Beyond the Standard Model

Lecture 3

Heather Logan
Carleton University
Plan

Lecture 1  Monday July 17
- Why BSM?
- Supersymmetry

Lecture 2  Monday July 17
- Supersymmetry continued: phenomenology

Lecture 3  Wednesday July 19
- Large extra dimensions: ADD
- Universal extra dimensions; particle spins and UED vs. SUSY

Lecture 4  Thursday July 20
- Deconstruction and the Little Higgs
- T-parity

Lecture 5  Friday July 21
- Warped extra dimensions: RS
- RS and Technicolour
The idea of extra dimensions is not new.

Kaluza & Klein
In 1919 the mathematician Kaluza was working with Einstein's equations for general relativity.
He discovered that if he put the equations in 5 dimensions, and made the extra dimension a small circle, then what came out in 4 dimensions was ordinary general relativity plus Maxwell’s equations (!).
The theoretical physicist Klein (of Klein-Gordon equation fame) is credited with inventing the idea that the extra dimensions could be physically real but curled up very small.
[The tower of excitations of particles in extra dimensions are called Kaluza-Klein (or KK) modes – more in a bit.]

String theory
String theory is only consistent in 26 dimensions (for bosonic string) or 10 dimensions (for superstring, with fermions and bosons) or maybe 11 dimensions (for supergravity).
The extra dimensions beyond our 3+1 are presumably rolled or folded up very small, so we can’t see them.
The “modern age” of extra-dimensional theories started in 1998:

Arkani-Hamed, Dimopoulos & Dvali (ADD) proposed a “solution” of the hierarchy problem: that the scale of quantum gravity is a TeV, not $10^{18}$ GeV $\sim M_{Pl}$ (!)

How could this be possible?

If there are $\delta$ new extra dimensions with size $R$, then below the scale $R$ the gravitational force law changes from $1/r^2$ to $1/r^{2+\delta}$!

This is just Gauss's law in more than 3 space dimensions: the gravitational field lines have more room to spread out:

$$V(r) \sim \frac{m_1 m_2}{M_*^{2+\delta} r^{1+\delta}} \quad (r \ll R)$$

$$V(r) \sim \frac{m_1 m_2}{M_*^{2+\delta} R^\delta r} \equiv \frac{m_1 m_2}{M_{Pl}^2} \frac{1}{r} \quad (r \gg R)$$

Get a relation between the true higher-dimensional Planck scale $M_*$ and our four-dim apparent Planck scale $M_{Pl} \sim 10^{18}$ GeV:

$$M_{Pl}^2 = M_*^{2+\delta} R^\delta$$
Let’s set $M_* \simeq \text{TeV}$ and see how big $R$ should be.

$\delta = 1 \rightarrow R \sim 10^{13} \text{ cm} = 10^8 \text{ km}$
Ruled out – we’ve measured the gravitational $1/r^2$ force law well on those scales!

$\delta = 2 \rightarrow R \sim 0.1–1 \text{ mm}$
A very interesting range!
Being probed by current direct gravity measurements.

“2 extra dimensions” band $\rightarrow$

$\delta = 6 \rightarrow R \sim 0.1 \text{ fm}$
This gives 10 total dimensions ( = # wanted by superstrings).

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TSI’06
But what about Standard Model (non-gravitational) physics? We’ve probed distance scales down to 0.01 fm with colliders (1 fm $\sim$ size of a proton; corresponds roughly to 1 GeV energy scale).

SM particles can’t propagate in the large extra dimensions. SM must “live on a brane”: particles confined to a 3+1-dim “surface” in the higher-dimensional space, which is no bigger than about 1/TeV.

This picture is called ADD [after the authors] or large extra dimensions.
The $\delta$ extra dimensions are compactified — have finite radius $R$. A particle propagating in such a space has its momentum in the compactified direction(s) quantized: just like particle-in-a-box.

$\delta = 1$: $E^{(n)} = p^{(n)} = 2\pi/\lambda^{(n)} = 2\pi n/L = n/R$ [definition of $R = L/2\pi$]

In this case the particles are relativistic, so $E \propto p$ instead of $p^2$ — for $\delta = 1$ the energy levels are evenly spaced $E \propto n$ (where $n$ is the mode number). A 4-dim observer will see the excited states as massive particles, with mass fixed by the quantized momentum in the extra dimension. Linear spacing of levels, $M^{(n)} = n/R$.

