Phenomenology of the Littlest Higgs Model

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Outline

- Introduction
- Littlest Higgs Model Description
- Phenomenology
- Outlook & Conclusions

The Standard Model

Higgs doublet Φ (color, weak, hypercharge)=$(1, 2, \frac{1}{2})$

$$V = -\mu^2\Phi^2 + \frac{\lambda}{4}\Phi^4 \qquad <\Phi>= v = \sqrt{\frac{2\mu^2}{\lambda}}$$

But Φ is unstable to quantum corrections:

top loop $\quad \sim -\frac{3}{8\pi^2}y_t^2\Lambda^2$

W, Z loops $\quad \sim \frac{1}{16\pi^2}g^2\Lambda^2$

h loops $\quad \sim \frac{1}{16\pi^2}\lambda^2\Lambda^2$

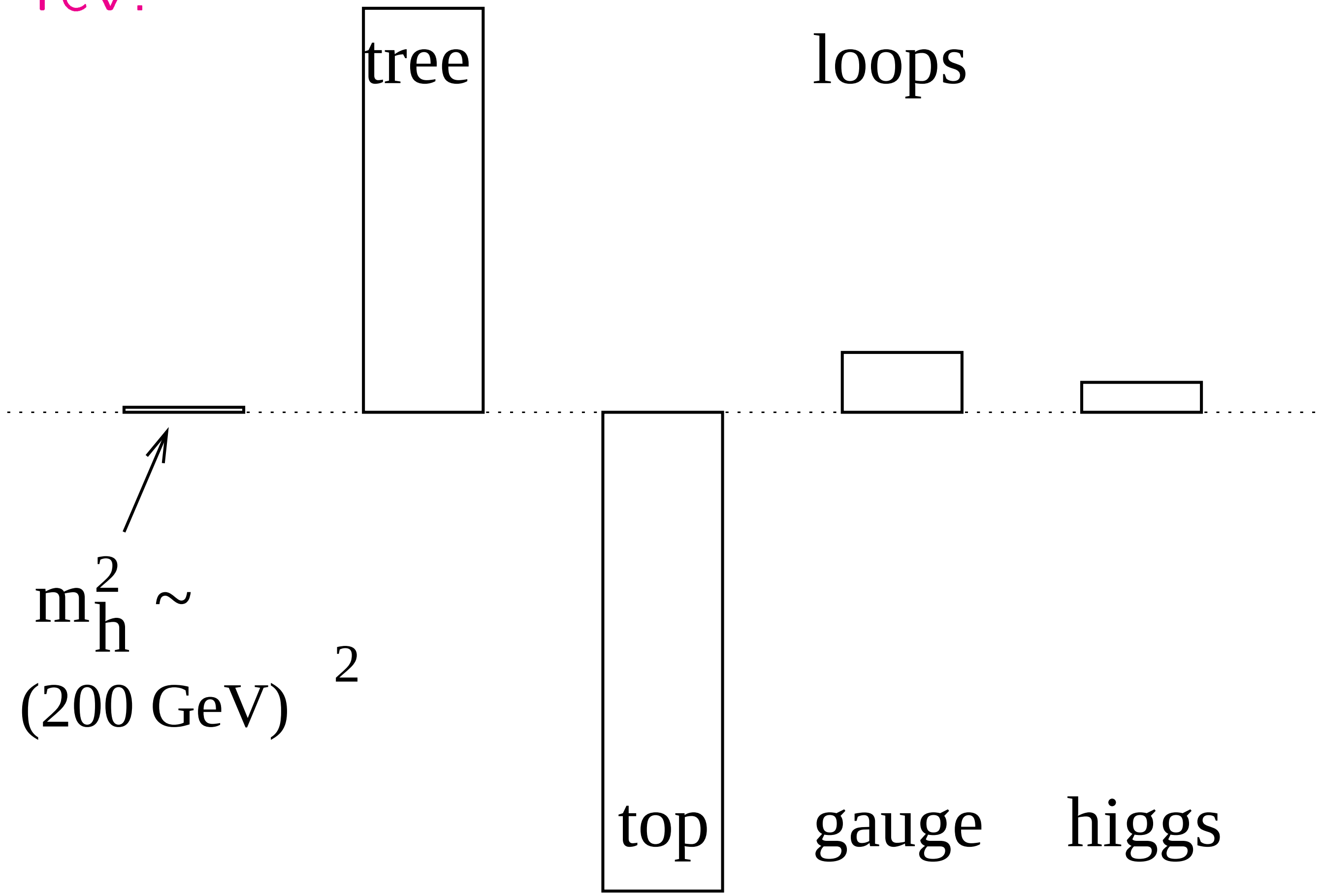
giving us a quantum-corrected Higgs mass:

$$m_h^2 = (m_h^2)_{\text{bare}} + \frac{3g^2\Lambda^2}{32\pi^2 m_W^2}\left(m_H^2 + 2m_W^2 + m_Z^2 - \frac{4}{3}N_C\sum_f m_f^2\right)$$

Only natural if $\Lambda < 1 - 10$TeV!

The Hierarchy Problem

$\Lambda = 10$ TeV:



Naturalness requirement: less than 90% cancellation on m_H^2

$\Lambda_t \lesssim 3$ TeV $\quad \Lambda_{W,Z} \lesssim 9$ TeV $\quad \Lambda_H \lesssim 12$ TeV

Cancellation Mechanisms?

- Veltman Point

$$2m_W^2 + m_Z^2 + m_H^2 - N_C \frac{4}{3} \sum_f m_f^2 = 0$$

Requires $m_h \simeq 315\text{GeV}$ – accidental?

- SUSY (Superpartners of *opposite* spin)

Natural cancellations: $\tilde{t}$ versus t

$\tilde{W}$ versus W

$\tilde{H}^0$ versus H,

lead to

$$\Delta m_H^2 \sim (M_{SUSY}^2 - M_{SM}^2) \frac{\lambda_f^2}{16\pi^2} \ln\left(\frac{\Lambda}{M_{SUSY}}\right)$$

Weak scale SUSY is natural $if\ M_{SUSY} \sim \mathcal{O}(1)$ TeV.

The Little Higgs Idea

An alternative way to keep H light naturally

- Higgs is a pseudo-Goldstone boson from global symmetry breaking at scale

$\Lambda \sim 4\pi f \sim 10 - 30$TeV;

- Higgs acquires a mass radiatively at the EW scale

$v \sim g^2 f/4\pi \sim 100 - 300$GeV;

- Quadratic divergences cancelled at one-loop level by new states

$M \sim gf \sim 1 - 3$TeV:

$$W, Z, B \leftrightarrow W_H, Z_H, B_H; \quad t \leftrightarrow T; \quad H \leftrightarrow \Phi.$$

(cancellation among same spin states!)

Since at least two operators are needed to explicitly break all the global symmetry, m_H^2 is smaller than Λ by two loop factors: a "little hierarchy" $\Lambda \gg f \gg v$.

The Littlest Higgs Model

A specific realization: $SU(5)$ Non-linear σ-model

Arkani-Hamed, Cohen, Katz, Nelson [hep-ph/0206021]

$$\mathcal{L}_\Sigma = \frac{1}{2}\frac{f^2}{4}\mathrm{Tr}|\mathcal{D}_\mu \Sigma|^2, \qquad \Sigma = e^{2i\Pi/f}\Sigma_0,$$

where f is the condensate scale (the Goldstone boson decay constant); Σ, Σ_0, Π are 5×5 matrices.

