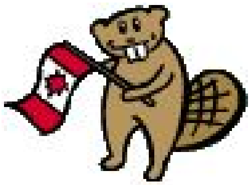
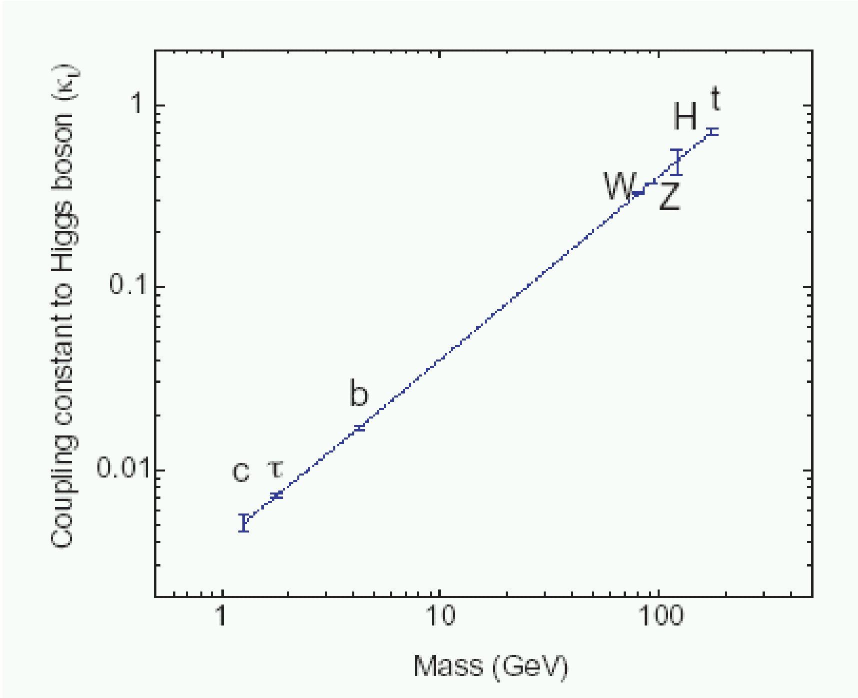
Theory uncertainties in ILC Higgs measurements

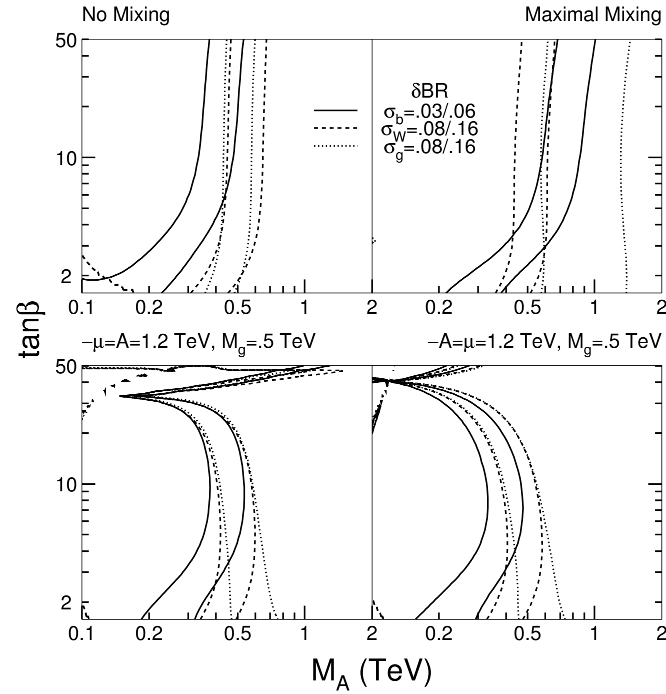
Heather Logan
(Carleton University)

