



How broken symmetries leak into everything: a tale of CP violation in the two-Higgs-doublet model

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C.H. de Lima & HEL, arXiv:2409.10603, 2403.17052



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Outline

CP and CP violation

Introducing the two-Higgs-doublet model

Radiative corrections and renormalizability

Ingredients for CP violation & our analysis

Practical consequences?

Conclusions

EDI advertisement

What is CP?

1) Parity operation (P): inversion of coordinate system

$$ec{r}
ightarrow -ec{r}$$
, $ec{p}
ightarrow -ec{p}$

$$\vec{L} = \vec{r} imes \vec{p} o + \vec{L}$$
 so $\vec{L} \cdot \vec{p} o - \vec{L} \cdot \vec{p}$ (helicity)

Up to 1956, parity was thought to be preserved in all physical processes – had been verified in gravitational, electromagnetic, and strong interactions, but not weak interactions (as flagged by Tsung-Dao Lee and Chen-Ning Yang)

Experimental test done by Chien-Shiung Wu (1957) in beta decay of cryogenic spin-polarized Cobalt-60 nuclei: demonstrated (maximal!) parity violation in weak interactions!

Discovery also explained how the weak-interaction decays $K^+ \rightarrow$ $\pi^+\pi^0$ (P-even) and $K^+\to\pi^+\pi^+\pi^-$ (P-odd) can coexist.

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What is CP?

2) Charge conjugation operation (C): swap each particle for its antiparticle

Weak interactions instead conserve the combined operation of charge conjugation and parity (CP): exchange each particle for its antiparticle and also flip all the helicities...

...or so it was thought for an entire 7 years until CP was also experimentally demonstrated to be (slightly!) violated in weakinteraction processes!

CP Violation

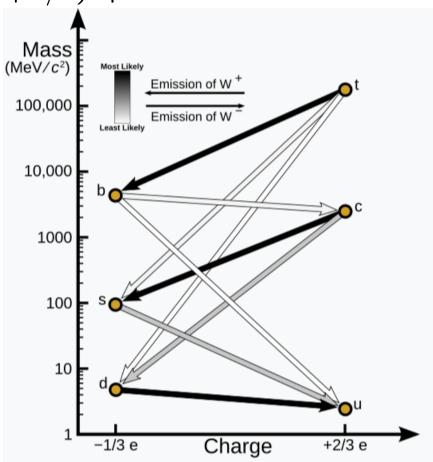
CP violation discovered 1964 (Cronin & Fitch) at the Alternating Gradient Synchrotron at Brookhaven National Lab (New York)

Neutral kaons: $K_S \to \pi\pi$ CP even $(c\tau \simeq 2.7 \text{ cm})$; $K_L \to 3\pi$ CP odd $(c\tau \simeq 15 \text{ m})$; but K_L also decays to $\pi\pi$ about 0.3% of the time!

Explained in the Standard Model by 3-generation CKM matrix (Kobayashi & Maskawa 1973); well-established by measurements at the "B-factories" BaBar and Belle during the '00s.

CP Violation in the Standard Model

CKM matrix V is a 3×3 unitary matrix that describes the mismatch between the mass eigenstates of the up-type (charge +2/3) quarks versus the down-type (charge -1/3) quarks.



Elements of V describe relative strengths of weak transitions.

Unitary 3×3 matrix: can absorb all but 3 angles and one complex phase into unphysical redefinitions of quark fields.

This complex phase is the source of the CP violation.

Niamh O'C, Wikipedia, CC BY-SA 3.0

Two Higgs Doublet Model (2HDM)

Add a second "doublet" of Higgs bosons to the one in the SM.

SM:
$$\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$
 \rightarrow physical Higgs boson h^0 ; extra bits of massive W^{\pm}, Z

2HDM:
$$\begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix}$$
, $\begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix}$ \rightarrow physical Higgs bosons h^0, H^0, A^0, H^\pm ; extra bits of massive W^\pm, Z

Have to restrict the doublets' interactions with fermions to avoid messing up CKM picture; easy to do using an extra symmetry.