A subgroup is gauged:

$$G_1 \otimes G_2 = [SU(2) \otimes U(1)]_1 \otimes [SU(2) \otimes U(1)]_2$$

with the covariant derivative

$$\mathcal{D}_\mu \Sigma = \partial_\mu \Sigma - i \sum_{j=1}^{2} \left[g_j W_j^a (Q_j^a \Sigma + \Sigma Q_j^{a\top}) + g'_j B_j (Y_j \Sigma + \Sigma Y_j^\top) \right]$$

The Littlest Higgs Model – Symmetry breaking

Spontaneous symmetry breaking by a vacuum expectation value f in the direction

$$\langle \Sigma \rangle = \Sigma_0 = \begin{pmatrix} & & \mathbb{1} \\ & 1 & \\ \mathbb{1} & & \end{pmatrix}$$

Global: $SU(5) \Rightarrow SO(5)$, leading to 14 Goldstone bosons;
Gauged: $[SU(2) \otimes U(1)]_1 \otimes [SU(2) \otimes U(1)]_2 \Rightarrow SU(2)_L \otimes U(1)_Y$

Fate of the Goldstone bosons:

$1_0 \oplus 3_0$	Longitudinal modes of $Z_H, W_H^\pm, A_H$
$2_{\pm\frac{1}{2}}$	h doublet
$3_{\pm 1}$	ϕ triplet

$$\Pi = \begin{pmatrix} 0 & h^\dagger/\sqrt{2} & \phi^\dagger \\ h/\sqrt{2} & 0 & h^\star/\sqrt{2} \\ \phi & h^T/\sqrt{2} & 0 \end{pmatrix}, \quad h = (h^+,\ h^0), \quad \phi = \begin{pmatrix} \phi^{++} & \phi^+/\sqrt{2} \\ \phi^+/\sqrt{2} & \phi^0 \end{pmatrix}$$

The Littlest Higgs Model – Gauge structure

Gauged $[SU(2) \times U(1)]^2$ subgroup:

$$Q_1^a = \begin{pmatrix} \sigma^a/2 & & \\ & & \\ & & \end{pmatrix} \qquad Q_2^a = \begin{pmatrix} & & \\ & & \\ & & -\sigma^a/2 \end{pmatrix}$$

$$Y_1 = \mathrm{diag}(-3,-3,2,2,2)/10 \qquad Y_2 = \mathrm{diag}(-2,-2,-2,3,3)/10$$

Note new gauge couplings $\{g_1,\ g_2,\ g_1',\ g_2'\}$ and mixing angles $\{c,\ s,\ c',\ s'\}$ to diagonalize to the SM gauge basis (EWSB introduces further mixing):

$$g = g_1 s = g_2 c, \qquad g' = g_1' s' = g_2' c'$$

Global Symmetries: (protect the Higgs mass)

$$SU(3)_1 \to \left(\begin{array}{c|c} 0_{2\times 2} & \\ \hline & V_3 \end{array}\right) \qquad SU(3)_2 \to \left(\begin{array}{c|c} V_3 & \\ \hline & 0_{2\times 2} \end{array}\right)$$

Gauge Loop Cancellation

Coupling of gauge bosons to Higgs is:

$$
\begin{aligned}
\mathcal{L}_{\Sigma}(W \cdot W) &= \frac{g^2}{4}\left[W_\mu^a W^{b\mu} - \frac{(c^2 - s^2)}{sc} W_\mu^a W'^{b\mu}\right] \mathrm{Tr}\left[h^\dagger h \delta^{ab}\right] \\
&- \frac{g^2}{4}\left[W'^a_\mu W'^{a\mu}\right] \mathrm{Tr}\left[h^\dagger h\right] \\
\mathcal{L}_{\Sigma}(B \cdot B) &= \frac{g'^2}{4}\left[B_\mu B^\mu - \frac{(c'^2 - s'^2)}{s'c'} B_\mu B'^\mu\right] \mathrm{Tr}\left[h^\dagger h\right] \\
&- \frac{g'^2}{4}\left[B'_\mu B'^\mu\right] \mathrm{Tr}\left[h^\dagger h\right]
\end{aligned}
$$

As in supersymmetry the global symmetry structure ensures that the new gauge bosons couple with $-g_i^2$!

Top Loop Cancellation

Add a vector-like pair of colored Weyl fermions:
$\tilde{t}$: $(3_c, 1_L)_{Y_i}$ and $\tilde{t}'^c$: $(3_c, 1_L)_{-Y_i}$.

The Lagrangian is:

$$\mathcal{L}_Y = \frac{1}{2}\lambda_1 f \epsilon_{ijk}\epsilon_{xy}\chi_i \Sigma_{jx}\Sigma_{ky} u_3'^c + \lambda_2 f \tilde{t}\tilde{t}'^c + \text{h.c.},$$

where $\chi_i = (b_3, t_3, \tilde{t})$, i, j, k run over 1..3 and x, y run over 4..5.
The bare mass term $\lambda_2 f \tilde{t}\tilde{t}'^c$ is *chosen* to be $\mathcal{O}(f)$.

The λ_1 term preserves global $SU(3)_2$, guaranteeing cancellation of the quadratic divergence:

$$-i\lambda_1(\sqrt{2}h^0 t_3 + if\tilde{t} - ih^0 h^{0*}\tilde{t}/f)\ u_3'^c + \text{h.c.}$$

The λ_2 term gives the mixing and the top quark mass:

$$m_t = \frac{\lambda_1\lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}}\, v; \quad M_T = \sqrt{\lambda_1^2 + \lambda_2^2}\ f.$$

To leading order,

$$t_3 \to t_L, \quad \tilde{t} \to T_L, \quad \{u_3'^c, \tilde{t}'^c\} \to \{t_R, T_R\}.$$

Fermion Yukawas and $U(1)$ Charges

Assume the same form as the top Yukawa coupling
Assume gauge invariance under $U(1)_1 \otimes U(1)_2$.

This fixes the fermion $U(1)$ charges up to two free continuous parameters y_u and y_e.

	Q	u'^c	d^c	L	e^c	$\tilde{t}$	$\tilde{t}'^c$
Y_1	$-\frac{3}{10} - y_u$	y_u	$\frac{3}{5} + y_u$	$\frac{3}{10} - y_e$	y_e	$\frac{1}{5} - y_u$	$-\frac{1}{5} + y_u$
Y_2	$\frac{7}{15} + y_u$	$-\frac{2}{3} - y_u$	$-\frac{4}{15} - y_u$	$-\frac{4}{5} + y_e$	$1 - y_e$	$\frac{7}{15} + y_u$	$-\frac{7}{15} - y_u$

Anomaly cancellation requires:

$$y_u = -2/5, \qquad y_e = 3/5,$$

but is not necessarily required for consistency of the model.