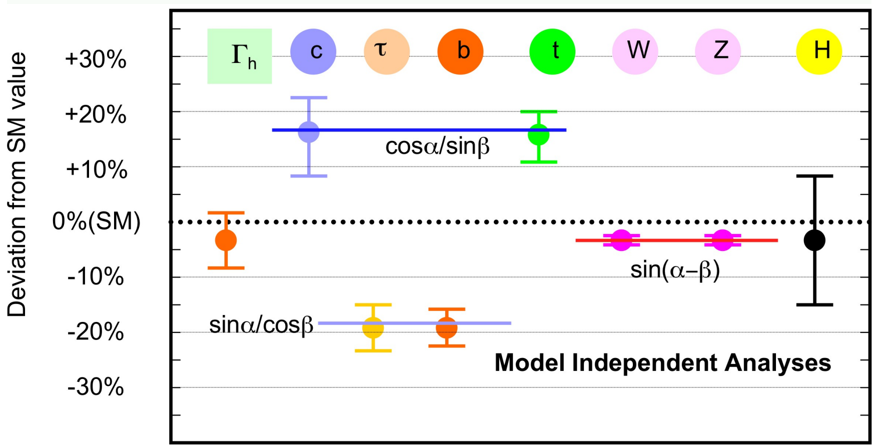


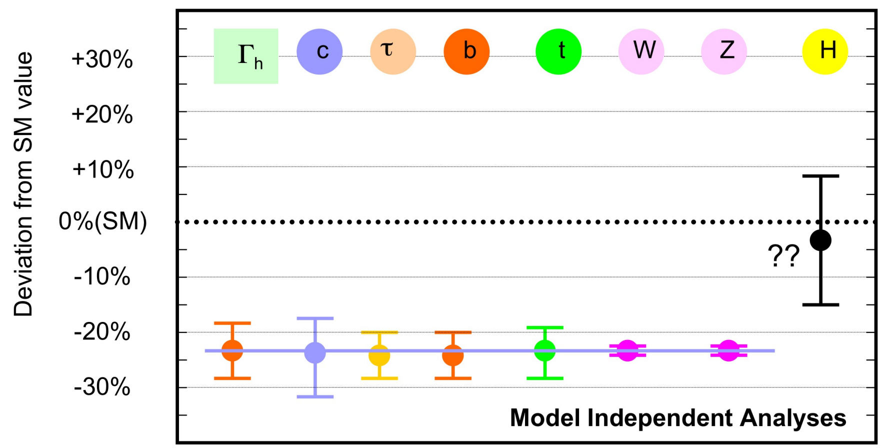
Based on A. Droll & HEL, hep-ph/0612317

Higgs coupling measurements are a big selling point for the ILC.









How do theory uncertainties affect this picture?

Conclusions

Theory uncertainties in Higgs couplings are around the percent-ish level.

Start to have a significant impact when experimental uncertainties get below the percent level.
This happens at high-energy / high-luminosity running (e.g., 1000 fb^{-1} at 1000 GeV).

Most important theory/parametric uncertainties are:
- m_b (current uncertainty 0.95%) – feeds into Γ_b calculation
- α_s (current uncertainty 1.7%) – feeds into Γ_b, Γ_c, Γ_g calculation

Expected experimental uncertainties

"Phase 1": 500 fb^{-1} at 350 GeV, no beam polarization

SM Higgs branching ratio uncertainties		
	$m_H = 120$ GeV	140 GeV
BR($b\bar{b}$)	2.4%	2.6%
BR($c\bar{c}$)	8.3%	19.0%
BR($\tau\tau$)	5.0%	8.0%
BR(WW)	5.1%	2.5%
BR(gg)	5.5%	14.0%

from K. Desch, hep-ph/0311092

"Phase 2": 1000 fb^{-1} at 1000 GeV, -80% e^- / $+60\%$ e^+ pol'n

SM Higgs cross section times BR statistical uncertainties			
	$m_H = 115$ GeV	120 GeV	140 GeV
$\sigma \times$ BR($b\bar{b}$)	0.3%	0.4%	0.5%
$\sigma \times$ BR(WW)	2.1%	1.3%	0.5%
$\sigma \times$ BR(gg)	1.4%	1.5%	2.5%
$\sigma \times$ BR($\gamma\gamma$)	5.3%	5.1%	5.9%

from T. Barklow, hep-ph/0312268

Theoretical & parametric uncertainties

Higgs observable	Theory uncertainty
$\Gamma_{b\bar{b}}$, $\Gamma_{c\bar{c}}$	1%
$\Gamma_{\tau\tau}$, $\Gamma_{\mu\mu}$	0.01%
Γ_{WW}, Γ_{ZZ}	0.5%
Γ_{gg}	3%
$\Gamma_{\gamma\gamma}$	0.1%
$\sigma_{e^+e^- \to \nu\bar{\nu}H}$	0.5%

Parameter	Value	Percent uncertainty
$\alpha_s(m_Z)$	0.1185 ± 0.0020	1.7%
$\overline{m_b}(M_b)$	4.20 ± 0.04 GeV	0.95%
$\overline{m_c}(M_c)$	1.224 ± 0.057 GeV	4.7%

α_s: world average from PDG

m_b and m_c: from fits to kinematic moments in inclusive semileptonic B meson decays. Uncertainties dominated by theory uncertainty in QCD corrections to HQET expansions.

Quantifying the impact of theory/param uncerts:

- "How well can you distinguish SM from BSM?"
- Construct a $\Delta\chi^2$ between the observables in the SM and the MSSM $m_h^{\rm max}$ scenario.
- Look at "reach" in M_A for a 5σ ($\Delta\chi^2 = 25$) discrepancy.

χ^2 observable

$$\chi^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} (Q_i^{M_1} - Q_i^{M_2}) [\sigma^2]_{ij}^{-1} (Q_j^{M_1} - Q_j^{M_2})$$

Q_i: the observables.
$[\sigma^2]_{ij}^{-1}$: inverse of the covariance matrix,

$$\sigma_{ij}^2 = \delta_{ij} u_i u_j + \sum_{k=1}^{m} c_i^k c_j^k$$

Straightforward to take into account both uncorrelated uncerts u_i and correlated uncerts c_i^k.

Have to propagate the theoretical and parametric uncertainties to the observables Q_i.

Propagation of theory/param uncertainties

Convenient to work entirely with fractional uncertainties.