$\delta = 2$: $E^{(n_1,n_2)} = \sqrt{(p^{(n_1)})^2 + (p^{(n_2)})^2} = \sqrt{(n_1/R_1)^2 + (n_2/R_2)^2}$

For common $R_1 = R_2 = R$, spacing [degeneracy] is:

$0$ [1], $1/R$ [2], $\sqrt{2}/R$ [1], $2/R$ [2], etc.

Energy spacing is $1/R \sim 1$ meV – 100 MeV (for $\delta = 2$ to 6)

Very dense spacing: Huge multiplicity; can’t resolve modes. Each KK graviton couples with $1/M_{Pl}$ strength; sum over huge number of modes makes cross sections respectable at energies near $M_\ast$. 

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TSI’06
Two types of collider signal processes:

1. Produce a KK graviton, which escapes from our brane

Monojet + missing energy signature

\[ pp \rightarrow g G^{(n)}, \quad q G^{(n)} \]

E.g.:

Because we don’t have a quantum theory of gravity, we don’t really know how to calculate near the cutoff \( M_\ast \) of the theory. Leads to some uncertainty in prediction for high-energy cross section:

$pp \rightarrow g \: G^{(n)}, \: q \: G^{(n)}$

Monojet + missing energy signature

Signal feature: tail at high $p_T^{\text{miss}}$


Limits from CDF Run II monojet search (1.1 fb$^{-1}$)

Data agrees with SM

$\delta = 2 \rightarrow M_D > 1.33 \: \text{TeV}$

$\delta = 6 \rightarrow M_D > 0.88 \: \text{TeV}$

diagram from Beauchemin, talk at SUSY’06 conference
Two types of collider signal processes:

2. Produce a pair of SM particles via exchange of a KK graviton

\[ pp \rightarrow G^{(n)} \rightarrow f \bar{f}, \quad gg \]

\[ \text{E.g.:} \]

- **Real Graviton Emission**
  - Monojets at hadron colliders

- **Virtual Graviton Emission**
  - Fermion or VB pairs at hadron or e+e- colliders

Broad excess in high-invariant-mass Drell-Yan cross section

Many many gravitons – no resonance.

Plot is CDF search in \( e^+e^- \) invariant mass:

Data agrees with SM.

Limits from DØ:

- \( \delta = 2 \rightarrow M_D > 1.67 \) TeV
- \( \delta = 6 \rightarrow M_D > 1.14 \) TeV

LHC should be sensitive up to \( M_D \simeq 10 \) TeV.
**Constraints on ADD:** Astrophysics provides some stringent constraints on the fundamental scale $M_\ast$ as a function of the number of extra dimensions $\delta$.

<table>
<thead>
<tr>
<th>Condition</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2$</td>
<td>$3$</td>
</tr>
<tr>
<td>Supernova Cooling (68)</td>
<td>$30$</td>
</tr>
<tr>
<td>Cosmic Diffuse $\gamma$-Rays:</td>
<td></td>
</tr>
<tr>
<td>Cosmic SNe (69)</td>
<td>$80$</td>
</tr>
<tr>
<td>$\nu\bar{\nu}$ Annihilation (71)</td>
<td>$110$</td>
</tr>
<tr>
<td>Re-heating (72)</td>
<td>$170$</td>
</tr>
<tr>
<td>Neutron Star Halo (73)</td>
<td>$450$</td>
</tr>
<tr>
<td>Overclosure of Universe (71)</td>
<td>$6.5/\sqrt{h}$</td>
</tr>
<tr>
<td>Matter Dominated Early Universe (75)</td>
<td>$85$</td>
</tr>
<tr>
<td>Neutron Star Heat Excess (73)</td>
<td>$1700$</td>
</tr>
</tbody>
</table>

Astro numbers are rough: depend on scheme for calculating near cutoff $M_\ast$!