Higgs Potential and EWSB

Intergrating out the heavy gauge bosons and T, the Coleman-Weinberg potential is generated. The effective Higgs potential is written as

$$V = \lambda_{\phi^2} f^2 \mathrm{Tr}(\phi^\dagger \phi) + i\lambda_{h\phi h} f \left(h\phi^\dagger h^T - h^* \phi h^\dagger \right) - \mu^2 h h^\dagger + \lambda_{h^4} (h h^\dagger)^2,$$

- In terms of fundamental parameters:

$$\begin{aligned}
\lambda_{\phi^2} &= \frac{a}{2}\left[\frac{g^2}{s^2c^2} + \frac{g'^2}{s'^2c'^2}\right] + 8a'\lambda_1^2, \\
\lambda_{h\phi h} &= -\frac{a}{4}\left[g^2\frac{(c^2-s^2)}{s^2c^2} + g'^2\frac{(c'^2-s'^2)}{s'^2c'^2}\right] + 4a'\lambda_1^2, \\
\lambda_{h^4} &= \frac{a}{8}\left[\frac{g^2}{s^2c^2} + \frac{g'^2}{s'^2c'^2}\right] + 2a'\lambda_1^2 = \frac{1}{4}\lambda_{\phi^2}.
\end{aligned}$$

μ^2 is viewed as free.

- EWSB: for $\mu^2 > 0$ (Note smallness of v'.)

$$v^2 = \frac{\mu^2}{\lambda_{h^4} - \lambda_{h\phi h}^2/\lambda_{\phi^2}}, \qquad v' = \frac{\lambda_{h\phi h}}{2\lambda_{\phi^2}}\frac{v^2}{f}. \qquad v/f \sim g/4\pi$$

Higgs Potential and EWSB, ctd.

- Higgs masses:

$$
\begin{aligned}
M_\Phi^2 &\simeq \lambda_{\phi^2} f^2, \\
m_H^2 &\simeq 2\mu^2 \sim v^2 \lesssim a_{1-\text{loop}} \frac{f^2}{16\pi^2} + a_{2-\text{loop}} \frac{f^2}{16\pi^2}
\end{aligned}
$$

- Naturalness:

If we require no more than 10% fine-tuning, then

$$
f \leq \frac{4\pi m_h}{\sqrt{0.1 a_{\text{max}}}} \simeq \frac{8 \text{ TeV}}{\sqrt{a_{\text{max}}}} \left(\frac{m_h}{200 \text{ GeV}} \right),
$$

where a_{max} is the larger of the 1-loop and 2-loop contributions.

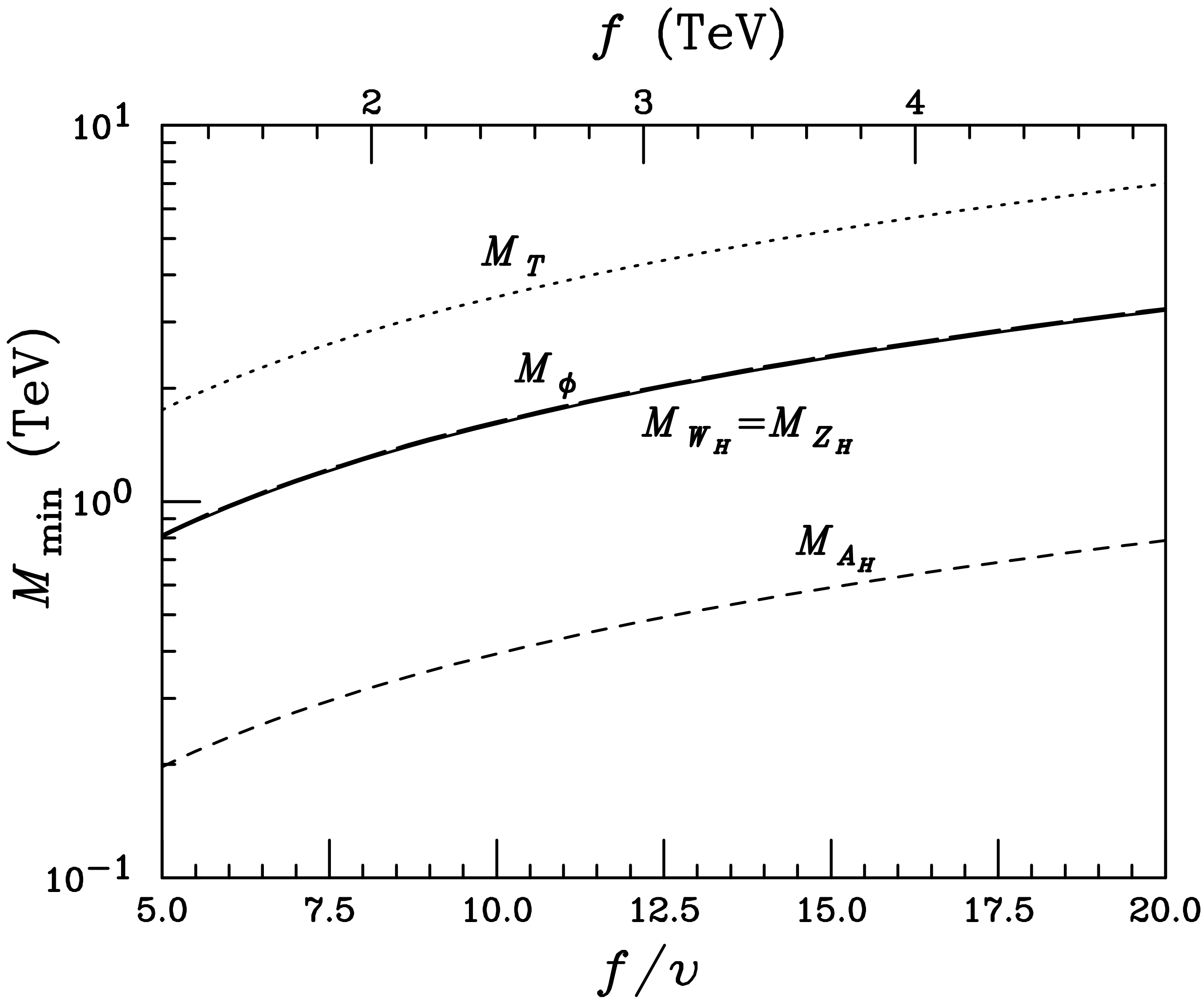
Littlest Higgs Parameter Space Summary

$\tan\theta = \frac{s}{c} = \frac{g_1}{g_2}$	New $SU(2)$ gauge coupling (or equivalently mixing angle θ)
$\tan\theta' = \frac{s'}{c'} = \frac{g'_1}{g'_2}$	New $U(1)$ gauge coupling (or equivalently mixing angle θ')
f	Symmetry breaking scale $\mathcal{O}$ (TeV)
v'	Triplet ϕ vacuum expectation value, $v'/v < v/4f$
m_h	Regular SM Higgs mass
m_T	Heavy vector top mass; we trade λ_2 for m_T

Littlest Higgs Physical State Summary

Particle	Mass
A_H	$m_Z^2 s_W^2 \left(\frac{f^2}{5s'^2c'^2v^2} + \mathcal{O}(1) \right)$
Z_H	$m_W^2 \left(\frac{f^2}{s^2c^2v^2} + \mathcal{O}(1) \right)$
W_H	$m_W^2 \left(\frac{f^2}{s^2c^2v^2} + \mathcal{O}(1) \right)$
ϕ^0	$\frac{2m_h^2 f^2}{v^2} \frac{1}{1-(4v'f/v^2)^2}$
ϕ^+	$\frac{2m_h^2 f^2}{v^2} \frac{1}{1-(4v'f/v^2)^2}$
ϕ^{++}	$\frac{2m_h^2 f^2}{v^2} \frac{1}{1-(4v'f/v^2)^2}$
T	$\sqrt{\lambda_1^2 + \lambda_2^2} f + \mathcal{O}\left(\frac{v^2}{f^2}\right)$

Masses of new particles (lower bounds)



We also worked out all the model couplings in order to do the phenomenology.