Uncertainty in BR_i due to theoretical uncertainty in Γ_k:

$$c_i^k = \frac{\Gamma_k}{\mathrm{BR}_i}\frac{\partial \mathrm{BR}_i}{\partial \Gamma_k}\sigma_{\Gamma_k} \quad \text{where} \quad \frac{\Gamma_k}{\mathrm{BR}_i}\frac{\partial \mathrm{BR}_i}{\partial \Gamma_k} = \begin{cases} -\mathrm{BR}_k & \text{for } i \neq k \\ (1-\mathrm{BR}_k) & \text{for } i = k. \end{cases}$$

Uncertainty in BR_i due to parametric uncertainty in input x_j:

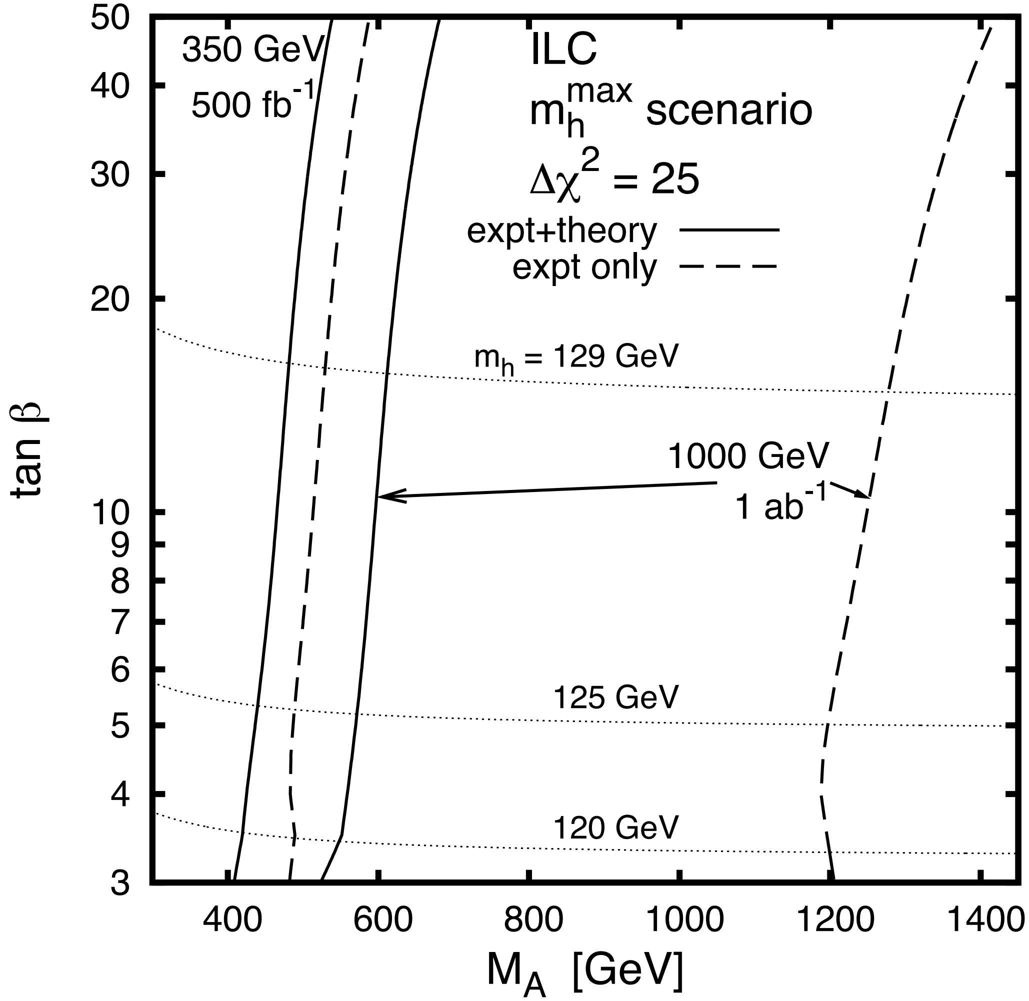
$$c_i^{x_j} = \frac{x_j}{\mathrm{BR}_i}\frac{\partial \mathrm{BR}_i}{\partial x_j}\sigma_{x_j} = \sum_{k=1}^{n}\left[\frac{\Gamma_k}{\mathrm{BR}_i}\frac{\partial \mathrm{BR}_i}{\partial \Gamma_k}\right]\left[\frac{x_j}{\Gamma_k}\frac{\partial \Gamma_k}{\partial x_j}\right]\sigma_{x_j}$$

Normalized derivatives $(x/\Gamma)(\partial\Gamma/\partial x)$:

Normalized derivatives of Higgs partial widths

	$\alpha_s(m_Z)$		$\overline{m_b}(M_b)$		$\overline{m_c}(M_c)$	
m_H	120 GeV	140 GeV	120 GeV	140 GeV	120 GeV	140 GeV
$\Gamma_{b\bar{b}}$	−1.177	−1.217	2.565	2.567	0.000	0.000
$\Gamma_{c\bar{c}}$	−4.361	−4.400	−0.083	−0.084	3.191	3.192
Γ_{gg}	2.277	2.221	−0.114	−0.112	−0.039	−0.032

