Theory uncertainties in ILC Higgs measurements

Heather Logan
(Carleton University)

Based on A. Droll & HEL, hep-ph/0612317
Higgs coupling measurements are a big selling point for the ILC.

How do theory uncertainties affect this picture?

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Conclusions

Theory uncertainties in Higgs couplings are around the percent-ish level.

Start to have a significant impact when experimental uncertainties get below the percent level. This happens at high-energy / high-luminosity running (e.g., 1000 fb$^{-1}$ at 1000 GeV).

Most important theory/parametric uncertainties are:
- $m_b$ (current uncertainty 0.95%) – feeds into $\Gamma_b$ calculation
- $\alpha_s$ (current uncertainty 1.7%) – feeds into $\Gamma_b$, $\Gamma_c$, $\Gamma_g$ calculation
### Expected experimental uncertainties

**“Phase 1”:** 500 fb$^{-1}$ at 350 GeV, no beam polarization

<table>
<thead>
<tr>
<th>SM Higgs branching ratio uncertainties</th>
<th>$m_H = 120$ GeV</th>
<th>140 GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>BR($bb$)</td>
<td>2.4%</td>
<td>2.6%</td>
</tr>
<tr>
<td>BR($cc$)</td>
<td>8.3%</td>
<td>19.0%</td>
</tr>
<tr>
<td>BR($\tau\tau$)</td>
<td>5.0%</td>
<td>8.0%</td>
</tr>
<tr>
<td>BR($WW$)</td>
<td>5.1%</td>
<td>2.5%</td>
</tr>
<tr>
<td>BR($gg$)</td>
<td>5.5%</td>
<td>14.0%</td>
</tr>
</tbody>
</table>

from K. Desch, hep-ph/0311092

**“Phase 2”:** 1000 fb$^{-1}$ at 1000 GeV, −80% $e^-$ / +60% $e^+$ pol’n

<table>
<thead>
<tr>
<th>SM Higgs cross section times BR statistical uncertainties</th>
<th>$m_H = 115$ GeV</th>
<th>120 GeV</th>
<th>140 GeV</th>
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</thead>
<tbody>
<tr>
<td>$\sigma \times$ BR($bb$)</td>
<td>0.3%</td>
<td>0.4%</td>
<td>0.5%</td>
</tr>
<tr>
<td>$\sigma \times$ BR($WW$)</td>
<td>2.1%</td>
<td>1.3%</td>
<td>0.5%</td>
</tr>
<tr>
<td>$\sigma \times$ BR($gg$)</td>
<td>1.4%</td>
<td>1.5%</td>
<td>2.5%</td>
</tr>
<tr>
<td>$\sigma \times$ BR($\gamma\gamma$)</td>
<td>5.3%</td>
<td>5.1%</td>
<td>5.9%</td>
</tr>
</tbody>
</table>

from T. Barklow, hep-ph/0312268

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Theoretical & parametric uncertainties

<table>
<thead>
<tr>
<th>Higgs observable</th>
<th>Theory uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma_{\bar{b}b}, \Gamma_{c\bar{c}}$</td>
<td>1%</td>
</tr>
<tr>
<td>$\Gamma_{\tau\tau}, \Gamma_{\mu\mu}$</td>
<td>0.01%</td>
</tr>
<tr>
<td>$\Gamma_{WW}, \Gamma_{ZZ}$</td>
<td>0.5%</td>
</tr>
<tr>
<td>$\Gamma_{gg}$</td>
<td>3%</td>
</tr>
<tr>
<td>$\Gamma_{\gamma\gamma}$</td>
<td>0.1%</td>
</tr>
<tr>
<td>$\sigma_{e^+e^-\rightarrow \nu\bar{\nu}H}$</td>
<td>0.5%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Percent uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_s(m_Z)$</td>
<td>$0.1185 \pm 0.0020$</td>
<td>1.7%</td>
</tr>
<tr>
<td>$m_b(M_b)$</td>
<td>$4.20 \pm 0.04$ GeV</td>
<td>0.95%</td>
</tr>
<tr>
<td>$m_c(M_c)$</td>
<td>$1.224 \pm 0.057$ GeV</td>
<td>4.7%</td>
</tr>
</tbody>
</table>

$\alpha_s$: world average from PDG

$m_b$ and $m_c$: from fits to kinematic moments in inclusive semileptonic $B$ meson decays. Uncertainties dominated by theory uncertainty in QCD corrections to HQET expansions.

