Custodial symmetry violation in the Georgi-Machacek model

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Workshop on Multi-Higgs Models
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Based on B. Keeshan, HEL & T. Pilkington, arXiv:1807.11511
Outline

Introduction

Georgi-Machacek model

Custodial symmetry violation

Our implementation

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Conclusions and outlook
Introduction

Can we constrain the possibility that “exotic” Higgs fields (isospin $> 1/2$) contribute to electroweak symmetry breaking?

Generically this is very strongly constrained by the $\rho$ parameter:

$$\rho \equiv \frac{\text{weak neutral current}}{\text{weak charged current}} = \frac{(g^2 + g'^2)/M_Z^2}{g^2/M_W^2} = \frac{v_\phi^2 + a\langle X^0 \rangle^2}{v_\phi^2 + b\langle X^0 \rangle^2}$$

$$a = 4 \left[ T(T+1) - Y^2 \right] c$$

$$b = 8Y^2$$

Expt: $\rho = 1.00037 \pm 0.00023$ (2016 PDG)

Need to do some model-building; otherwise $v_{\text{exotic}} \ll v_{\text{doublet}}$. 

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There are only two known approaches:

1) Use the septet \((T,Y) = (3,2)\): \(\rho = 1\) by accident!
   Doublet \(\left( \frac{1}{2}, \frac{1}{2} \right) \) + septet \((3,2)\): Scalar septet model
   
   Hisano & Tsumura, 1301.6455; Kanemura, Kikuchi & Yagyu, 1301.7303

2) Use global \(SU(2)_L \times SU(2)_R\) imposed on the scalar potential
   Global \(SU(2)_L \times SU(2)_R \rightarrow\) custodial \(SU(2)\) ensures tree-level \(\rho = 1\)
   Doublet + triplets \((1,0) + (1,1)\): Georgi-Machacek model

   Georgi & Machacek 1985; Chanowitz & Golden 1985

   Doublet + quartets \(\left( \frac{3}{2}, \frac{1}{2} \right) + \left( \frac{3}{2}, \frac{3}{2} \right)\):
   Generalized Georgi-Machacek models

   Doublet + quintets \((2,0) + (2,1) + (2,2)\):
   Machacek models

   Doublet + sextets \(\left( \frac{5}{2}, \frac{1}{2} \right) + \left( \frac{5}{2}, \frac{3}{2} \right) + \left( \frac{5}{2}, \frac{5}{2} \right)\):
   
   Galison 1984; Robinett 1985; HEL 1999; Chang et al 2012; HEL & Rentala 2015

   Larger than sextets \(\rightarrow\) too many large multiplets, violates perturbativity

   Can also have duplications, combinations \(\rightarrow\) ignore that here.
Both approaches have theoretical “issues”:

1) Can’t give the septet a vev through spontaneous breaking without generating a physical massless Goldstone boson. Have to couple it to the SM doublet through a dimension-7 $X\Phi^*\Phi^5$ term  

Need the UV completion to be nearby!

2) Global $\text{SU}(2)_L \times \text{SU}(2)_R$ is broken by gauging hypercharge. Special relations among params of full gauge-invariant scalar potential can only hold at one energy scale: violated by running due to hypercharge.  

Need the UV completion to be nearby!

This talk: quantify (2) in the Georgi-Machacek model.
Georgi-Machacek model  Georgi & Machacek 1985; Chanowitz & Golden 1985

SM Higgs (bi-)doublet + triplets \((1, 0) + (1, 1)\) in a bi-triplet:

\[
\Phi = \begin{pmatrix} \phi^0 & \phi^+ \\ -\phi^{++} & \phi^0 \end{pmatrix} \quad X = \begin{pmatrix} \chi^0 & \xi^+ & \chi^{++} \\ -\chi^{++} & \xi^0 & \chi^+ \\ \chi^{++} & -\xi^{++} & \chi^0 \end{pmatrix}
\]

Global \(SU(2)_L \times SU(2)_R \rightarrow\) custodial symmetry \(\langle \chi^0 \rangle = \langle \xi^0 \rangle \equiv v_\chi\)