Describe Higgs masses & interactions with the Higgs potential:

$$\begin{split} V &= m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left[m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right] \\ &+ \frac{1}{2} \lambda_1 \left(\Phi_1^\dagger \Phi_1 \right)^2 + \frac{1}{2} \lambda_2 \left(\Phi_2^\dagger \Phi_2 \right)^2 + \lambda_3 \left(\Phi_1^\dagger \Phi_1 \right) \left(\Phi_2^\dagger \Phi_2 \right) + \lambda_4 \left| \Phi_1^\dagger \Phi_2 \right|^2 \\ &+ \left\{ \frac{1}{2} \lambda_5 \left(\Phi_1^\dagger \Phi_2 \right)^2 + \text{h.c.} \right\}. \end{split}$$

QM: must be a Hermitian operator!

Two Higgs Doublet Model (2HDM)

New source of CP violation: relative phase of m_{12}^2 and $\lambda_5!$

$$V = m_{11}^{2} \Phi_{1}^{\dagger} \Phi_{1} + m_{22}^{2} \Phi_{2}^{\dagger} \Phi_{2} - \left[m_{12}^{2} \Phi_{1}^{\dagger} \Phi_{2} + \text{h.c.} \right]$$

$$+ \frac{1}{2} \lambda_{1} \left(\Phi_{1}^{\dagger} \Phi_{1} \right)^{2} + \frac{1}{2} \lambda_{2} \left(\Phi_{2}^{\dagger} \Phi_{2} \right)^{2} + \lambda_{3} \left(\Phi_{1}^{\dagger} \Phi_{1} \right) \left(\Phi_{2}^{\dagger} \Phi_{2} \right) + \lambda_{4} \left| \Phi_{1}^{\dagger} \Phi_{2} \right|^{2}$$

$$+ \left\{ \frac{1}{2} \lambda_{5} \left(\Phi_{1}^{\dagger} \Phi_{2} \right)^{2} + \text{h.c.} \right\}.$$

Usual approach: choose m_{12}^2 real; then ${\rm Im}(\lambda_5)$ contains the CPV; constrained by electron's electric dipole moment (upper bound): $|d_e| < 4.1 \times 10^{-30} e\,{\rm cm}$ (JILA 2022).

In the 2HDM, this is uncomfortably fine-tuned: even for quite heavy extra Higgs boson masses $M \sim \text{TeV}$, imaginary part of λ_5 has to be $\lesssim 10^{-2}$ while real part can be $\mathcal{O}(1)$.

$$\left(\frac{1 \text{ TeV}}{M}\right)^2 \operatorname{Im}(\lambda_5) \times f(\sin^2\beta, \cos^2\beta) \lesssim 0.5 - 1\%$$

Altmannshofer, Gori, Hamer, & Patel, 2020

Avoid the problem by imposing CP: force m_{12}^2 and λ_5 to be real!

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Real Two Higgs Doublet Model (2HDM)

Very nice model; 3–4 decades of theoretical studies and experimental searches

Prototype model for LHC searches for charged Higgs bosons & additional neutral Higgs bosons, and for model-specific fits of Higgs coupling measurements

Well-established infrastructure of theoretical computer codes for calculating spectrum, theoretical & experimental constraints, & decay predictions, including radiative corrections up to 2 loops

But... is it consistent?

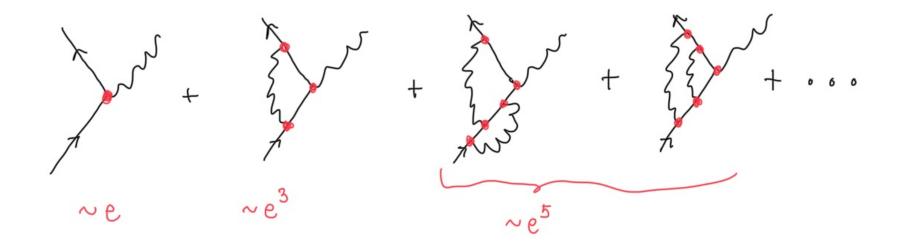
D. Fontes, M. Löschner, J.C. Romão, & J.P. Silva, 2103.05002 (EPJC)

- We know that CP is violated in the weak interactions of quarks.
- Known feature of quantum field theory: such symmetry violation must eventually propagate to all parts of the theory via radiative corrections.