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Quantifying the impact of theory/param uncerts:
- “How well can you distinguish SM from BSM?”
- Construct a $\Delta \chi^2$ between the observables in the SM and the MSSM $m_{h_{\text{max}}}$ scenario.
- Look at “reach” in $M_A$ for a $5\sigma$ ($\Delta \chi^2 = 25$) discrepancy.

$\chi^2$ observable

$$\chi^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} (Q_{i}^{M_1} - Q_{i}^{M_2})[\sigma^2]_{ij}^{-1} (Q_{j}^{M_1} - Q_{j}^{M_2})$$

$Q_i$: the observables.
$[\sigma^2]_{ij}^{-1}$: inverse of the covariance matrix,

$$\sigma^2_{ij} = \delta_{ij} u_i u_j + \sum_{k=1}^{m} c_i^k c_j^k$$

Straightforward to take into account both uncorrelated uncerts $u_i$ and correlated uncerts $c_i^k$.

Have to propagate the theoretical and parametric uncertainties to the observables $Q_i$. 

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Propagation of theory/param uncertainties
Convenient to work entirely with fractional uncertainties.

Uncertainty in $\text{BR}_i$ due to theoretical uncertainty in $\Gamma_k$:

$$c^k_i = \frac{\Gamma_k \partial\text{BR}_i}{\text{BR}_i \partial \Gamma_k} \sigma_{\Gamma_k}$$

where

$$\frac{\Gamma_k \partial\text{BR}_i}{\text{BR}_i \partial \Gamma_k} = \begin{cases} -\text{BR}_k & \text{for } i \neq k \\ (1 - \text{BR}_k) & \text{for } i = k. \end{cases}$$

Uncertainty in $\text{BR}_i$ due to parametric uncertainty in input $x_j$:

$$c^x_j = \frac{x_j \partial\text{BR}_i}{\text{BR}_i \partial x_j} \sigma_{x_j} = \sum_{k=1}^{n} \left[ \frac{\Gamma_k \partial\text{BR}_i}{\text{BR}_i \partial \Gamma_k} \right] \left[ \frac{x_j \partial \Gamma_k}{\Gamma_k \partial x_j} \right] \sigma_{x_j}$$

Normalized derivatives ($x/\Gamma)(\partial \Gamma/\partial x$):

<table>
<thead>
<tr>
<th>$m_H$</th>
<th>$\alpha_s(m_Z)$</th>
<th>$\bar{m}_b(M_b)$</th>
<th>$\bar{m}_c(M_c)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>120 GeV</td>
<td>140 GeV</td>
<td>120 GeV</td>
<td>140 GeV</td>
</tr>
<tr>
<td>$\Gamma_{bb}$</td>
<td>$-1.177$</td>
<td>$-1.217$</td>
<td>$2.565$</td>
</tr>
<tr>
<td>$\Gamma_{cc}$</td>
<td>$-4.361$</td>
<td>$-4.400$</td>
<td>$-0.083$</td>
</tr>
<tr>
<td>$\Gamma_{gg}$</td>
<td>$2.277$</td>
<td>$2.221$</td>
<td>$-0.114$</td>
</tr>
</tbody>
</table>

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Results

Phase 1: Reach $\sim 500$ GeV without thy/param uncerts. Reduced by about 10% by including thy/param uncerts.

Phase 2: Reach $\sim 1200$ GeV without thy/param uncerts. Reduced by about $2 \times$ to $\sim 600$ GeV including thy/param uncerts.
Phase 1:

Effect is mostly due to $m_b$ and $\alpha_s$ input uncertainties.