Most general scalar potential invariant under \(SU(2)_L \times SU(2)_R\):

\[
V(\Phi, X) = \frac{\mu_2^2}{2} \text{Tr}(\Phi^\dagger \Phi) + \frac{\mu_3^2}{2} \text{Tr}(X^\dagger X) + \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2
\]
\[
+ \lambda_2 \text{Tr}(\Phi^\dagger \Phi) \text{Tr}(X^\dagger X) + \lambda_3 \text{Tr}(X^\dagger X X^\dagger X)
\]
\[
+ \lambda_4 [\text{Tr}(X^\dagger X)]^2 - \lambda_5 \text{Tr}(\Phi^\dagger \tau^a \Phi \tau^b) \text{Tr}(X^\dagger t^a X t^b)
\]
\[
- M_1 \text{Tr}(\Phi^\dagger \tau^a \Phi \tau^b) (UXU^\dagger)_{ab} - M_2 \text{Tr}(X^\dagger t^a X t^b) (UXU^\dagger)_{ab}
\]

9 parameters, 2 fixed by \(G_F\) and \(m_h\) \(\rightarrow\) 7 free parameters.  Aoki & Kanemura, 0712.4053

Chiang & Yagyu, 1211.2658; Chiang, Kuo & Yagyu, 1307.7526

Hartling, Kumar & HEL, 1404.2640

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Georgi-Machacek model  
Georgi & Machacek 1985; Chanowitz & Golden 1985

SM Higgs (bi-)doublet $+ \,$ triplets $(1, 0) + (1, 1)$ in a **bi-triplet**:

$$\Phi = \begin{pmatrix} \phi^0 & \phi^+ \\ -\phi^+ & \phi^0 \end{pmatrix} \quad X = \begin{pmatrix} \chi^0 & \xi^+ & \chi^{++} \\ -\chi^{++} & \xi^0 & \chi^+ \\ \chi^{+++} & -\xi^{++} & \chi^0 \end{pmatrix}$$

**Global SU(2)$_L \times$SU(2)$_R \rightarrow$ custodial symmetry $\langle \chi^0 \rangle = \langle \xi^0 \rangle \equiv v_\chi$**

**Physical spectrum:**

**Bi-doublet:** $2 \otimes 2 \rightarrow 1 \oplus 3$

**Bi-triplet:** $3 \otimes 3 \rightarrow 1 \oplus 3 \oplus 5$

- Two custodial singlets mix $\rightarrow h^0, \, H^0 \, m_h, \, m_H, \,$ angle $\alpha$

  Usually identify $h^0 = h(125)$

- Two custodial triplets mix $\rightarrow (H_3^+, H_3^0, H_3^-) \, m_3 +$ Goldstones

  Phenomenology very similar to $H^\pm, \, A^0$ in 2HDM Type I, $\tan \beta \rightarrow \cot \theta_H$

- Custodial fiveplet $(H_5^{++, H_5^+, H_5^0, H_5^-, H_5^{--}) \, m_5$

  Fermiophobic; $H_5 VV$ couplings $\propto s_H \equiv \sqrt{8} v_\chi / v_{\text{SM}}$

  $s_{H}^2 \equiv$ exotic fraction of $M^2_W, \, M^2_Z$

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Smoking-gun processes:

\[ \text{VBF} \to H^\pm \to W^\pm Z \quad \text{VBF + } q\ell\ell; \text{VBF + } 3\ell + \text{MET} \]

\[ \text{VBF} \to H^\pm \to W^\pm W^\pm \quad \text{VBF + like-sign dileptons + MET} \]

Cross section \( \propto s_H^2 \equiv \text{fraction of } M_W^2, M_Z^2 \) due to exotic scalars
Searches

SM VBF $\rightarrow W^{\pm}W^{\pm} \rightarrow \ell^{\pm}\ell^{\pm} + \text{MET}$ cross section measurement

ATLAS Run 1 1405.6241, PRL 2014

Recast to constrain VBF $\rightarrow H_{5}^{\pm\pm} \rightarrow W^{\pm}W^{\pm} \rightarrow \ell^{\pm}\ell^{\pm} + \text{MET}$

Chiang, Kanemura, Yagyu, 1407.5053
Searches

VBF $H_5^{\pm\pm} \rightarrow W^\pm W^\pm \rightarrow \ell^\pm \ell^\pm + \text{MET}$ (CMS Run 1)

CMS 1410.6315, PRL 2015

Translated using VBF $\rightarrow H^{\pm\pm}$ cross sections from LHCHXSWG-2015-001 (M. Zaro + HEL)