Radiative corrections and renormalization

Interacting quantum field theories can almost never be solved exactly.

Instead, use perturbation theory to calculate order-by-order in powers of the interaction coupling.



Radiative corrections and renormalization

Loop diagrams are computed using Feynman's "sum over histories" method: integrate over any momenta that aren't fixed by external kinematics, and sum up all contributing diagrams.



Complication: the momentum integrals can be divergent!

$$\int d^4k \, \frac{1}{k^2} \, \frac{k^{\mu}}{k^2 - m^2} \, \frac{k_{\mu}}{k^2 - m^2} \sim \int d^4k \, \frac{1}{k^4} \sim \int d\Omega \int^{\Lambda} k^3 dk \, \frac{1}{k^4} \sim \ln \Lambda \to \infty$$

The divergence comes from the high-momentum \leftrightarrow short-distance part of the integral.

Radiative corrections and renormalization

But we don't measure parameters like e at infinitely short distances; we measure them "in the lab" (specifically in the "Thomson limit" of zero momentum transfer).

→ Re-express the calculation in terms of *measurable* input processes: all the divergences cancel in physical predictions! (theory is "renormalizable")

Organize the math by defining $e_0 = e + \delta e$: e_0 is the "bare coupling" that appears in the Lagrangian. e is the "renormalized coupling" that we measure physically. δe is the "counterterm" that cancels the divergence in calculations of renormalized predictions.

Finite parts of the calculation are meaningful and can be tested (e.g. electron g-2 vs. Rydberg constant: good to $1/10^8$).

Problem with the Real 2HDM

We get the Real 2HDM by imposing CP invariance on m_{12}^2 and λ_5 (i.e., requiring them to be real): removes a degree of freedom.

$$V = m_{11}^{2} \Phi_{1}^{\dagger} \Phi_{1} + m_{22}^{2} \Phi_{2}^{\dagger} \Phi_{2} - \left[m_{12}^{2} \Phi_{1}^{\dagger} \Phi_{2} + \text{h.c.} \right]$$

$$+ \frac{1}{2} \lambda_{1} \left(\Phi_{1}^{\dagger} \Phi_{1} \right)^{2} + \frac{1}{2} \lambda_{2} \left(\Phi_{2}^{\dagger} \Phi_{2} \right)^{2} + \lambda_{3} \left(\Phi_{1}^{\dagger} \Phi_{1} \right) \left(\Phi_{2}^{\dagger} \Phi_{2} \right) + \lambda_{4} \left| \Phi_{1}^{\dagger} \Phi_{2} \right|^{2}$$

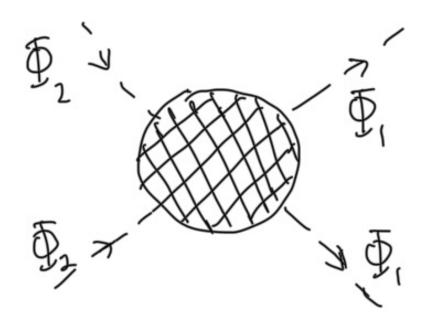
$$+ \left\{ \frac{1}{2} \lambda_{5} \left(\Phi_{1}^{\dagger} \Phi_{2} \right)^{2} + \text{h.c.} \right\}.$$

If there are divergent radiative corrections to the operators $(\Phi_1^{\dagger}\Phi_2)^2$ or $\Phi_1^{\dagger}\Phi_2$ with imaginary parts, we are now in big trouble because there is no counterterm available to cancel the divergence!

- Problem first flagged by D. Fontes et al., arXiv:2103.05002, but their state-of-the-art 3-loop calculation found no imaginary divergent contribution.
- We demonstrated that imaginary divergent contributions do show up, but that they first appear at 7 loops.

Ingredients for CP Violation (& our analysis)

We want to identify the simplest possible diagrams that can contribute an imaginary divergent correction to $\left(\Phi_1^\dagger\Phi_2\right)^2$.



First ingredient: need CP violation! Get it from the known CP violation in the quark mass matrices.