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Parametric and theoretical uncertainties make all the measurements a little worse.

Sample point on experimental uncert only $\Delta \chi^2 = 25$ contour:

<table>
<thead>
<tr>
<th>Observable</th>
<th>Shift</th>
<th>Expt uncert</th>
<th>Pull</th>
<th>Thy+par uncert</th>
<th>Total uncert</th>
<th>Pull</th>
</tr>
</thead>
<tbody>
<tr>
<td>BR($bb$)</td>
<td>8.1%</td>
<td>2.5%</td>
<td>3.25</td>
<td>1.6%</td>
<td>3.0%</td>
<td>2.71</td>
</tr>
<tr>
<td>BR($cc$)</td>
<td>−12.0%</td>
<td>1.2%</td>
<td>−0.90</td>
<td>16.1%</td>
<td>20.8%</td>
<td>−0.57</td>
</tr>
<tr>
<td>BR($\tau\tau$)</td>
<td>10.0%</td>
<td>6.4%</td>
<td>1.56</td>
<td>1.8%</td>
<td>6.6%</td>
<td>1.51</td>
</tr>
<tr>
<td>BR($WW$)</td>
<td>−11.6%</td>
<td>3.9%</td>
<td>−2.96</td>
<td>1.8%</td>
<td>4.3%</td>
<td>−2.68</td>
</tr>
<tr>
<td>BR($gg$)</td>
<td>−14.7%</td>
<td>9.4%</td>
<td>−1.56</td>
<td>5.8%</td>
<td>11.1%</td>
<td>−1.33</td>
</tr>
</tbody>
</table>

$\sum$(Pull)$^2$:
- 25 with experimental uncertainties only
- 18.9 summing “Total uncert” pulls above
- 17.4 including correlations
Phase 2:

Effect is again mostly due to $m_b$ and $\alpha_s$ uncertainties.

Theory uncertainties in $\Gamma_b$ (and $\Gamma_g$ at low $\tan\beta$) also moderately important.

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Parametric and theoretical uncertainties have a huge impact on the measurements, especially the most precise Phase 2 rates.

Sample point on experimental uncert only $\Delta \chi^2 = 25$ contour:

<table>
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<tr>
<th>Observable</th>
<th>Shift</th>
<th>Expt uncert</th>
<th>Pull</th>
<th>Thy+par uncert</th>
<th>Total uncert</th>
<th>Pull</th>
</tr>
</thead>
<tbody>
<tr>
<td>BR($bb$)</td>
<td>1.7%</td>
<td>2.5%</td>
<td>0.67</td>
<td>1.7%</td>
<td>3.0%</td>
<td>0.55</td>
</tr>
<tr>
<td>BR($cc$)</td>
<td>−2.5%</td>
<td>13.3%</td>
<td>−0.19</td>
<td>16.1%</td>
<td>20.8%</td>
<td>−0.12</td>
</tr>
<tr>
<td>BR($\tau\tau$)</td>
<td>2.1%</td>
<td>6.4%</td>
<td>0.34</td>
<td>1.8%</td>
<td>6.6%</td>
<td>0.32</td>
</tr>
<tr>
<td>BR($WW$)</td>
<td>−2.1%</td>
<td>3.9%</td>
<td>−0.53</td>
<td>1.8%</td>
<td>4.3%</td>
<td>−0.48</td>
</tr>
<tr>
<td>BR($gg$)</td>
<td>−4.6%</td>
<td>9.4%</td>
<td>−0.48</td>
<td>5.8%</td>
<td>11.1%</td>
<td>−0.41</td>
</tr>
<tr>
<td>$\sigma \times$ BR($bb$)</td>
<td>1.7%</td>
<td>0.45%</td>
<td>3.72</td>
<td>1.7%</td>
<td>1.8%</td>
<td>0.93</td>
</tr>
<tr>
<td>$\sigma \times$ BR($WW$)</td>
<td>−2.1%</td>
<td>0.93%</td>
<td>−2.22</td>
<td>1.9%</td>
<td>2.1%</td>
<td>−0.98</td>
</tr>
<tr>
<td>$\sigma \times$ BR($gg$)</td>
<td>−4.6%</td>
<td>2.0%</td>
<td>−2.32</td>
<td>5.8%</td>
<td>6.2%</td>
<td>−0.74</td>
</tr>
<tr>
<td>$\sigma \times$ BR($\gamma\gamma$)</td>
<td>0.27%</td>
<td>5.5%</td>
<td>0.05</td>
<td>1.9%</td>
<td>5.8%</td>
<td>0.05</td>
</tr>
</tbody>
</table>