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Searches

**VBF** $H_5^± \rightarrow W^± Z \rightarrow ℓ^± ℓ^+ ℓ^- + \text{MET} \ (\text{ATLAS Run 2})$

<table>
<thead>
<tr>
<th>$m(H_2)$ (GeV)</th>
<th>ATLAS -1</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>0.1</td>
</tr>
<tr>
<td>300</td>
<td>0.2</td>
</tr>
<tr>
<td>400</td>
<td>0.3</td>
</tr>
<tr>
<td>500</td>
<td>0.4</td>
</tr>
<tr>
<td>600</td>
<td>0.5</td>
</tr>
<tr>
<td>700</td>
<td>0.6</td>
</tr>
<tr>
<td>800</td>
<td>0.7</td>
</tr>
<tr>
<td>900</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Stronger upper bound on $s_H$ for $m_5 \in (700, 900)$ GeV compared to $H_5^{±±}$

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Searches: $H_5^0 \rightarrow \gamma \gamma$ at low mass

Drell-Yan $pp \rightarrow H_5^0 H_5^{\pm}$ depends only on $m_5$ and gauge couplings!

If $W$ loop contribution dominates $H_5^0 \rightarrow \gamma \gamma, Z \gamma$, tree and loop decays scale the same way with $s_H$ and $m_5 \lesssim 110$ GeV is excluded.

Vega, Vega-Morales & Xie, 1805.01970

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Custodial symmetry violation in the GM model: a long history

Gunion, Vega & Wudka 1991 showed that computing the $T$ parameter in the GM model yields infinity due to an uncancelled UV divergence caused by hypercharge violating the custodial symmetry at 1-loop. Full gauge-invariant but $\text{SU}(2)_L \times \text{SU}(2)_R$-violating scalar potential yields the needed counterterm.

Englert, Re & Spannowsky 1302.6505 applied $S,T$ parameter constraints by subtracting a counterterm for $T$ (just the divergent part? not clear)

Chiang, Kuo & Yagyu 1804.02633 calculated 1-loop renormalized predictions for $h$ couplings in GM model and used measured $T$ parameter as input to fix the relevant custodial-symmetry-violating counterterm

Blasi, De Curtis & Yagyu 1704.08512 computed the RGEs and studied custodial violation from running up from custodial-symmetric theory at the weak scale (RGEs independently checked by us)

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Full gauge-invariant potential:

\[
V(\phi, \chi, \xi) = \tilde{\mu}_2^2 \phi^\dagger \phi + \tilde{\mu}_3^2 \chi^\dagger \chi + \frac{\tilde{\mu}_3^2}{2} \xi^\dagger \xi \\
+ \tilde{\lambda}_1 (\phi^\dagger \phi)^2 + \tilde{\lambda}_2 |\tilde{\chi}^\dagger \chi|^2 + \tilde{\lambda}_3 (\phi^\dagger \tau^a \phi)(\chi^\dagger t^a \chi) \\
+ \left[ \tilde{\lambda}_4 (\tilde{\phi}^\dagger \tau^a \phi)(\chi^\dagger t^a \xi) + \text{h.c.} \right] + \tilde{\lambda}_5 (\phi^\dagger \phi)(\chi^\dagger \chi) \\
+ \tilde{\lambda}_6 (\phi^\dagger \phi)(\xi^\dagger \xi) + \tilde{\lambda}_7 (\chi^\dagger \chi)^2 + \tilde{\lambda}_8 (\xi^\dagger \xi)^2 \\
+ \tilde{\lambda}_9 |\chi^\dagger \xi|^2 + \tilde{\lambda}_{10} (\chi^\dagger \chi)(\xi^\dagger \xi) \\
- \frac{1}{2} \left[ \tilde{M}_1 \phi^\dagger \Delta_2 \tilde{\phi} + \text{h.c.} \right] + \frac{\tilde{M}_1}{\sqrt{2}} \phi^\dagger \Delta_0 \phi - 6 \tilde{M}_2 \chi^\dagger \overline{\Delta}_0 \chi
\]

where

\[
\Delta_2 \equiv \sqrt{2} \tau^a U_{ai} \chi_i = \begin{pmatrix}
\chi^+/\sqrt{2} & -\chi^{++} \\
\chi^0 & -\chi^+/\sqrt{2}
\end{pmatrix},
\]

\[
\Delta_0 \equiv \sqrt{2} \tau^a U_{ai} \xi_i = \begin{pmatrix}
\xi^0/\sqrt{2} & -\xi^+ \\
-\xi^{++} & -\xi^0/\sqrt{2}
\end{pmatrix},
\]

\[
\overline{\Delta}_0 \equiv -t^a U_{ai} \xi_i = \begin{pmatrix}
-\xi^0 & \xi^+ & 0 \\
\xi^{++} & 0 & \xi^+
\end{pmatrix}.
\]