Other Little Higgs Models

- Arkani-Hamed, Cohen, Katz, Nelson, Gregoire, Wacker "Minimal Moose" [hep-ph/0206020]

  $SU(3) \times SU(2) \times U(1)$ two-site, 4-link moose
  Low energy theory: a 2HDM + complex scalar weak triplet + singlet

- Arkani-Hamed, Cohen, Katz, Nelson, "The Littlest Higgs" [hep-ph/0206021]

  $[SU(2) \times U(1)]^2$ gauged non-linear σ-model
  Low energy theory: 1 Higgs doublet

- Low, Skiba, Smith "Antisymmetric Condensate" [hep-ph/0207243]

  $SU(6)/Sp(6)$ group structure. Low energy theory: a 2HDM

- Kaplan, Schmaltz "Simple Group" [hep-ph/0302049]

  $SU(4) \times U(1)$ group structure, Low energy theory: a 2HDM
  No new gauge parameters, many extra gauge bosons,
  more natural top cancellation

- S. Chang and J. Wacker "Little Higgs and Custodial $SU(2)$" [hep-ph/0303001]

  A simple modification of the "minimal moose";
  $SU(5)$ group structure; a 2HDM

What Are Generic Features?

So many Little Higgs models...so little time...

Any Little Higgs model *must* have the following features at 1−few TeV:

- W' and Z' bosons to cancel W and Z loops

  Amost pure $SU(2)$ gauge bosons. Standard Model fermions transform under $SU(2)_1$ only – couplings are universal $\sim g_1 = gc/s \to$ LHC cross section. Decays are more model dependent, since non-fermionic decays can play a role.

- T heavy top to cancel t loop

  T production and decays are fixed by the generic form of $\mathcal{L}_{t,T}$, which appears in many little Higgs models. Cross section $\sim \lambda_1^2/\sqrt{\lambda_1^2+\lambda_2^2}$.

  Masses: $m_t \simeq v\lambda_1\lambda_2/\sqrt{\lambda_1^2+\lambda_2^2}, \quad M_T \simeq f\sqrt{\lambda_1^2+\lambda_2^2}$.

Will do pheno of Littlest Higgs with emphasis on generic features.

Electroweak Precision Constraints

Csaki, Hubisz, Kribs, Meade, Terning [hep-ph/0211124]

also Hewett, Petriello, Rizzo [hep-ph/0211218]

They found $f > 4$ TeV at 95% CL. These results apply to non-canonical fermion Yukawas, generated by some operator with dim > 4.

The strength of their constraint comes primarily from neutrino scattering, which is strongly affected by choice of y_e, y_u

- Both $U(1)$'s violate the custodial $SU(2)$ symmetry

$$\frac{M^2_{W^\pm_L}}{M^2_{Z_L}} = c^2_{\mathrm{w}} \left[1 + \frac{v^2}{f^2} \frac{5}{4} (c'^2 - s'^2)^2 - 4 \frac{v'^2}{v^2} \right].$$

So, evade precision constraint from M_W by tuning parameters:

$$c'^2 \approx s'^2, \qquad v'^2 \ll v^2.$$

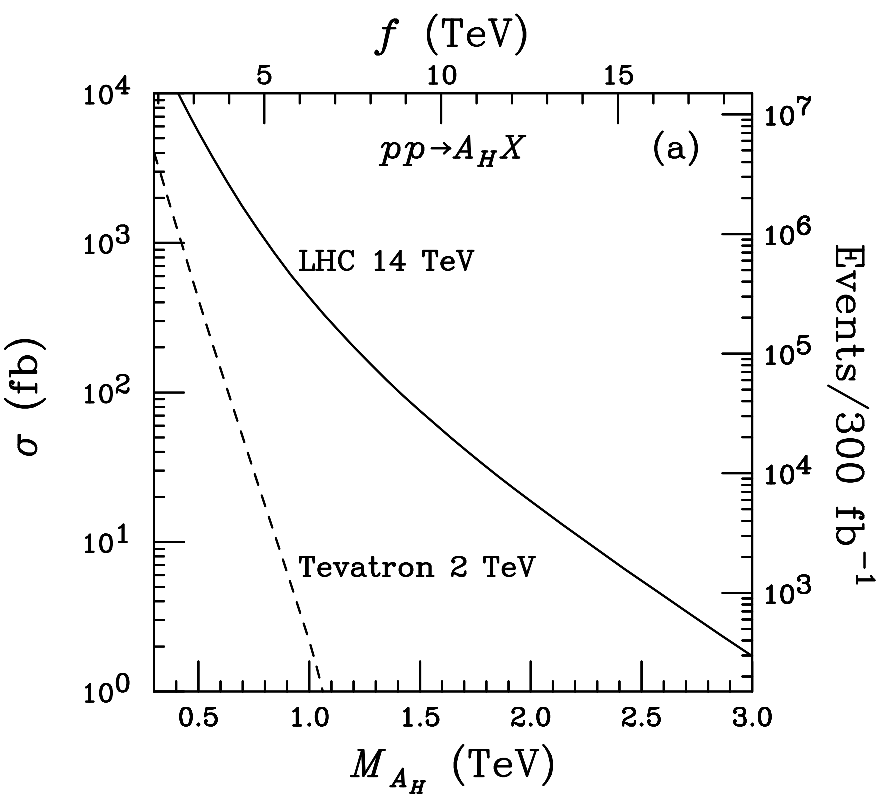
Electroweak Precision Constraints, ctd.