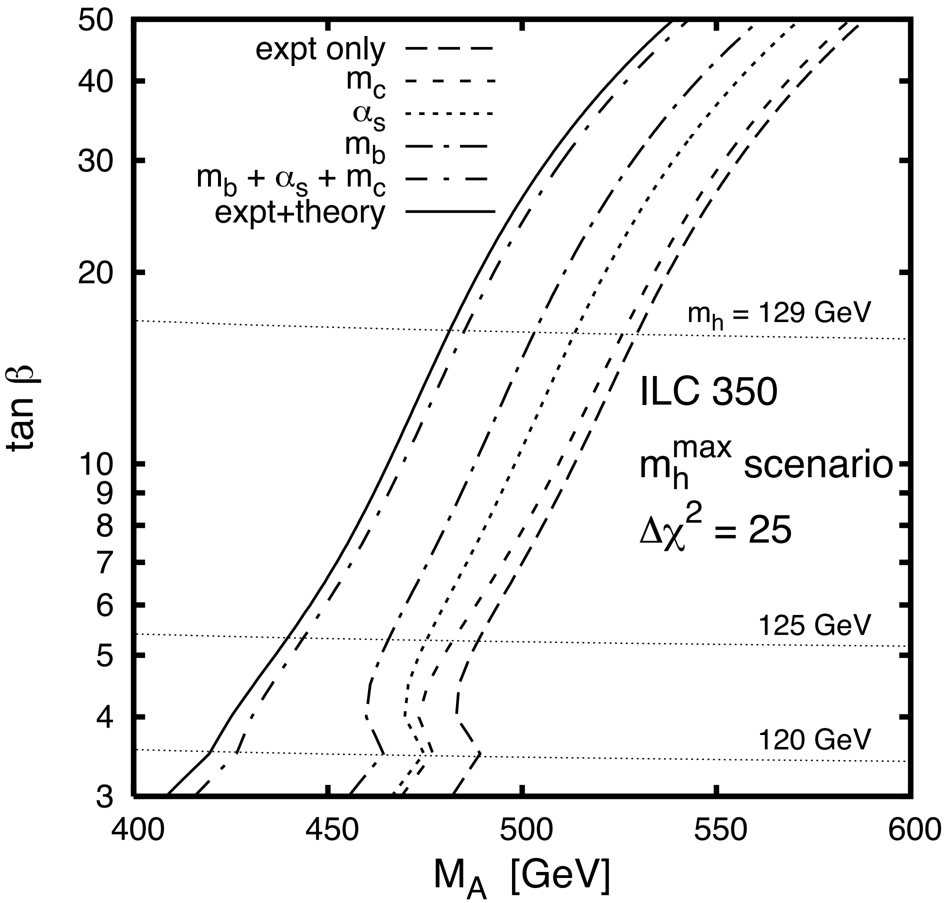
Results

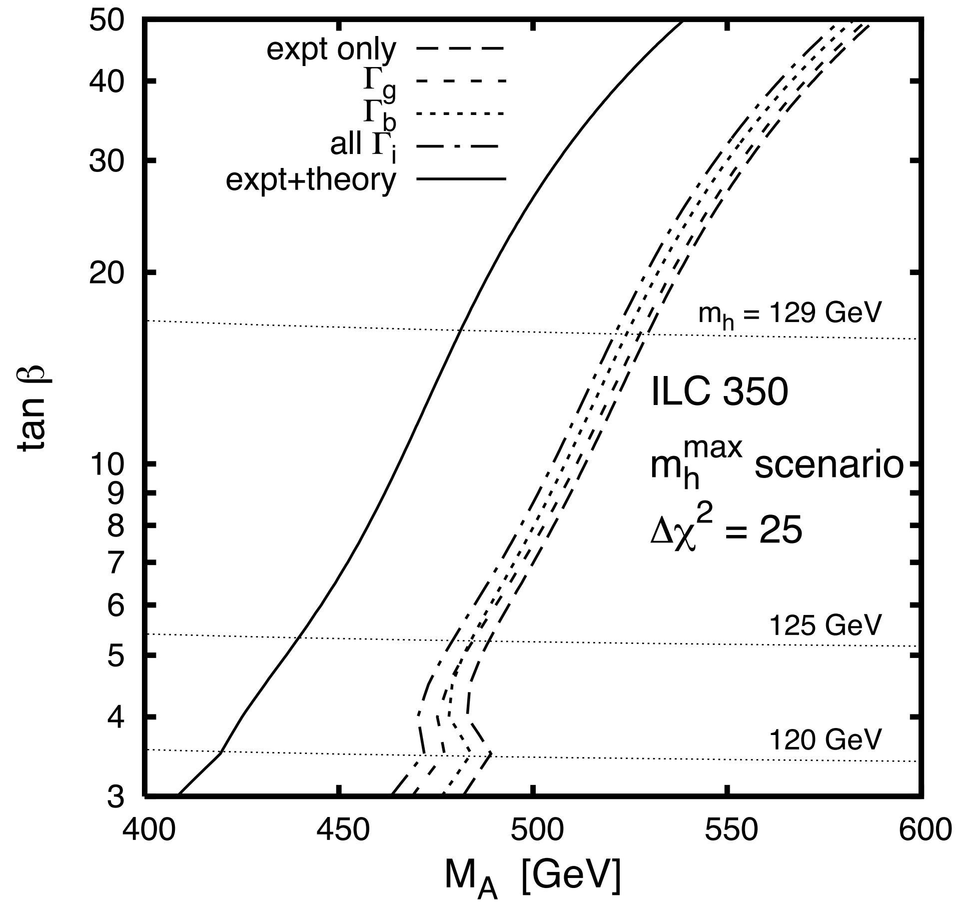


Phase 1: Reach $\sim$ 500 GeV without thy/param uncerts.
Reduced by about 10% by including thy/param uncerts.

Phase 2: Reach $\sim$ 1200 GeV without thy/param uncerts.
Reduced by about 2$\times$ to $\sim$ 600 GeV including thy/param uncerts.