Minimize potential, compute mass matrices, etc.
16 Lagrangian parameters compared to 9 in original GM model: Matching gauge-invariant potential to original GM model yields

\[
\begin{align*}
\tilde{\mu}_2^2 &= \mu_2^2 \\
\tilde{\mu}_3^2 &= \mu_3^2 \\
\tilde{\mu}_3^2 &= \mu_3^2 \\
\tilde{\lambda}_1 &= 4\lambda_1 \\
\tilde{\lambda}_2 &= 2\lambda_3 \\
\tilde{\lambda}_3 &= -2\lambda_5 \\
\tilde{\lambda}_4 &= -\sqrt{2}\lambda_5 \\
\tilde{\lambda}_5 &= 4\lambda_2 \\
\tilde{\lambda}_6 &= 2\lambda_2 \\
\tilde{\lambda}_7 &= 2\lambda_3 + 4\lambda_4 \\
\tilde{\lambda}_8 &= \lambda_3 + \lambda_4 \\
\tilde{\lambda}_9 &= 4\lambda_3 \\
\tilde{\lambda}_{10} &= 4\lambda_4 \\
\tilde{M}_1' &= M_1 \\
\tilde{M}_1 &= M_1 \\
\tilde{M}_2 &= M_2
\end{align*}
\]

RGEs with \( g' = 0 \) preserve these relations.

Keeping \( g' \neq 0 \) violates these relations and introduces custodial symmetry violation through the RGE running.

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Our implementation

Basic idea:

- Assume custodial symmetry at some high scale $\Lambda$
  (accidental SU(2)$_L \times$SU(2)$_R$ coming from UV completion e.g. composite Higgs)

- Run down to weak scale $\Rightarrow$ custodial violation generated
  (1-loop RGEs, tree-level matching $\equiv$ leading log approximation)
  Measured value of $\rho$ will put an upper bound on scale $\Lambda$

- Subject to $\rho$ constraint (and perturbativity at $\Lambda$), quantify maximum allowed custodial symmetry violation and its phenomenological consequences
Our implementation

Details:

- Start with a benchmark scenario at the weak scale* (for concreteness, and to get \( G_F, m_h \) close to their correct values)
  *“weak scale” = \( m_5 \)

### H5plane benchmark
(introduced by HXSWG for \( H_5 \) LHC searches)

<table>
<thead>
<tr>
<th>Fixed Parameters</th>
<th>Variable Parameters</th>
<th>Dependent Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>( G_F = 1.1663787 \times 10^{-5} \text{ GeV}^{-2} )</td>
<td>( m_5 \in [200, 3000] \text{ GeV} )</td>
<td>( \lambda_2 = 0.4m_5/(1000 \text{ GeV}) )</td>
</tr>
<tr>
<td>( m_h = 125 \text{ GeV} )</td>
<td>( s_H \in (0, 1) )</td>
<td>( M_1 = \sqrt{2}s_H(m_5^2 + v^2)/v )</td>
</tr>
<tr>
<td>( \lambda_3 = -0.1 )</td>
<td></td>
<td>( M_2 = M_1/6 )</td>
</tr>
<tr>
<td>( \lambda_4 = 0.2 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Run up with \( g' = 0 \) (custodial symmetric!) to some scale \( \Lambda \); check perturbativity of quartic couplings (avoid Landau pole)
  ⇒ upper bound on \( \Lambda \) to avoid perturbativity violation

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Our implementation

Details (continued):

- Run back down with $g' \neq 0$ to get the custodial-violating Lagrangian parameters at the weak scale

- Compute vevs $\rightarrow G_F$ and mass matrices $\rightarrow m_h$; adjust original weak-scale inputs and iterate until these match experiment in custodial violating theory