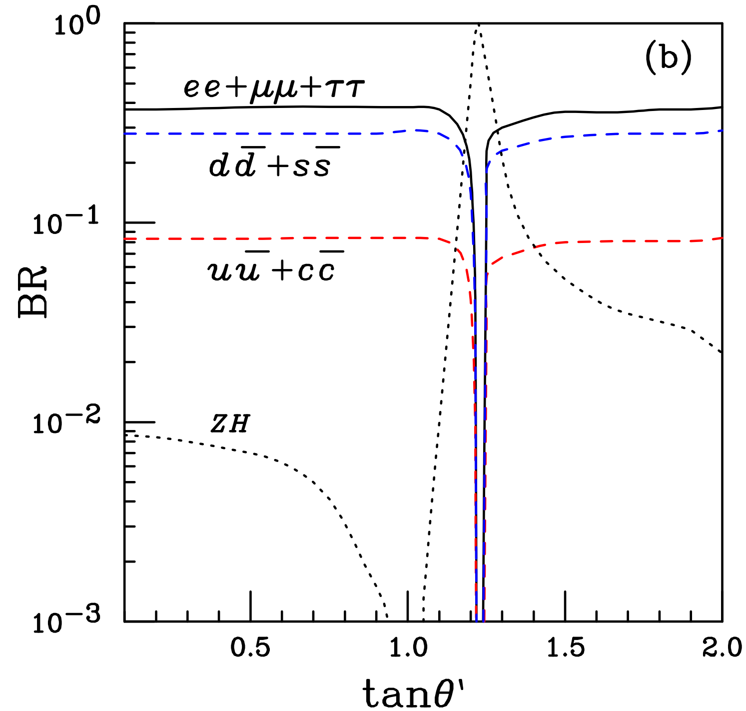
	$SU(2)_H$	$U(1)_H$	$< \phi^0 >$
$M^2_{W^\pm_L}$	$-\frac{5}{12}\frac{v^2}{f^2} + c^2 s^2 \frac{v^2}{f^2}$	0	$4\frac{v'^2}{v^2}$
$M^2_{Z_L}$	$-\frac{5}{12}\frac{v^2}{f^2} + c^2 s^2 \frac{v^2}{f^2}$	$-\frac{5}{4}\frac{v^2}{f^2}(c'^2 - s'^2)^2$	$8\frac{v'^2}{v^2}$
G_F	$\frac{5}{12}\frac{v^2}{f^2}$	0	$-4\frac{v'^2}{v^2}$
$M^2_{Z_L} G_F$	$c^2 s^2 \frac{v^2}{f^2}$	$-\frac{5}{4}\frac{v^2}{f^2}(c'^2 - s'^2)^2$	$4\frac{v'^2}{v^2}$
δg_{ff}	$\propto c^2 \frac{v^2}{f^2}$	$\propto (-c'^2 Y_1 + s'^2 Y_2)\frac{v^2}{f^2}$	0

- reduce the triplet vev: $v' \ll v$;
- reduce $SU(2)$ heavy gauge boson contributions: $c \ll 1$;
- reduce $U(1)$ heavy gauge boson contribution: $c'^2 \approx s'^2$;
- reduce SM fermion contributions: $c'^2 Y_1 \approx s'^2 Y_2$. [Csaki: $Y_2 = 0$]

Doable, but need symmetries (otherwise it's a fine-tuning)

A_H Production and Decay (the lightest new particle)