$\sum$(Pull)$^2$:
- 25 with experimental uncertainties only
- 3.2 summing “Total uncert” pulls above
- 1.7 including correlations

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Outlook: $\alpha_s$

ILC measurements will improve the precision on $\alpha_s(m_Z)$ by $\gtrsim 2\times$:
- Event shape observables
- $\sigma_{\bar{t}t}/\sigma_{\mu^+\mu^-}$ above $2m_t$
- $\Gamma^\text{had}_Z/\Gamma^\text{lept}_Z$ at $Z$ pole (GigaZ option)

Effect of improving $\Delta \alpha_s(m_Z)$
from 0.0020 (1.7%) [current PDG]
to 0.0009 (0.76%) [Tesla TDR]
(includes GigaZ).

Not much impact unless
$\Delta \bar{m}_b(M_b)$ is also improved.
Outlook: other observables

Phase 2 experimental precision dominated by three channels:
\( \sigma \times \text{BR}(b\bar{b}), \sigma \times \text{BR}(gg) \): suffer directly from large par/thy uncerts.
\( \sigma \times \text{BR}(WW) \): affected indirectly through Higgs total width.

A brief foray into the MSSM:
Study characteristic features of MSSM Higgs couplings:
\[
\begin{align*}
\frac{g_{h^0\bar{t}t}}{g_{H_{SM}\bar{t}t}} &= \frac{g_{h^0\bar{c}c}}{g_{H_{SM}\bar{c}c}} = \sin(\beta - \alpha) + \cot \beta \cos(\beta - \alpha) \\
\frac{g_{H_{SM}\bar{t}t}}{g_{H_{SM}\bar{c}c}} &= \sin(\beta - \alpha) - \tan \beta \cos(\beta - \alpha) \\
\frac{g_{h^0\tau\bar{t}}}{g_{H_{SM}\tau\tau}} &= \frac{g_{h^0\tau\tau}}{g_{H_{SM}\tau\tau}} = \sin(\beta - \alpha) \\
\end{align*}
\]

Interested in the approach to decoupling:
\[
\cos(\beta - \alpha) \simeq \frac{1}{2} \sin 4\beta \frac{m_Z^2}{M_A^2} \rightarrow 0 \text{ for } M_A \gg m_Z
\]

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Plug in and keep leading term in $m_Z^2/M_A^2$:

$$\frac{\delta \Gamma_W}{\Gamma_W} = \frac{\delta \Gamma_Z}{\Gamma_Z} \simeq -\frac{1}{4} \sin^2 4\beta \frac{m_Z^4}{M_A^4} \simeq -4 \cot^2 \beta \frac{m_Z^4}{M_A^4}$$

$$\frac{\delta \Gamma_b}{\Gamma_b} \simeq \frac{\delta \Gamma_\tau}{\Gamma_\tau} \simeq -\tan \beta \sin 4\beta \frac{m_Z^2}{M_A^2} \simeq +4 \frac{m_Z^2}{M_A^2}$$

$$\frac{\delta \Gamma_c}{\Gamma_c} \simeq \cot \beta \sin 4\beta \frac{m_Z^2}{M_A^2} \simeq -4 \cot^2 \beta \frac{m_Z^2}{M_A^2}$$

(Last equality: used large $\tan \beta$ approximation $\sin 4\beta \simeq -4 \cot \beta$.)