- Compute $\rho$; adjust upper bound on $\Lambda$ if necessary
  
  $\rho = 1.00037 \pm 0.00023$ (2016 PDG) [require within $\pm 2\sigma$]

- Compute weak-scale predictions for custodial-violating observables ($\lambda^h_{WZ}$, mass splittings, mixings)
Results (within H5plane benchmark): cutoff scale

Left: Scale of Landau pole

Right: Highest scale at which perturbative unitarity constraints on custodial-symmetric \( \lambda_i \) remain satisfied

UV completion must appear below 10s to 100s of TeV

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Results (within H5plane benchmark): $\rho$ parameter

Left: Maximum cutoff scale including $\rho$ parameter constraint (dashed) + perturbative unitarity (solid)

Right: Weak-scale value of $\rho$, for $\Lambda$ as large as possible

$\rho$ samples full $2\sigma$ allowed range

$\Delta \rho$ is positive in most of H5plane benchmark parameter space

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Results (within H5plane benchmark): $hWW/hZZ$

Test of custodial symmetry violation in $h_{125}$ couplings:

$$\lambda_{hWZ}^h \equiv \frac{\kappa_{hW}^h}{\kappa_{hZ}^h}$$

Plot: $\delta\lambda_{hWZ}^h \equiv \lambda_{hWZ}^h - 1$

for $\Lambda$ as large as possible

Deviation from SM prediction ($\lambda_{hWZ}^h = 1$) at most half a percent

Current LHC precision: $\lambda_{hWZ}^h = 0.88^{+0.10}_{-0.09}$ ATLAS + CMS Run 1, 1606.02266

Future precision based on $hWW$ and $hZZ$ projections:
HL-LHC: a few percent
ILC: about half a percent
FCC-ee: about 0.2 percent

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Results (within H5plane benchmark): mass splittings

Plot: \( m_{H_3^\pm} - m_{H_3^0} \)
for \( \Lambda \) as large as possible
(negative values: \( H_3^\pm \) is lighter)

Custodial-violating mass splitting of \( H_3^0, \pm \) is at most 5.3 GeV.
\( m_{H_3^0} > m_{H_3^\pm} \) everywhere in H5plane benchmark.

Measurement prospects: \( H_3^0 \rightarrow b\bar{b}, t\bar{t}; \ H_3^+ \rightarrow t\bar{b} \)
Couplings as in Type-I 2HDM: down-type decays not enhanced
Mass splitting too small to detect at LHC

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Results (within H5plane benchmark): mass splittings

Left:  $m_{H^{±±}_5} - m_{H^0_5}$

Right:  $m_{H^{±}_5} - m_{H^0_5}$

for Λ as large as possible

Custodial-violating mass splitting of $H^{0,±,±±}_5$ is at most 7.2 GeV. $m_{H^{±±}_5} > m_{H^{±}_5} > m_{H^0_5}$ everywhere in H5plane benchmark.

Decays are to $VV$ – similar challenges to detect small mass splittings at LHC.

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Results (within H5plane benchmark): mixing

Custodial sym violation mixes doublet into fermiophobic $H_5$

Left: induced $\kappa_f$ for $H_5^0$
for $\Lambda$ as large as possible

Right: BR($H_5^0 \rightarrow f \bar{f}$)

Custodial-violation-induced BR of $H_5^0$ to fermions ($t\bar{t}$) reaches almost half a percent in H5plane benchmark ($m_5 > 200$ GeV).
Effect at low mass < $2M_W$ may be much more interesting:
competition with powerful $H_5^0 \rightarrow \gamma\gamma$ channel $\Rightarrow$ future work

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Results (within H5plane benchmark): mixing

Custodial sym violation mixes doublet into fermiophobic $H_5$

Left: induced $\kappa_f$ for $H_5^\pm$ for $\Lambda$ as large as possible

Right: $\text{BR}(H_5^+ \rightarrow f \bar{f})$

Custodial-violation-induced BR of $H_5^+$ to fermions ($t\bar{b}$) reaches 1.2% in H5plane benchmark ($m_5 > 200$ GeV).

At low mass $< M_W + M_Z$ this can compete with $W\gamma$ decay.

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Conclusions and outlook

Hypercharge interactions violate custodial symmetry in the GM model beyond tree level.

We studied the impact of this assuming a custodial-symmetric theory at some high scale \( \Lambda \) and running down to the weak scale.