Ingredients for CP Violation: Jarlskog invariant

Reparameterization-invariant measure of the CP violation in the CKM matrix Jarlskog, ZPhysC, PRL 1985

$$J = \left| \operatorname{Im}(V_{\alpha i} V_{\beta j} V_{\alpha j}^* V_{\beta i}^*) \right|, \qquad (\alpha \neq \beta, i \neq j)$$

More useful here to express it in terms of the quark couplings to the Higgs fields that give rise to the mass matrices.

Define combinations of quark coupling (Yukawa) matrices:

$$\widehat{H}_u = Y_u Y_u^{\dagger} \qquad \widehat{H}_d = Y_d Y_d^{\dagger}$$

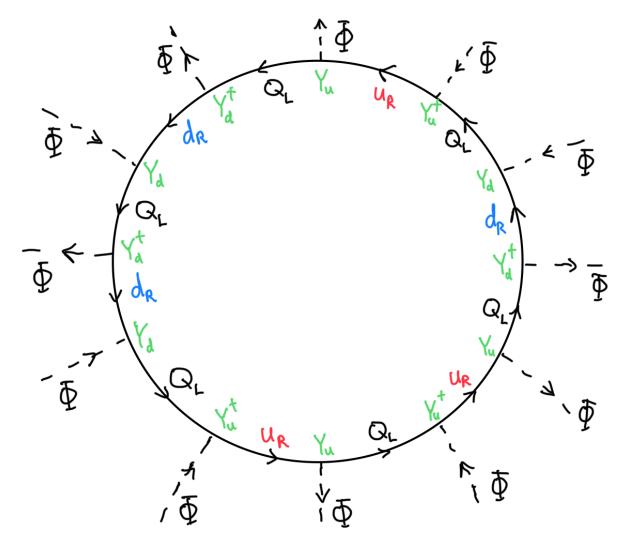
Minimal combination that yields an imaginary part involves 12 powers of Y's: Botella & Silva, PRD 1995

$$\mathcal{J} = \operatorname{Tr}\left(\widehat{H}_u \widehat{H}_d \widehat{H}_u^2 \widehat{H}_d^2\right)$$
 where $\operatorname{Im}(\mathcal{J}) \propto J$

 \Rightarrow Draw the simplest loop diagrams that involve $\mathcal J$ or $\mathcal J^*$.

(Focus on "Type I" 2HDM. We also do "Type II" in our paper.)

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12 Yukawa insertions \Rightarrow 12 scalar "legs". Connect 8 incoming and outgoing scalars so that only 4 "legs" are left: another 4 loops. \Rightarrow 5 loops total so far.

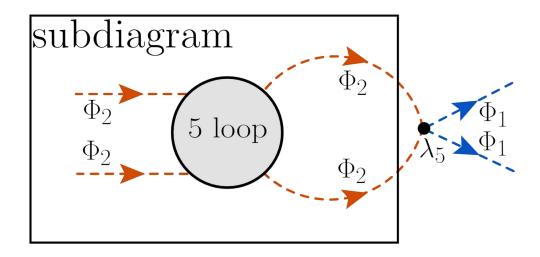
Type I:

only Φ_2 couples to quarks:

6 incoming Φ_2 's 6 outgoing Φ_2 's

But want $(\Phi_1^{\dagger}\Phi_2)^2$: need to convert two outgoing Φ_2 's into Φ_1 's!

Can do this by inserting a λ_5 vertex. Second ingredient!



\Rightarrow Creates a 6th loop.

Deeper understanding:

The 2HDM with $\lambda_5=0$ has an extra symmetry which is obeyed by all parts of the theory. This guarantees that loop diagrams cannot generate any divergent contributions to λ_5 , either real or imaginary. So any such contributions have to "know about" nonzero λ_5 .