Biggest deviations from SM are in $\Gamma_b$ and $\Gamma_\tau$.

Picture not dramatically altered by radiative corrections.
Phase 2 experimental precision dominated by three channels: 
\(\sigma \times \text{BR}(b\bar{b}), \sigma \times \text{BR}(gg)\): suffer directly from large par/thy uncerts.
\(\sigma \times \text{BR}(WW)\): affected indirectly through Higgs total width.

Parametric & theoretical uncertainties are washing out sensitivity to shift in \(\Gamma_b\) relative to \(\Gamma_W\)!

Want another non-hadronic final state to restore sensitivity. 
\(\sigma \times \text{BR}(\tau\tau)\) would be perfect.

Sensitivity would come from the ratio:
\[
\frac{\sigma \times \text{BR}(\tau\tau)}{\sigma \times \text{BR}(WW)} = \frac{\Gamma_\tau}{\Gamma_W}
\]

- \(m_b, \alpha_s\), QCD uncertainties in total width cancel
- Ratio \(\Gamma_\tau/\Gamma_W\) exhibits large deviation from SM

Using correlation matrix in the \(\chi^2\) means we don’t need to play with ratios: everything is automatic.
Going from Phase 1 to Phase 2, precision on key final states improves:

- $b\bar{b}$: 5–6×
- $WW$: 4–5×
- $gg$: 3.5–5.5×

“Reasonable” to expect similar improvement in $\tau\tau$:
assume 4× and see what happens.

“Phase 2”: 1000 fb$^{-1}$ at 1000 GeV, $-80\% e^- / +60\% e^+$ pol’n

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<td>0.3%</td>
<td>0.4%</td>
<td>0.5%</td>
</tr>
<tr>
<td>$\sigma \times BR(WW)$</td>
<td>2.1%</td>
<td>1.3%</td>
<td>0.5%</td>
</tr>
<tr>
<td>$\sigma \times BR(gg)$</td>
<td>1.4%</td>
<td>1.5%</td>
<td>2.5%</td>
</tr>
<tr>
<td>$\sigma \times BR(\gamma\gamma)$</td>
<td>5.3%</td>
<td>5.1%</td>
<td>5.9%</td>
</tr>
<tr>
<td>$\sigma \times BR(\tau\tau)$</td>
<td>–</td>
<td>1.3%</td>
<td>2.0%</td>
</tr>
</tbody>
</table>

Original selection required $\sum \text{vis} = m_H$; have to change this for $\tau\tau$. 
Effect of adding a measurement of $\sigma \times \text{BR}(\tau\tau)$ in Phase 2:

Not a big effect on expt-only reach.
Much bigger effect once param/theory uncertainties are included.

$\tan \beta$ vs. $M_A$ [GeV]

-$\Delta \chi^2 = 25$
-$m_h^\text{max}$ scenario
-$m_H = 120 (140)$ GeV
-$m_h = 125$ GeV
-$129$ GeV
-$120$ GeV

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Conclusions

Theory uncertainties are at the level of a couple of percent.

Start to have a significant impact when experimental uncertainties get below the percent level – **big impact on Phase 2.**

Most important theory/parametric uncertainties are:
- $m_b$ (current uncertainty 0.95%) – feeds into $\Gamma_b$ calculation
  *Improving this is important!*
- $\alpha_s$ (current uncertainty 1.7%) – feeds into $\Gamma_b$, $\Gamma_c$, $\Gamma_g$ calculation
  *Will improve by $\gtrsim 2\times$ at ILC. GigaZ valuable here.*

Understanding the pattern of theory/parametric uncertainties points out the most valuable new experimental channels.
*Adding $\sigma \times BR(\tau\tau)$: small impact with only expt uncerts; huge impact after thy+param uncerts included.*