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**Compare LHC and ILC reach in $M_\ast$ [TeV]**

From monojet/mono-photon:

<table>
<thead>
<tr>
<th>Process</th>
<th>$\sqrt{s}$ (TeV)</th>
<th>$M_H$ (TeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^+e^- \to \gamma + G_n$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LC $P_{-,+} = 0$</td>
<td>$5.9$</td>
<td>$2.5$</td>
</tr>
<tr>
<td>LC $P_+ = 0.8$</td>
<td>$8.3$</td>
<td>$2.9$</td>
</tr>
<tr>
<td>LC $P_{-} = 0.8, P_+ = 0.6$</td>
<td>$10.4$</td>
<td>$3.3$</td>
</tr>
<tr>
<td>$pp \to g + G_n$</td>
<td>$2$</td>
<td>$4$</td>
</tr>
<tr>
<td>LHC</td>
<td>$4 - 8.9$</td>
<td>$5.0 - 5.8$</td>
</tr>
</tbody>
</table>

From s-channel $G^{(n)}$:

<table>
<thead>
<tr>
<th>Process</th>
<th>$\sqrt{s}$ (TeV)</th>
<th>$M_H$ (TeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^+e^- \to ff$</td>
<td>$0.5$</td>
<td>$4.1$</td>
</tr>
<tr>
<td>$\gamma\gamma \to ff$</td>
<td>$1.0$</td>
<td>$7.2$</td>
</tr>
<tr>
<td>$\gamma\gamma \to WW$</td>
<td>$1.0$</td>
<td>$3.5$</td>
</tr>
<tr>
<td>$e\gamma \to e\gamma$</td>
<td>$1.0$</td>
<td>$8.0$</td>
</tr>
<tr>
<td>$pp \to \ell^+\ell^-$</td>
<td>$14.0$</td>
<td>$7.5$</td>
</tr>
<tr>
<td>$pp \to \gamma\gamma$</td>
<td>$14.0$</td>
<td>$7.1$</td>
</tr>
</tbody>
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3 tables from Hewett & Spiropulu, hep-ph/0205106

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Black holes at colliders (!)

If the fundamental scale $M_*$ is as low as a TeV, then LHC collisions will have “trans-Planckian” energies! Offers the possibility to probe quantum gravity directly.

Two colliding partons will come within their mutual higher-dimensional Schwarzschild radius – can form a mini black hole!

Spectrum of black holes produced at LHC with decay to final states tagged with $e$ or $\gamma$:

from Dimopoulos & Landsberg, hep-ph/0106295, via a talk by Landsberg
Is ADD really a solution of the hierarchy problem?
Only “sort of”.

ADD solves the Higgs mass divergence problem.
\[ \Delta m^2 \sim \left( \frac{g^2}{16\pi^2} \right) \Lambda^2 \] is not fine-tuned if \( \Lambda \sim \text{TeV} \).

But, ADD trades a large energy hierarchy for a large size hierarchy: the ratio of \( 1/R \) to the fundamental scale \( M_* \).

\[ \delta = 2: \; R \sim 0.1 \text{ mm} \rightarrow 1/R \sim \text{meV} = 10^{-15} \text{ TeV} \] awful!

\[ \ldots \]

\[ \delta = 6: \; R \sim \text{fm} \rightarrow 1/R \sim 100 \text{ MeV} = 10^{-4} \text{ TeV} \] better; still looks tuned.

ADD is not a terrific solution to the hierarchy problem, but experiment will decide.
ADD puts only gravity in the extra dimensions.

What if the SM particles can also propagate in extra dimensions? This is called **Universal Extra Dimensions (UED)**

We’ve probed SM particle interactions up to the 100’s of GeV scale: extra dimension must be roughly $1/(100's \text{ of GeV})$ size or smaller.

Particle-in-a-box for all SM fields:
Get KK towers of SM particles!

UED is not an attempt to solve the hierarchy problem. However, it gives some very interesting phenomenology and leads the way to other approaches to the hierarchy. It also provides an interesting “straw-man” model to compare to SUSY.
Universal extra dimensions: 5th dimension is a line segment. Again get particle-in-a-box KK excitations: $M^{(n)} = n/R$ (5-dim)

Gauge boson KK modes: search for resonances, same as $Z'$ search: e.g., look in dileptons. $Z_{KK}, W_{KK}, g_{KK}$ resonances at LHC energies if $R \sim 1/\text{TeV}$. Tevatron limits in the several-hundred-GeV range from $ee, \mu\mu$ resonance search.

Indirect constraints from precision EW meas: $1/R \gtrsim 6$ TeV
Strong limits due to tree-level exchange of KK gauge bosons. E.g., new contribution to muon decay with a $W^{(1)}$ instead of a $W$ exchange.