Type I:

only Φ_2 couples to quarks:

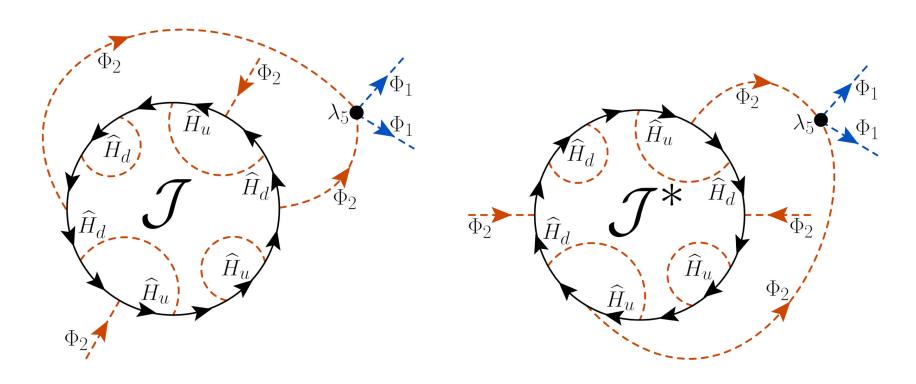
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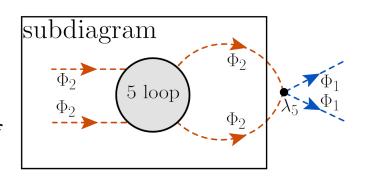
Have to also consider the complex conjugate diagrams: reverse the direction of all quark and Φ_2 arrows (replaces \mathcal{J} with \mathcal{J}^*) and move the λ_5 insertion to the other pair of Φ_2 legs.

The only structural difference between these diagrams is that the injection of momentum flowing in the 6th loop happens in a different place. Does the imaginary part cancel?



Our analysis goes as follows.

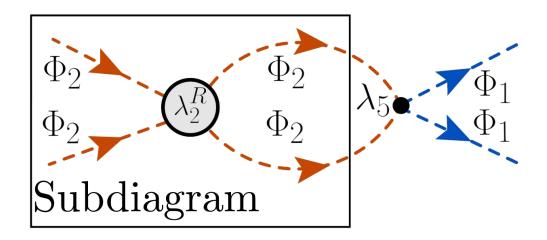
- Treat the sum of all 5-loop subdiagrams proportional to \mathcal{J} as a formfactor, which depends on the momenta of its four legs.



- Any parts of this formfactor that will later contribute to divergences of the 6-loop diagram must be an analytic dimensionless function of Lorentz-invariant combinations of the momenta of its four legs.
- If this formfactor has a piece which is antisymmetric when the relevant momenta are swapped, then the imaginary divergence will not cancel. Ex: $(k_a^2 k_b^2)/fn(k_i^2)$
- But an antisymmetric piece of a dimensionless formfactor lacks a well-defined zero-momentum limit: unphysical, must = 0!

Deeper understanding:

The formfactor for the 5-loop subdiagram is "actually" just the renormalization of the operator $(\Phi_2^{\dagger}\Phi_2)^2$, which is hermitian and thus cannot acquire an imaginary part (divergent or otherwise) in the zero-momentum limit.



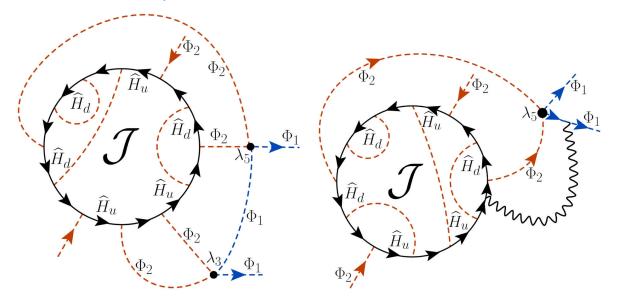
The coefficient of this operator being dimensionless guarantees that momentum dependence cannot circumvent this conclusion.

We need a third ingredient to break the subdiagram structure.

Ingredients for CP Violation: Breaking the subdiagram structure

To break the subdiagram structure, have to attach something to both the 5-loop formfactor and an external Φ_1 leg.

⇒ Creates a 7th loop!



Limited possibilities: $\lambda_3 \left(\Phi_1^{\dagger} \Phi_1 \right) \left(\Phi_2^{\dagger} \Phi_2 \right)$ or $\lambda_4 \left| \Phi_1^{\dagger} \Phi_2 \right|^2$, or an SU(2) or U(1) gauge boson \rightarrow Predict the parameter dependence of the imaginary divergence!

$$\operatorname{Im}(\mathcal{J}) \lambda_5 \left\{ b_1 \lambda_3 + b_2 \lambda_4 + b_3 g'^2 + b_4 g^2 \right\} / (16\pi^2)^7$$

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Practical consequences?