Phase 1:





Effect is mostly due to m_b and α_s input uncertainties.

Parametric and theoretical uncertainties make all the measurements a little worse.

Sample point on experimental uncert only $\Delta\chi^2 = 25$ contour:

Phase 1 sample point: $M_A = 537.6$ GeV, $\tan\beta = 20$						
Observable	Shift	Expt uncert	Pull	Thy+par uncert	Total uncert	Pull
BR($b\bar{b}$)	8.1%	2.5%	3.25	1.6%	3.0%	2.71
BR($c\bar{c}$)	−12.0%	13.2%	−0.90	16.1%	20.8%	−0.57
BR($\tau\tau$)	10.0%	6.4%	1.56	1.8%	6.6%	1.51
BR(WW)	−11.6%	3.9%	−2.96	1.8%	4.3%	−2.68
BR(gg)	−14.7%	9.4%	−1.56	5.8%	11.1%	−1.33

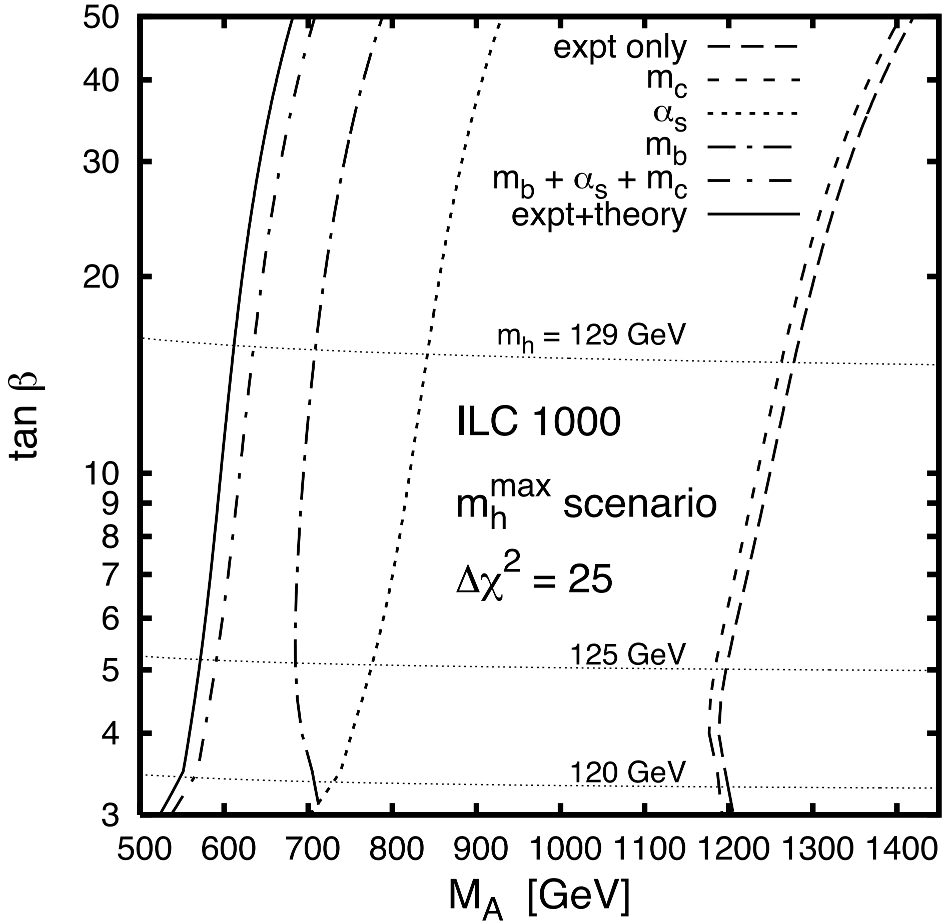
$\sum(\text{Pull})^2$:

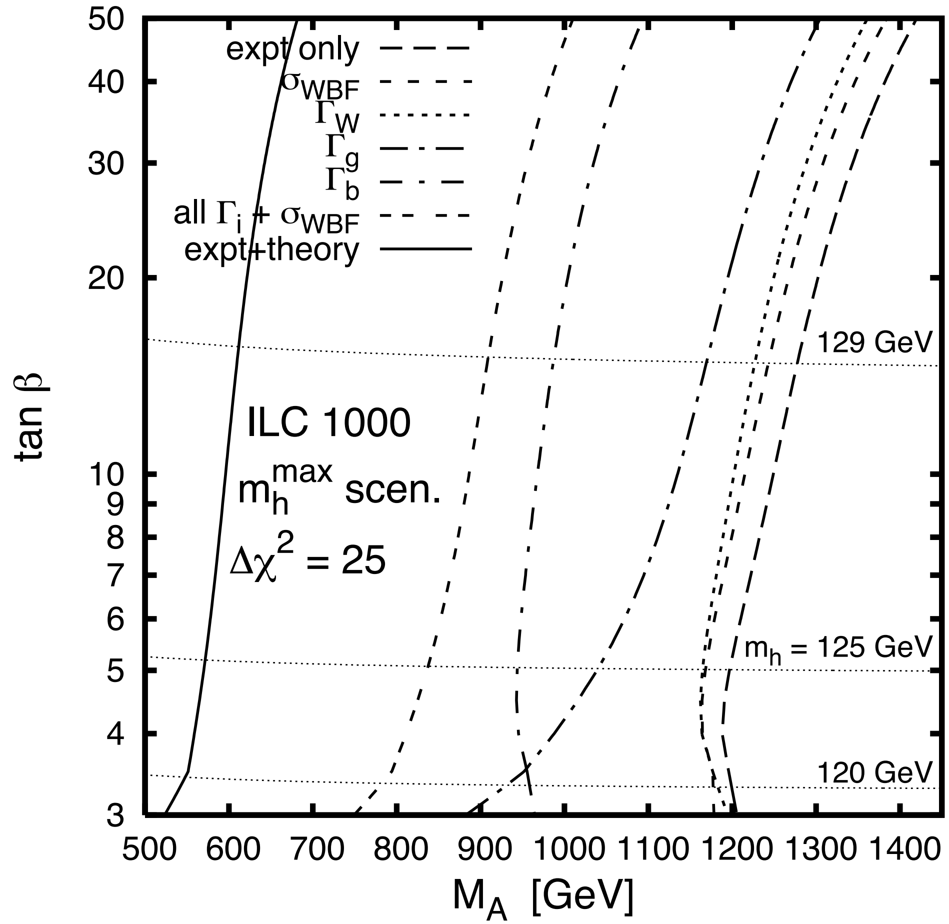
25 with experimental uncertainties only

18.9 summing "Total uncert" pulls above

17.4 including correlations

Phase 2:





Effect is again mostly due to m_b and α_s uncertainties.

Theory uncertainties in Γ_b (and Γ_g at low $\tan\beta$) also moderately important.

Parametric and theoretical uncertainties have a huge impact on the measurements, especially the most precise Phase 2 rates.

Sample point on experimental uncert only $\Delta\chi^2 = 25$ contour:

Phase 2 sample point: $M_A = 1302.4$ GeV, $\tan\beta = 20$						
Observable	Shift	Expt uncert	Pull	Thy+par uncert	Total uncert	Pull
BR($b\bar{b}$)	1.7%	2.5%	0.67	1.7%	3.0%	0.55
BR($c\bar{c}$)	−2.5%	13.3%	−0.19	16.1%	20.8%	−0.12
BR($\tau\tau$)	2.1%	6.4%	0.34	1.8%	6.6%	0.32
BR(WW)	−2.1%	3.9%	−0.53	1.8%	4.3%	−0.48
BR(gg)	−4.6%	9.4%	−0.48	5.8%	11.1%	−0.41
$\sigma \times$ BR($b\bar{b}$)	1.7%	0.45%	3.72	1.7%	1.8%	0.93
$\sigma \times$ BR(WW)	−2.1%	0.93%	−2.22	1.9%	2.1%	−0.98
$\sigma \times$ BR(gg)	−4.6%	2.0%	−2.32	5.8%	6.2%	−0.74
$\sigma \times$ BR($\gamma\gamma$)	0.27%	5.5%	0.05	1.9%	5.8%	0.05

$\sum$(Pull)^2:

25 with experimental uncertainties only

3.2 summing “Total uncert” pulls above

1.7 including correlations

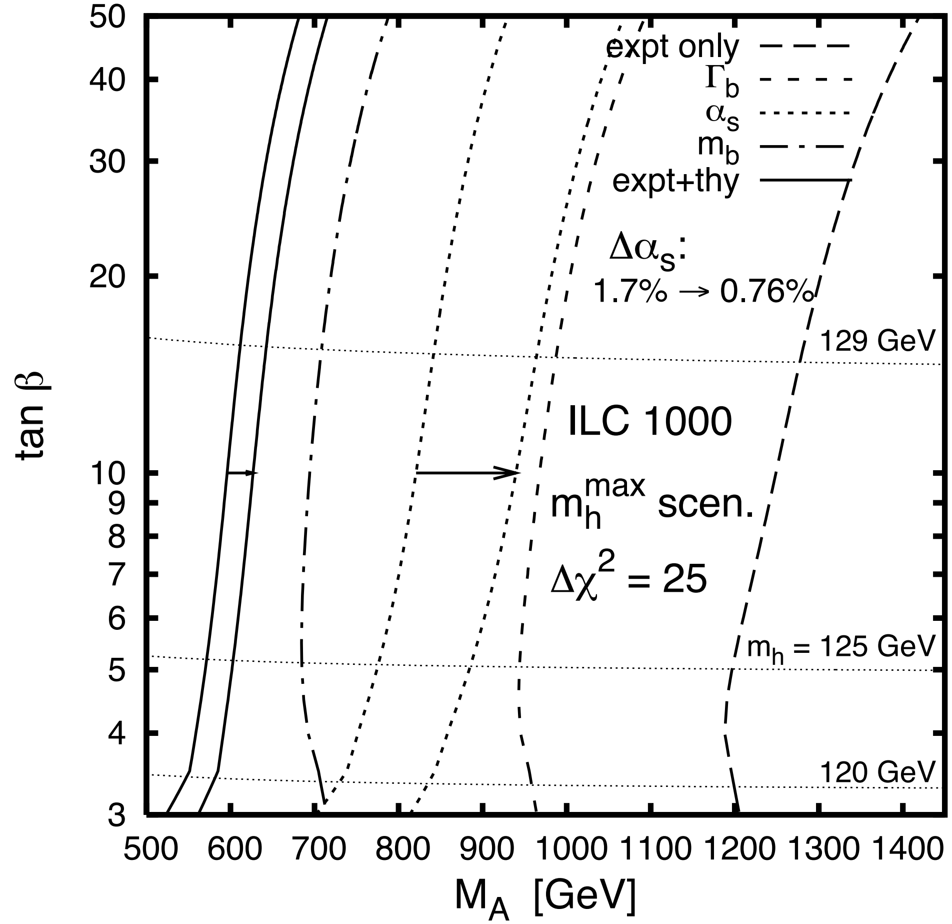
Outlook: α_s

ILC measurements will improve the precision on $\alpha_s(m_Z)$ by $\gtrsim 2\times$:

- Event shape observables
- $\sigma_{t\bar{t}}/\sigma_{\mu^+\mu^-}$ above $2m_t$
- $\Gamma_Z^{had}/\Gamma_Z^{lept}$ at Z pole (GigaZ option)

Effect of improving $\Delta\alpha_s(m_Z)$ from 0.0020 (1.7%) [current PDG] to 0.0009 (0.76%) [Tesla TDR] (includes GigaZ).

Not much impact unless $\Delta\overline{m_b}(M_b)$ is also improved.



Outlook: other observables

Phase 2 experimental precision dominated by three channels:
$\sigma \times \text{BR}(b\bar{b}), \sigma \times \text{BR}(gg)$: suffer directly from large par/thy uncerts.
$\sigma \times \text{BR}(WW)$: affected indirectly through Higgs total width.

A brief foray into the MSSM:
Study characteristic features of MSSM Higgs couplings:

$$\frac{g_{h^0\bar{t}t}}{g_{H_{\text{SM}}\bar{t}t}} = \frac{g_{h^0\bar{c}c}}{g_{H_{\text{SM}}\bar{c}c}} = \sin(\beta-\alpha) + \cot\beta\cos(\beta-\alpha)$$

$$\frac{g_{h^0\bar{b}b}}{g_{H_{\text{SM}}\bar{b}b}} = \frac{g_{h^0\tau\tau}}{g_{H_{\text{SM}}\tau\tau}} = \sin(\beta-\alpha) - \tan\beta\cos(\beta-\alpha)$$

$$\frac{g_{h^0WW}}{g_{H_{\text{SM}}WW}} = \frac{g_{h^0ZZ}}{g_{H_{\text{SM}}ZZ}} = \sin(\beta-\alpha)$$

Interested in the approach to decoupling:

$$\cos(\beta-\alpha) \simeq \frac{1}{2}\sin 4\beta \frac{m_Z^2}{M_A^2} \longrightarrow 0 \text{ for } M_A \gg m_Z$$

Plug in and keep leading term in m_Z^2/M_A^2:

$$\begin{aligned} \frac{\delta\Gamma_W}{\Gamma_W} = \frac{\delta\Gamma_Z}{\Gamma_Z} &\simeq -\frac{1}{4}\sin^2 4\beta \frac{m_Z^4}{M_A^4} \simeq -4\cot^2\beta \frac{m_Z^4}{M_A^4} \\ \frac{\delta\Gamma_b}{\Gamma_b} \simeq \frac{\delta\Gamma_\tau}{\Gamma_\tau} &\simeq -\tan\beta \sin 4\beta \frac{m_Z^2}{M_A^2} \simeq +4\frac{m_Z^2}{M_A^2} \\ \frac{\delta\Gamma_c}{\Gamma_c} &\simeq \cot\beta \sin 4\beta \frac{m_Z^2}{M_A^2} \simeq -4\cot^2\beta \frac{m_Z^2}{M_A^2} \end{aligned}$$

(Last equality: used large $\tan\beta$ approximation $\sin 4\beta \simeq -4\cot\beta$.)

Biggest deviations from SM are in Γ_b and Γ_τ.
Picture not dramatically altered by radiative corrections.

Phase 2 experimental precision dominated by three channels:
$\sigma \times \mathrm{BR}(b\bar{b}), \sigma \times \mathrm{BR}(gg)$: suffer directly from large par/thy uncerts.
$\sigma \times \mathrm{BR}(WW)$: affected indirectly through Higgs total width.

Parametric & theoretical uncertainties are washing out sensitivity to shift in Γ_b relative to Γ_W!

Want another non-hadronic final state to restore sensitivity.
$\sigma \times \mathrm{BR}(\tau\tau)$ would be perfect.

Sensitivity would come from the ratio:

$$\frac{\sigma \times \mathrm{BR}(\tau\tau)}{\sigma \times \mathrm{BR}(WW)} = \frac{\Gamma_\tau}{\Gamma_W}$$

- m_b, α_s, QCD uncertainties in total width cancel
- Ratio Γ_τ/Γ_W exhibits large deviation from SM

Using correlation matrix in the χ^2 means we don't need to play with ratios: everything is automatic.