Fermion KK modes: letting fermions into the 5th dimension complicates things. A chiral 5-dim fermion corresponds to a left- and right-handed 4-dim fermion! Need to get rid of the extra components of the zero-modes, so SM fermions stay chiral.
Fermions are dealt with by “orbifolding”:
Compactify the 5th dimension on a circle (of radius $R$): $S_1$
Identify the top half of the circle with the bottom half: $S_1/Z_2$
[this is the orbifolding]
Fields odd under the $Z_2$ orbifold do not have zero modes:
Set up the boundary conditions so that only desired chiral fermions
(and gauge/Higgs bosons) have zero modes.
Fermion KK modes are vectorlike (Dirac): $e_L^{(1)}$, $e_R^{(1)}$ separate

5th dimension compactified on $S_1$: 5th-dim momentum is conserved! $\sum$ KK number must stay the same in any reaction.

After orbifolding, the 5th-dim momentum is no longer conserved.
There are two special points, the “boundaries” or fixed points of the orbifold.
Lagrangian can include interactions localized at the boundaries: lead to mass splittings between KK modes.
Left over is a “KK parity” $= (-1)^n$ ($n$ is KK number).

UED with KK parity: Electroweak precision constraints much weaker. No tree-level exchange of level-1 KK excitations.
Limits on KK quark masses $\sim$ few hundred GeV from direct searches.
[It’s beginning to look a lot like SUSY....]
UED phenomenology

KK mode masses get radiative corrections from loops of SM particles. Get splitting in spectrum:

This spectrum is for a common boundary mass [like $m_0$ in CMSSM] Coloured particles get largest radiative corrections: get shifted upwards.
Lightest odd-parity particle (LKP) is stable: dark matter candidate; missing energy in decay chains.
LKP is naturally $\gamma^{(1)}$ for common boundary terms.

This is looking a lot like SUSY!
Discovery reach at LHC is comparable to SUSY from Cheng, Matchev, & Schmaltz, hep-ph/0205314.

LHC sensitive up to $1/R \approx 1.5$ TeV with 300 fb$^{-1}$.

from Cheng, Matchev, & Schmaltz, hep-ph/0205314

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Because of KK parity, get cascade decay chains:

from Cheng, Matchev, & Schmaltz, hep-ph/0205314

Spectrum tends to be more degenerate than SUSY, but collider signals are similar. Jets, leptons, missing $p_T$
Couplings related to corresponding SM couplings, just like SUSY. KK-odd particles must be pair-produced.

Major difference is particle spins!
SUSY: partners have opposite spin.
UED: partners have same spin.
Sure-fire way to tell you’ve got UED and not SUSY: detect the 2nd KK level!

Nevertheless, UED gives us a wonderful “straw-man” model to compare to SUSY:
Want to measure spins of SUSY particles to tell they are not KK excitations with some weird spectrum where we’ve missed the 2nd level!
Measuring spin at an $e^+e^-$ collider:
Use threshold dependence of cross section.
Consider pair production of slepton pairs; compare to pair production of KK lepton pairs.

Scalar pairs: $e^+e^- \rightarrow \tilde{\ell}^+\tilde{\ell}^-$ (sleptons): $\sigma \propto \beta^3$
Fermion pairs: $e^+e^- \rightarrow \ell_1^+\ell_1^-$ (KK leptons): $\sigma \propto \beta(3 - \beta^2)$

where $\beta = p/E = \sqrt{1 - 4m^2/s}$ is the velocity of the produced particle.

Can do a threshold scan:
Get the spin from $\beta$ dependence of threshold.
Also get very precise mass measurement.

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Fig. 4.7. Comparison of spin-0 and spin-$\frac{1}{2}$ particle pair production in $e^+e^-$ collisions, for particles of mass $m = 15$ GeV.
Another way to measure spin:
Look at angular distribution of pair production.
Consider pair production of slepton pairs; compare to pair production of KK lepton pairs.

Scalar pairs: $e^+e^- \rightarrow \tilde{\ell}^+\tilde{\ell}^-$ (sleptons): $\frac{d\sigma}{d\cos\theta} \propto 1 - \cos^2\theta$

Fermion pairs: $e^+e^- \rightarrow \ell_1^+\ell_1^-$ (KK leptons): $\frac{d\sigma}{d\cos\theta} \propto 1 + \left(\frac{E^2 - M^2}{E^2 + M^2}\right)\cos^2\theta$

Plot shows SUSY, UED, and pure “phase space”, $d\sigma/d\cos\theta = \text{constant}$ for comparison.

from Barr, hep-ph/0511115
It gets a little more tricky because SUSY/KK particles decay:
\[ \tilde{\ell}^+ \tilde{\ell}^- \rightarrow \ell^+ \tilde{N}_1 \ell^- \tilde{N}_1 \text{ or } \ell^+_1 \ell^-_1 \rightarrow \ell^+ \gamma_1 \ell^- \gamma_1 \]

At ILC this is not a problem: can still reconstruct $\tilde{\ell}^+ \tilde{\ell}^-$ directions.