For this first pass we used the H5plane benchmark \((m_5 > 200 \text{ GeV})\)

Main results:
- **UV completion must lie below 10s to 100s of TeV**
  forced by perturbative unitarity + measured \( \rho \) parameter
- **Custodial-violating effects are small!** (too small to see at LHC)
  assumption of custodial-symmetric GM is good for LHC searches

Custodial-violation-induced fermion couplings of otherwise fermiophobic \( H_5 \) may become important for masses below \( 2M_W \), where tree-level decays go offshell and loop decays become important.

Competition with powerful \( H_5^0 \to \gamma\gamma \) channel? \( \Rightarrow \) future work

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BACKUP SLIDES
Most general $SU(2)_L \times SU(2)_R$-invariant scalar potential:  
Aoki & Kanemura, 0712.4053
Chiang & Yagyu, 1211.2658; Chiang, Kuo & Yagyu, 1307.7526
Hartling, Kumar & HEL, 1404.2640

$$V(\Phi, X) = \frac{\mu_2^2}{2} \text{Tr}(\Phi^\dagger \Phi) + \frac{\mu_3^2}{2} \text{Tr}(X^\dagger X) + \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2$$
$$+ \lambda_2 \text{Tr}(\Phi^\dagger \Phi) \text{Tr}(X^\dagger X) + \lambda_3 \text{Tr}(X^\dagger XX^\dagger X)$$
$$+ \lambda_4 [\text{Tr}(X^\dagger X)]^2 - \lambda_5 \text{Tr}(\Phi^\dagger \tau^a \Phi \tau^b) \text{Tr}(X^\dagger t^a X t^b)$$
$$- M_1 \text{Tr}(\Phi^\dagger \tau^a \Phi \tau^b) (UXU^\dagger)_{ab} - M_2 \text{Tr}(X^\dagger t^a X t^b) (UXU^\dagger)_{ab}$$

9 parameters, 2 fixed by $M_W$ and $m_h \to$ free parameters are $m_H, m_3, m_5, v_\chi, \alpha$ plus two triple-scalar couplings.

**Dimension-3 terms usually omitted by imposing $Z_2$ sym. on $X$.**

These dim-3 terms are essential for the model to possess a decoupling limit!

$(UXU^\dagger)_{ab}$ is just the matrix $X$ in the Cartesian basis of SU(2), found using

$$U = \begin{pmatrix}
\frac{-1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\
\frac{-i}{\sqrt{2}} & 0 & \frac{-i}{\sqrt{2}} \\
\frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2}
\end{pmatrix}$$
Fields and vevs for full gauge-invariant potential:

\[
\phi = \begin{pmatrix} \phi^+ \\ \phi_0 \end{pmatrix}, \quad \chi = \begin{pmatrix} \chi^{++} \\ \chi^+ \\ \chi^0 \end{pmatrix}, \quad \xi = \begin{pmatrix} \xi^+ \\ \xi^0 \\ -\xi^{++} \end{pmatrix},
\]

(1)

\[
\tilde{\phi} \equiv C_2 \phi^* = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \phi^* = \begin{pmatrix} \phi^{0*} \\ -\phi^{+*} \end{pmatrix},
\]

\[
\tilde{\chi} \equiv C_3 \chi^* = \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \chi^* = \begin{pmatrix} \chi^{0*} \\ -\chi^{++*} \\ \chi^{+++*} \end{pmatrix}.
\]

(2)

\[
\phi^0 \rightarrow \frac{\tilde{v}_\phi}{\sqrt{2}} + \frac{\phi^{0,r} + i\phi^{0,i}}{\sqrt{2}}, \quad \chi^0 \rightarrow \tilde{v}_\chi + \frac{\chi^{0,r} + i\chi^{0,i}}{\sqrt{2}}, \quad \xi^0 \rightarrow \tilde{v}_\xi + \xi^{0,r}.
\]

(3)

\[
\rho = \frac{\tilde{v}_\phi^2 + 4\tilde{v}_\chi^2 + 4\tilde{v}_\xi^2}{\tilde{v}_\phi^2 + 8\tilde{v}_\chi^2} = \frac{v^2}{v^2 + 4(\tilde{v}_\chi^2 - \tilde{v}_\xi^2)}.
\]

(4)