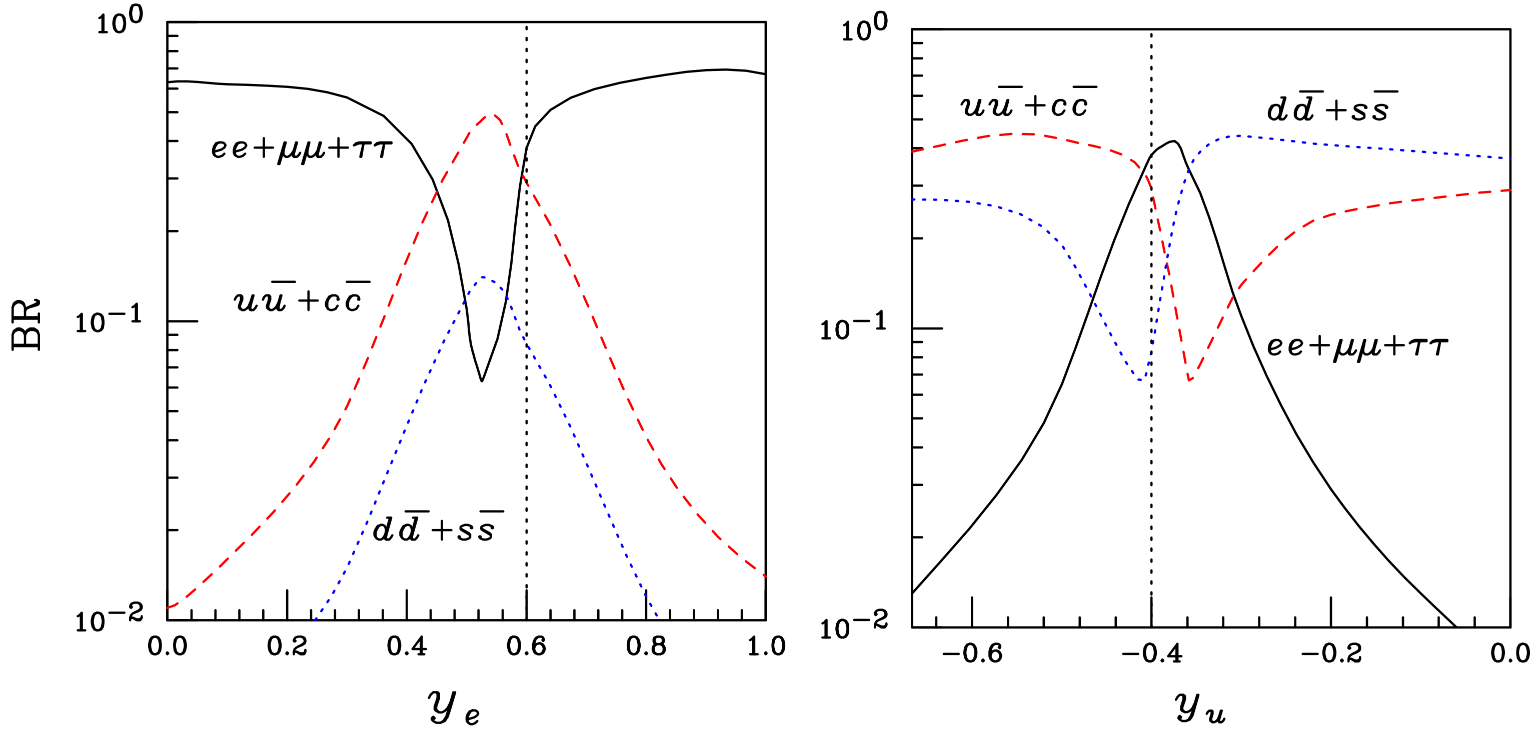




Anomaly cancellation imposed: $y_u = -2/5$, $y_e = 3/5$

(a) has $\tan(\theta') = 1$ or equivalently $g'_1 = g'_2$

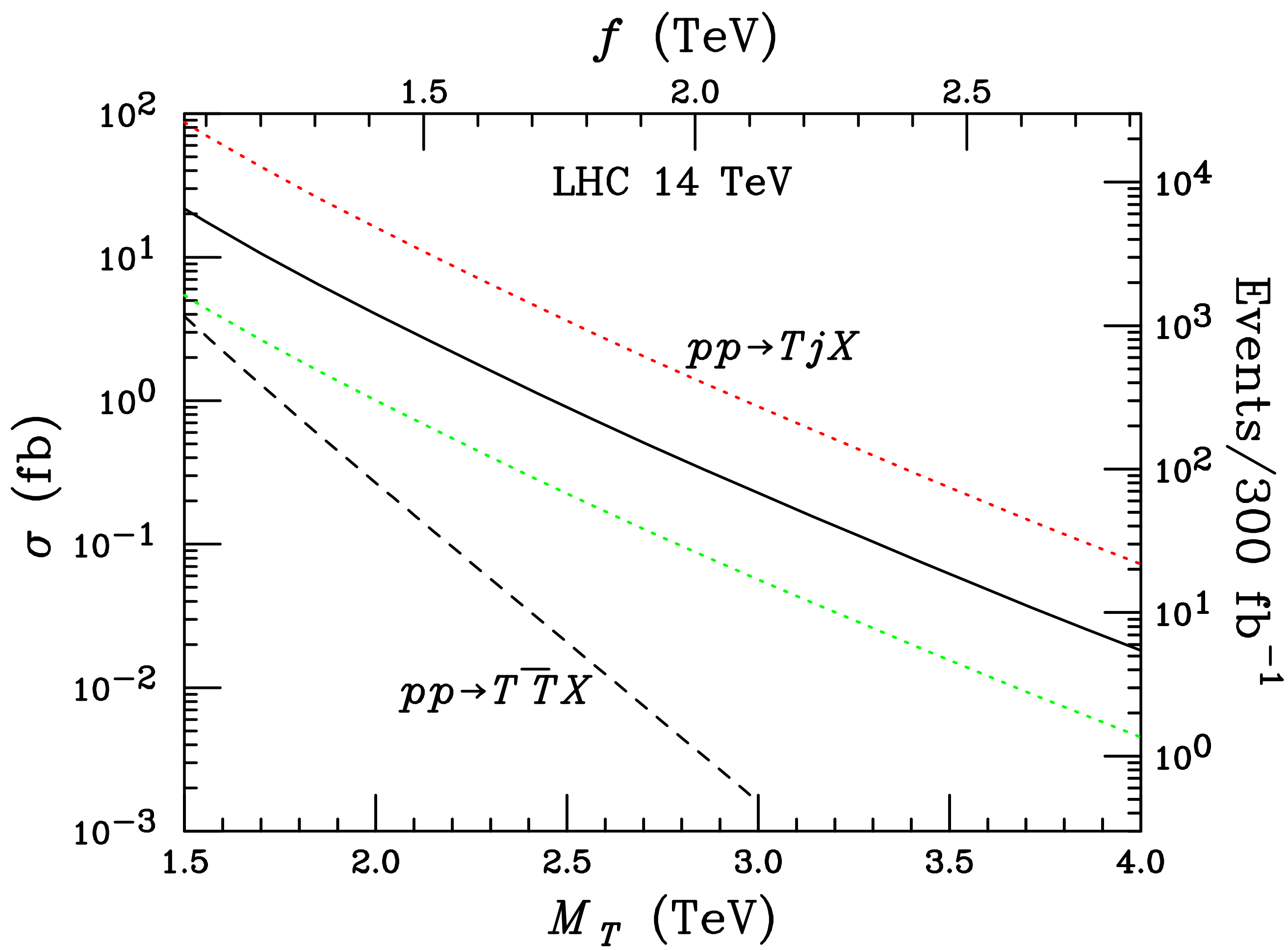
A_H Branching Ratios vs. y_e and y_u



Both plots have $\tan(\theta') = 1$, or equivalently $g_1' = g_2'$.

Model dependence: Fermion $U(1)$ charges; A_H mass.
(Don't even have to gauge the second $U(1)$...)

Heavy T Signatures



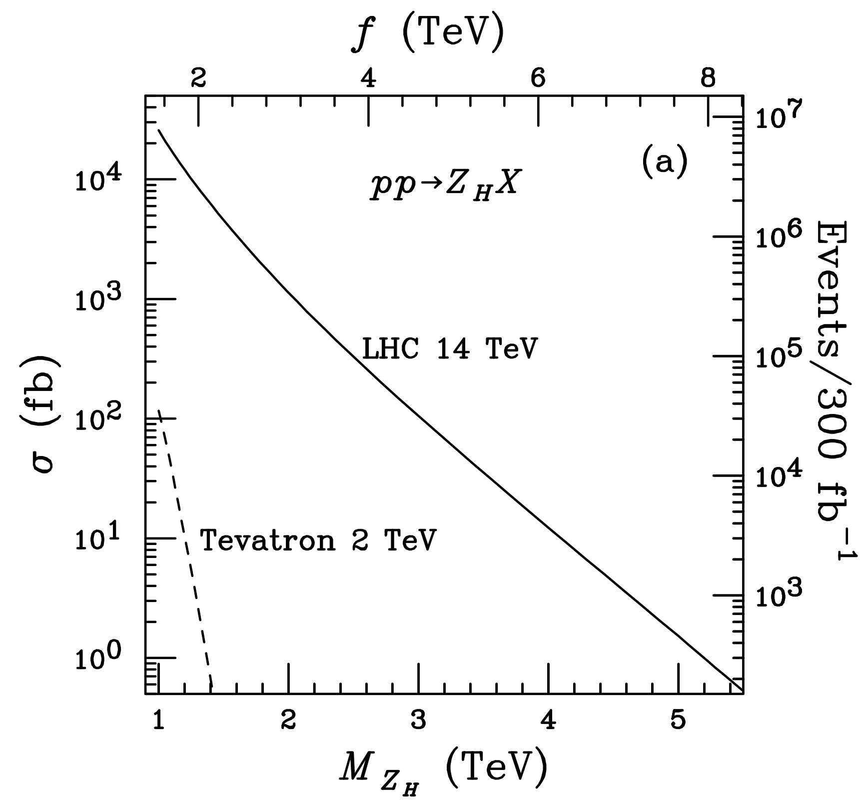
Solid line: $\lambda_1/\lambda_2 = 1$; Dotted: $\lambda_1/\lambda_2 = 2$, $1/2$

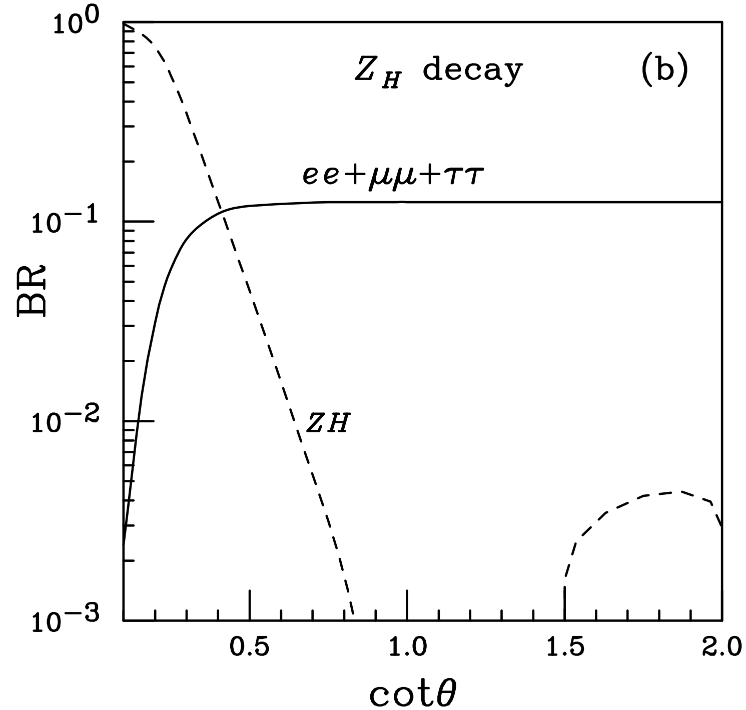
T decays: $\Gamma(T \to tH) = \Gamma(T \to tZ) = \frac{1}{2}\Gamma(T \to bW) = \frac{\kappa^2}{32\pi} M_T$

where $\kappa = \lambda_1^2/\sqrt{\lambda_1^2 + \lambda_2^2}$.