The coefficient of the divergence also controls the energy-scale dependence of the associated "running" coupling ("renormalizationgroup equation").

Can estimate the size of the imaginary part if one assumes that it is zero (for some unknown reason) at the scale of quantum gravity (where particle physicists normally stash our ignorance): get $\text{Im}(\lambda_5) \sim 10^{-22}$ (versus $\text{Re}(\lambda_5) \sim 1$).

So tiny that it has no conceivable effect! Can continue to use the Real 2HDM... but have to accept that the question of why $Im(\lambda_5) = 0$ at the Planck scale is not answered.

Probably better to accept that the 2HDM is more likely to be complex (and somewhat tuned) than approximately real.

Conclusions

A principle of quantum field theory (and physics in general): You can't truly preserve a symmetry in one part of a theory if it is violated in another (interacting) part of the theory.

In some cases, the transmitted symmetry violation is small and calculable (nice).

In others (like the Real 2HDM), the transmitted symmetry violation is divergent and setting it to be zero "by hand" is totally artificial.

By carefully following the symmetries, it's possible to pin down at what order the divergent symmetry violation occurs and what parameters it must depend on, even when doing the full calculation is not technically feasible.

Equity/Diversity/Inclusion Advertisement:

Guide on "How to make your research group more inclusive for autistic trainees" (by me)

- 6 pages, \lesssim 15-minute read
- concrete, actionable steps and up-to-date information
- lots of references
- includes a poster

arXiv:2410.17929 \rightarrow



BACKUP SLIDES

CP Violation

CP violation discovered 1964 (Cronin & Fitch) at the Alternating Gradient Synchrotron at Brookhaven National Lab (New York)

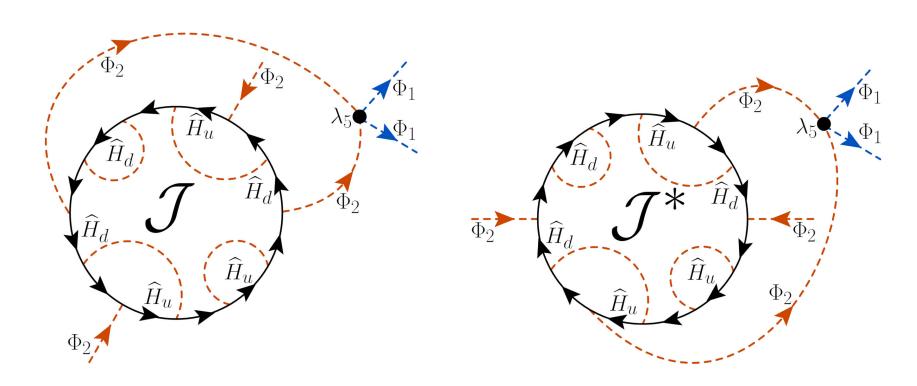
Neutral kaons: $K_S \to \pi\pi$ CP even $(c\tau \simeq 2.7 \text{ cm})$; $K_L \to 3\pi$ CP odd $(c\tau \simeq 15 \text{ m})$; but K_L also decays to $\pi\pi$ about 0.3% of the time!

Explained in the Standard Model by 3-generation CKM matrix (Kobayashi & Maskawa 1973); quantitatively established by the "B-factories" BaBar and Belle during the '00s.

CPV is one of the key ingredients needed to dynamically give rise to the baryon asymmetry of the universe (Sakharov 1967), but there's not enough CPV in the SM to achieve observed asymmetry. → Beyond-the-SM sources? Most new sources of CPV are severely constrained by limits on electric dipole moments (EDMs) of electron and various nuclei: have to be careful.