Going from Phase 1 to Phase 2, precision on key final states improves:

$b\bar{b}$: 5–6× WW: 4–5× gg: 3.5–5.5×

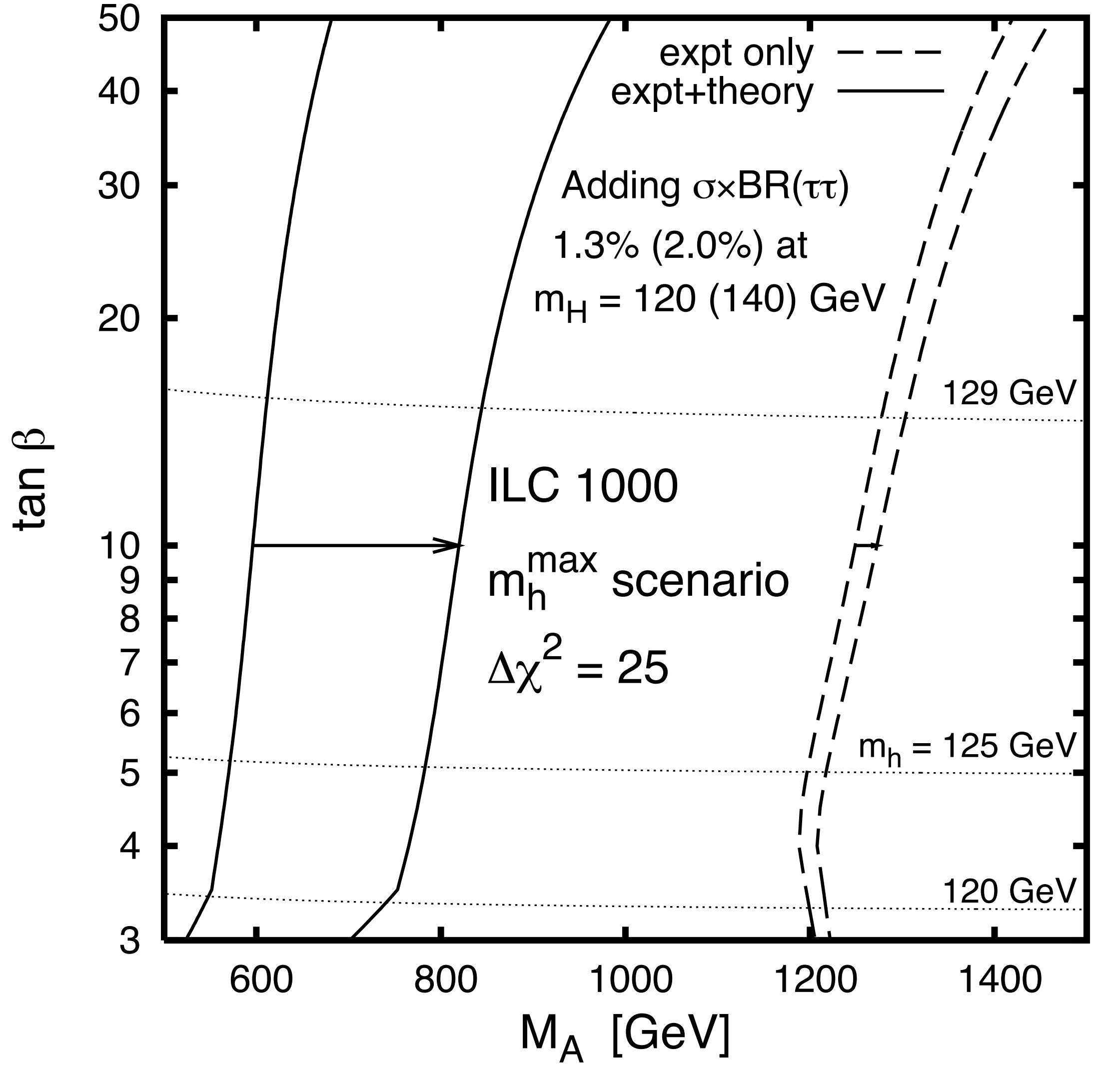
"Reasonable" to expect similar improvement in $\tau\tau$:
assume 4× and see what happens.

"Phase 2": 1000 fb^{-1} at 1000 GeV, -80% e^- / $+60\%$ e^+ pol'n

SM Higgs cross section times BR statistical uncertainties			
	$m_H = 115$ GeV	120 GeV	140 GeV
$\sigma \times$ BR($b\bar{b}$)	0.3%	0.4%	0.5%
$\sigma \times$ BR(WW)	2.1%	1.3%	0.5%
$\sigma \times$ BR(gg)	1.4%	1.5%	2.5%
$\sigma \times$ BR($\gamma\gamma$)	5.3%	5.1%	5.9%
$\sigma \times$ BR($\tau\tau$)	–	1.3%	2.0%

Original selection required $\sum$vis $= m_H$; have to change this for $\tau\tau$.

Effect of adding a measurement of $\sigma \times BR(\tau\tau)$ in Phase 2:



Not a big effect on expt-only reach.

Much bigger effect once param/theory uncertainties are included.

Conclusions

Theory uncertainties are at the level of a couple of percent.

Start to have a significant impact when experimental uncertainties get below the percent level – big impact on Phase 2.

Most important theory/parametric uncertainties are:
- m_b (current uncertainty 0.95%) – feeds into Γ_b calculation
Improving this is important!
Need more QCD theory work on semileptonic B decay spectra.
- α_s (current uncertainty 1.7%) – feeds into Γ_b, Γ_c, Γ_g calculation
Will improve by $\gtrsim 2\times$ at ILC. GigaZ valuable here.

Understanding the pattern of theory/parametric uncertainties points out the most valuable new experimental channels.
Adding $\sigma \times$ BR($\tau\tau$): small impact with only expt uncerts; huge impact after thy+param uncerts included.