Robust!

Z_H Production and Decay





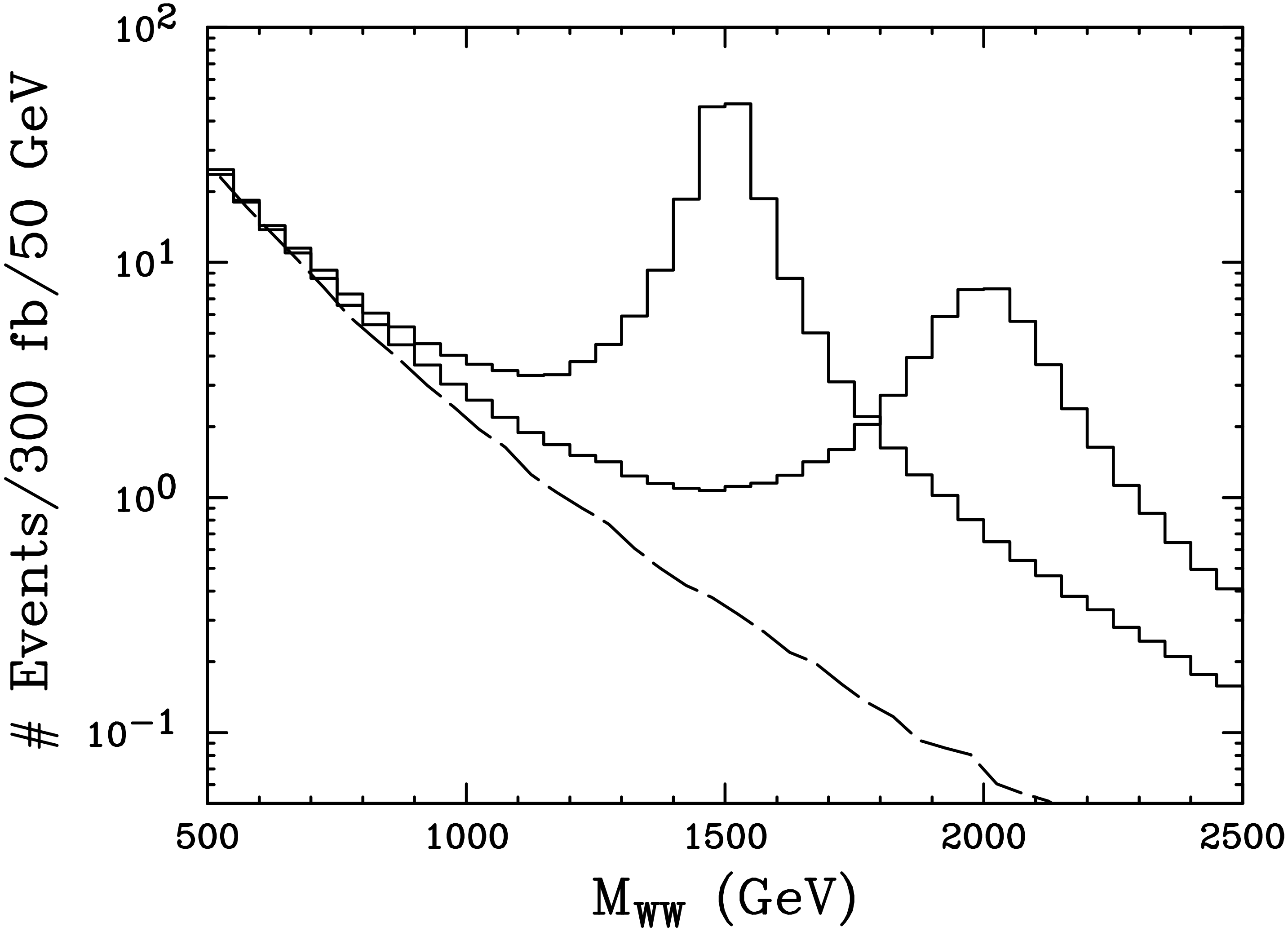
(a) has $\cot\theta = 1$.
xsec $\sim g^2 \cot^2\theta$. (EW precision favors small $\cot\theta$...)

Z_H decays: Equipartition among fermions.

Robust!

See also Burdman, Perelstein & Pierce [hep-ph/0212228]

Resonant $W^+W^+ \to \Phi^{++} \to W^+W^+$



Cross Section is proportional to $(v'/v)^2 = (0.1)^2$ for this plot

Model dependence: v'; content of scalar sector.

Effects on WWZ and WWH couplings

- WWZ coupling

$$\mathcal{L}_{WWV} = ig_{WWV}\left[g_1^V(W^+_{\mu\nu}W^{-\mu} - W^{+\mu}W^-_{\mu\nu})V^\nu + \kappa_V W^+_\mu W^-_\nu V^{\mu\nu} + \frac{\lambda_V}{m_W^2}W^{+\nu}_\mu W^{-\rho}_\nu V^\mu_\rho\right]$$

$$g_1^Z = \kappa_Z = 1 + \frac{1}{2(c_W^2 - s_W^2)}\left[\frac{1}{4}\frac{v^2}{f^2}\left(-1 + (c^2 - s^2)^2 + 5(c'^2 - s'^2)^2\right) - 4\frac{v'^2}{v^2}\right]$$

$$f \gtrsim \frac{v}{\sqrt{4 \times 5\%}} \approx 2.3\ v, \qquad v' \lesssim \frac{\sqrt{5\%}\ v}{2} \approx 10\%\ v$$

At future linear collider, $10^{-3} - 10^{-4}$ reachable:

$f \sim (15 - 50)v \sim 3.5 - 12$ TeV.

- WWH coupling at future linear collider

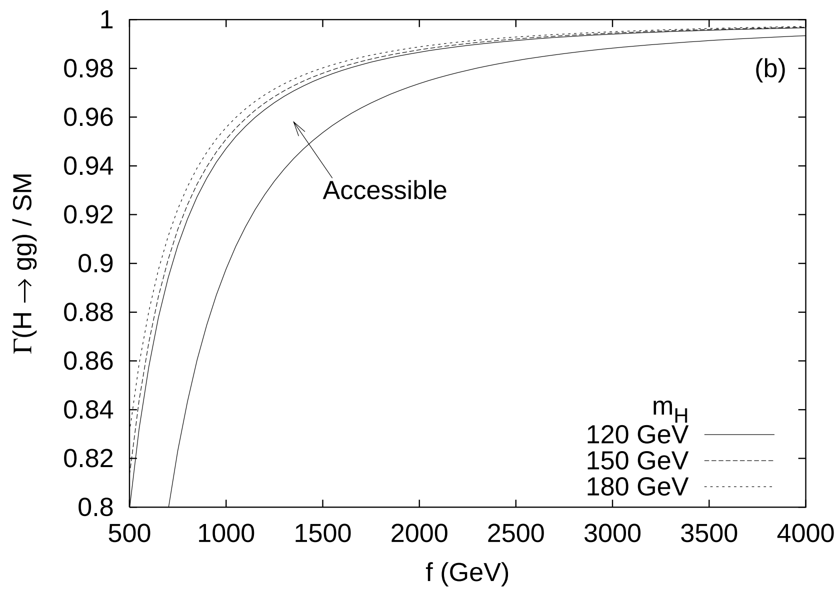
$$f > \frac{v}{\sqrt{3 \times 1\%}} \approx 1.4 \text{ TeV}$$

