_{ZH}, M_{WH} \geq 2$ TeV usually required.

**Top-partner mass is linked to $f$:**
Tends to be pushed above 1–3 TeV by EW precision constraints on $f$.

But we need new physics by 1 TeV to cancel $\Lambda^2$ Higgs mass radiative correction before the fine tuning becomes too severe!
If we could eliminate tree-level exchange of $W_H$, $Z_H$, the EW precision constraints would become much looser. Then new particles can be light enough to cancel the Higgs mass divergence without fine-tuning.

Is there an analogue of KK-parity for the little Higgs?

Yes: generically, $T$-parity (short for “TeV-scale parity”).

Construct the Little Higgs model with a $Z_2$ symmetry of the Lagrangian.

Generally have to set some couplings equal, sometimes add a few more particles so that a $Z_2$ parity is conserved.
Phenomenology of the Littlest Higgs with T-parity:
Very similar to UED phenomenology!

Still have the $Z_H$, $W_H$, $A_H$ gauge bosons of Littlest Higgs model
Now they are T-odd: must be pair-produced.
$A_H$ is the lightest: “LTP” (lightest T-odd particle)
Missing energy signatures
Dark matter candidate

Still have the $T$ of Littlest Higgs model
Two versions of the T-parity model: one with $T_+$ (T-even) and
one with $T_-$ (T-odd).
$T_+$: single-production is the same; decays are the same.
$T_-$: must be pair-produced; decays to top and LTP.

Get extra T-odd fermion “partners” of each SM generation
They are needed to make model T-symmetric
Can mix in general: flavour-changing issue (as in SUSY!)
Need to assume T-odd fermions do not mix between generations
To distinguish SUSY from UED or Little Higgs with T-parity, we have to measure the spins.
Consider a decay chain:
\[ \tilde{q} \rightarrow q \tilde{N}_2 \rightarrow q \ell^\pm \tilde{\ell}^\mp \rightarrow q \ell^+ \ell^- \tilde{N}_1 \text{ in SUSY} \]
\[ q_1 \rightarrow qZ_1 \rightarrow q \ell^\pm \ell_1^\mp \rightarrow q \ell^+ \ell^- \gamma_1 \text{ in UED} \]

Form \( M_{q\ell} \) invariant mass dist’n with first (near) lepton
Shape depends on spin of intermediate particle:
\( \tilde{N}_2 \) in SUSY – spin 1/2; \( Z_1 \) in UED – spin 1

Problem: hard to distinguish the first (near) lepton from the second (far) lepton. Tends to wash out spin correlations.
Solution: use a charge asymmetry between \( q\ell^+ \) and \( q\ell^- \)
\( \tilde{N}_2 \) typically mostly \( \tilde{W}^0 \): couples to LH fermions / RH anti-fermions.
Helicity conservation leads to different \( M_{\ell q} \) shape for \( \ell^+ \) vs. \( \ell^- \):

for SUSY

Summing over \( \tilde{q} + \tilde{q}^* \) would wash this out again EXCEPT:
LHC is a \( pp \) collider: more \( q \) than \( \bar{q} \) in PDFs.

from Barr, hep-ph/0405052
Make a lepton charge asymmetry:

\[ A^{+-} = \frac{s^+ - s^-}{s^+ + s^-}, \quad s^\pm = \frac{d\sigma}{d(M_{\ell^\pm q})} \]

Charge asymmetry depends on \( M_{\ell q} \) differently for SUSY, for UED, and for pure “phase space” (flat distribution):

from Battaglia, Datta, De Roeck, Kong, & Matchev, hep-ph/0507284

This tests the spin of \( \tilde{N}_2 \) or \( Z^{(1)} \).
There are many other spin observables in other decay chains.

**SUSY:** $\tilde{g} \rightarrow \tilde{b}_1 \rightarrow \tilde{N}_2 \rightarrow \ell_1 \rightarrow \tilde{N}_1$. Final state is $b\bar{b}\ell^+\ell^-p_T^{\text{miss}}$.

**UED:** $g_1 \rightarrow b_{L1} \rightarrow Z_1 \rightarrow \ell_{R1} \rightarrow \gamma_1$. Final state is $b\bar{b}\ell^+\ell^-p_T^{\text{miss}}$.

UED spectrum can match SUSY spectrum: have only the spins to distinguish them.

Lepton charge asym. vs. $M_{b\ell}$ (softer $b$).

Azimuthal angle dist’n between the two $b$ jets.

---

**Graphs:**

- **SUSY** and **UED** spectra with cuts for different values of $\alpha(1)$.
- Mass spectra for SPS1a with $L = 100$ fb$^{-1}$ and $L = 200$ fb$^{-1}$.

*From Alves, Éboli, & Plehn, hep-ph/0605067*
Summary

The main motivations for introducing physics beyond the Standard Model are
- Dark matter (experimental evidence via gravity)
- Hierarchy problem (theoretical disaster with Higgs mass scale)

We discussed two classes of experimental signatures:
- Resonances
  - Technicolour
  - Higgsless models
  - Little Higgs models
- Decay chains to an invisible dark matter particle
  - SUSY
  - Universal extra dimensions
  - Little Higgs with T-parity
  - Any model with a sector odd under a $Z_2$ symmetry