At LHC it's more difficult: CM frame is boosted longitudinally.

Measure instead the lepton polar angle in the $\ell^+ \ell^-$ CM frame.

Still has some sensitivity.

Figure 3.2.3: Angular distribution of smuons (two entries per event) in the reaction $e^- R e^+_L \rightarrow \tilde{\mu}_R \tilde{\mu}_R \rightarrow \mu^- \tilde{\chi}_1^0 \mu^+ \tilde{\chi}_1^0$. The hatched histogram represents the false solution.

From Tesla TDR, hep-ph/0106315

From Barr, hep-ph/0511115
Another method for LHC: look at a decay chain.

Example:
\[
\tilde{q} \rightarrow q\tilde{N}_2 \rightarrow q\ell^\pm \tilde{\ell}^\mp \rightarrow q\ell^+\ell^-\tilde{N}_1 \text{ in SUSY}
\]
\[
q_1 \rightarrow qZ_1 \rightarrow q\ell^\pm \ell_1^\mp \rightarrow q\ell^+\ell^-\gamma_1 \text{ in UED}
\]

Diagram from Battaglia, Datta, De Roeck, Kong, & Matchev, hep-ph/0507284

Form $M_{q\ell}$ invariant mass dist’n with first (near) lepton
Shape depends on spin of intermediate particle:
$\tilde{N}_2$ in SUSY – spin 1/2; $Z_1$ in UED – spin 1

Problem: hard to distinguish the first (near) lepton from the second (far) lepton. Tends to wash out spin correlations.
Solution: use a charge asymmetry between $q\ell^+$ and $q\ell^-$
$
\tilde{N}_2$ typically mostly $\tilde{W}^0$: couples to LH fermions / RH anti-fermions.

Helicity conservation leads to different $M_{\ell q}$ shape for $\ell^+$ vs. $\ell^-$:

Summing over $\bar{q} + \bar{q}^*$ would wash this out again EXCEPT:

LHC is a $pp$ collider: more $q$ than $\bar{q}$ in PDFs.
Make a lepton charge asymmetry:

\[ A^{+-} = \frac{s^+ - s^-}{s^+ + s^-}, \quad s^\pm = \frac{d\sigma}{d(M_{\ell\pm q})} \]

Charge asymmetry depends on \( M_{\ell q} \) differently for SUSY, for UED, and for pure “phase space” (flat distribution):

![Graph showing the comparison of charge asymmetry \( A^{+-} \) for SUSY and UED, with a phase space comparison.](image)

from Battaglia, Datta, De Roeck, Kong, & Matchev, hep-ph/0507284

This tests the spin of \( \tilde{N}_2 \) or \( Z^{(1)} \).
There are many other spin observables in other decay chains.

**SUSY:** $\tilde{g} \rightarrow \tilde{b}_1 \rightarrow \tilde{N}_2 \rightarrow \ell_1 \rightarrow \tilde{N}_1$. Final state is $b\bar{b}\ell^+\ell^- p_T^{\text{miss}}$.

**UED:** $g_1 \rightarrow b_{L1} \rightarrow Z_1 \rightarrow \ell_{R1} \rightarrow \gamma_1$. Final state is $b\bar{b}\ell^+\ell^- p_T^{\text{miss}}$.

Choose UED spectrum to match SUSY spectrum!

Have only the spins to distinguish them!

Lepton charge asym. vs. $M_{b\ell}$ (softer $b$).

Azimuthal angle dist’n between the two $b$ jets.

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from Alves, Éboli, & Plehn, hep-ph/0605067
Summary:

We talked about:

**ADD**
Large extra dimensions, only gravity in the bulk.
Quantum gravity at the TeV scale: mini black holes at colliders!?

Universal extra dimensions (UED)
1/TeV size extra dimension.
All the SM particles in the bulk ("universal") – signatures similar to SUSY! [Because of KK parity.]
Challenge: determine spins of new particles to distinguish UED from SUSY.