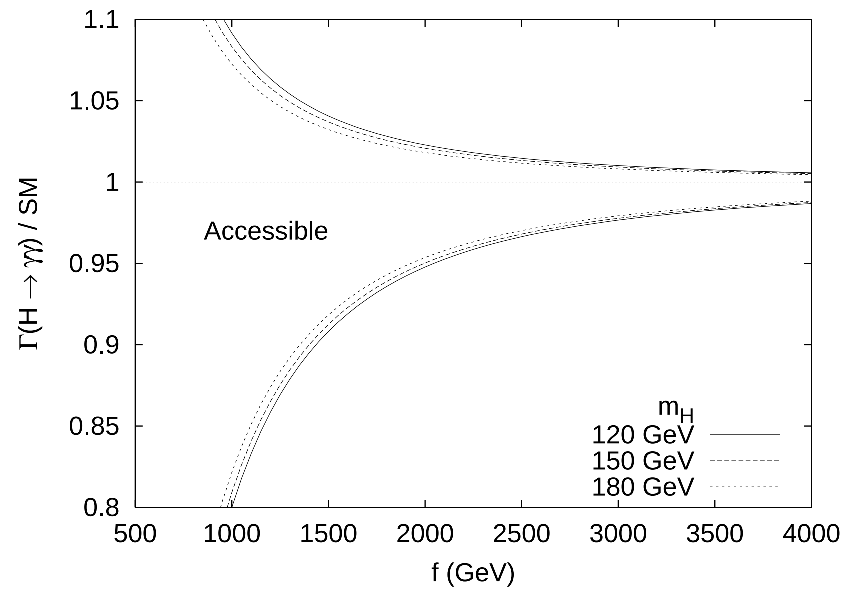
- Should also see model signatures in improved precision EW, e.g. GigaZ.

Loop induced Higgs decay: $H \to gg,\ \gamma\gamma$

Robust Higgs signatures:

- T affects $H \to gg$
- $W_H^{\pm}$, T affect $H \to \gamma\gamma$



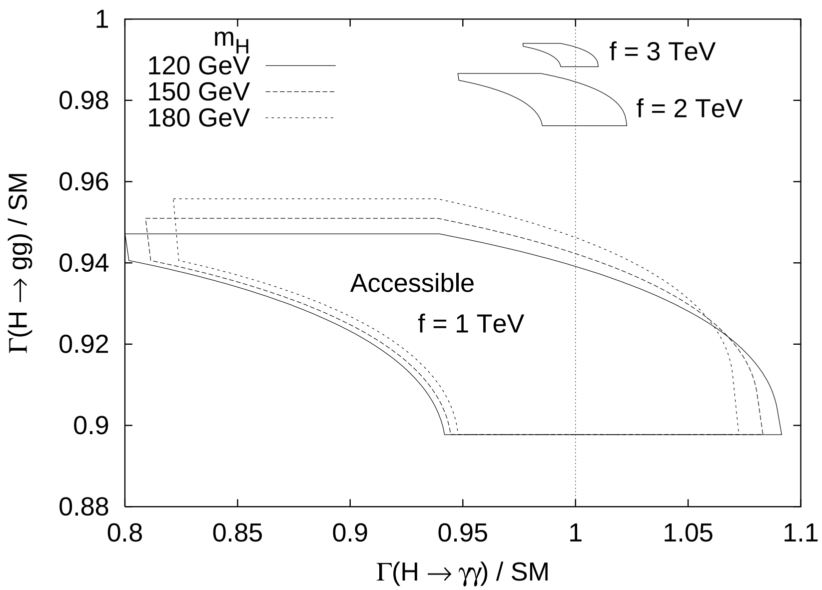


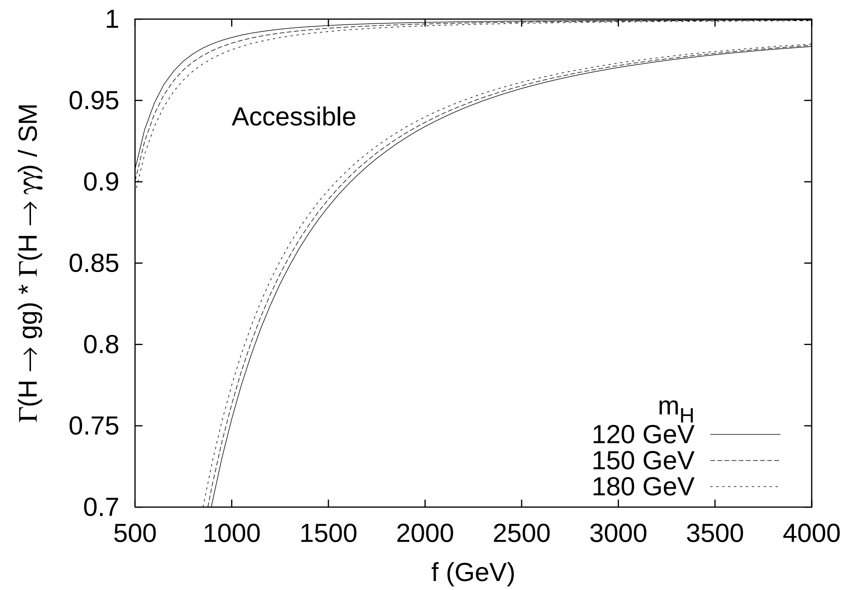
$H \to gg$:
LHC: $25-30\% \oplus 15\%$ thy.
Only very low f values.
LC: $6-20\% \oplus 15\%$ thy.
Probes $f < 750$ GeV at 1σ.

$H \to \gamma\gamma$:
LHC: $15-20\%$. Probes $f < 1.0$ TeV at 1σ ($f < 750$ GeV at 2σ).
LC: $15-20\%$. Same as LHC.
$\gamma\gamma$: 3%. Probes $f < 2.7$ TeV at 1σ ($f < 1.8$ TeV at 2σ)
($f < 1.2$ TeV at 5σ).

New effects scale like $1/f^2$, thus decouple at high scales.

$H \to gg$ and $H \to \gamma\gamma$ correlated





- when $H \to gg$ maximal, then $H \to \gamma\gamma$ minimal, and vice versa.
  $\Rightarrow$ maximum contribution from T to $H \to gg$ leads to maximum cancellation in $H \to \gamma\gamma$ between W_L & T loops.

Model dependence: Assumes one Higgs doublet. Big effects in 2HDM from 2nd doublet, mixing.

Conclusions

- The Little Higgs idea provides a new way to address the Hierarchy problem by letting the Higgs be a pseudo-Goldstone boson, and pushing off the problem to $\mathcal{O}(10\text{TeV})$.

- EW precision constraints may be evaded by tuning the model parameters.

- Finding generic features is an important consideration: There must be new states Z_H, W_H, T and the LHC has a good chance to find them.

- The LHC and linear e^+e^- collider can probe the parameter space up to f of a few TeV, enough to relegate the idea to "fine tuned".