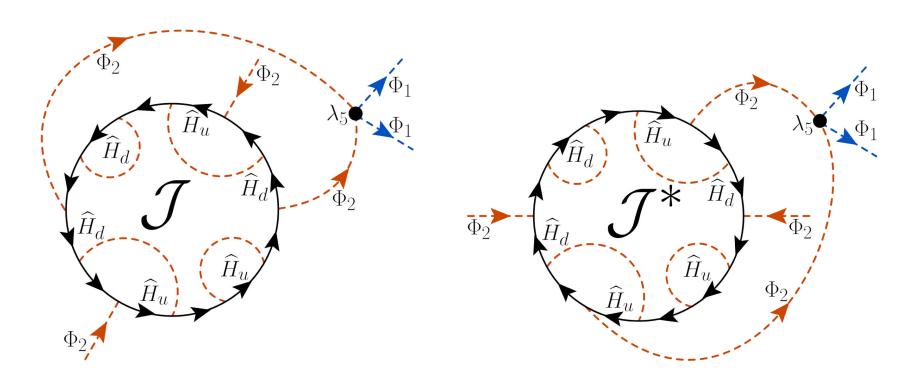
But is this enough?

Have to also consider the contribution of the complex conjugate diagrams: reverse the direction of all quark and Φ_2 arrows (this replaces \mathcal{J} with \mathcal{J}^*) and move the λ_5 insertion to the other pair of Φ_2 legs. E.g.:



If each such pair of diagrams give an identical divergent contribution, then the imaginary part cancels in their sum: no CP violation!

The only structural difference between these diagrams is that the injection of momentum flowing in the 6th loop happens in a different place. How to get a handle on this?



In the SM Lagrangian there are very few "opportunities" for CP violation: need operators that are not self-Hermitian.

- The quark mixing matrix V_{CKM} : 2 × 2 not enough (phases can all be rotated away by field redefinitions); in 3×3 one physical CPV phase remains \rightarrow original motivation for 3 quark generations
- $G^{\mu\nu}\tilde{G}_{\mu\nu}$ operator (strong interaction): Strong CP problem coefficient of this operator constrained by neutron EDM to be $< 10^{-10}$. Very fine tuned! \rightarrow most popular solution is Peccei-Quinn axion; beyond the scope of this talk.
- Massive neutrinos (technically BSM): 3×3 lepton mixing matrix (PMNS) has its own CPV phase; also possibility for two additional Majorana phases.

Beyond the SM, any term in the Lagrangian that is not self-Hermitian is a new possible source of CP violation.

$$\mathcal{L} \supset \left\{ C_i \mathcal{O}_i + C_i^* \mathcal{O}_i^{\dagger} \right\}$$

- + opportunity to explain baryon asymmetry of the universe!
- generally strongly constrained by EDMs \rightarrow fine-tuning
- → Consider the two Higgs doublet model (2HDM)

The most general gauge-invariant scalar potential for the 2HDM:

$$V = m_{11}^{2} \Phi_{1}^{\dagger} \Phi_{1} + m_{22}^{2} \Phi_{2}^{\dagger} \Phi_{2} - \left[m_{12}^{2} \Phi_{1}^{\dagger} \Phi_{2} + \text{h.c.} \right]$$

$$+ \frac{1}{2} \lambda_{1} \left(\Phi_{1}^{\dagger} \Phi_{1} \right)^{2} + \frac{1}{2} \lambda_{2} \left(\Phi_{2}^{\dagger} \Phi_{2} \right)^{2} + \lambda_{3} \left(\Phi_{1}^{\dagger} \Phi_{1} \right) \left(\Phi_{2}^{\dagger} \Phi_{2} \right) + \lambda_{4} \left| \Phi_{1}^{\dagger} \Phi_{2} \right|^{2}$$

$$+ \left\{ \frac{1}{2} \lambda_{5} \left(\Phi_{1}^{\dagger} \Phi_{2} \right)^{2} + \left[\lambda_{6} \left(\Phi_{1}^{\dagger} \Phi_{1} \right) + \lambda_{7} \left(\Phi_{2}^{\dagger} \Phi_{2} \right) \right] \left(\Phi_{1}^{\dagger} \Phi_{2} \right) + \text{h.c.} \right\}$$

(10 parameters, 4 of them complex)

Yukawa Lagrangian: two copies of that of the SM:

$$\mathcal{L}_{Yuk} = -Y_{ij}^{d1} \bar{Q}_{Li} \Phi_1 d_{Rj} - Y_{ij}^{u1} \bar{Q}_{Li} \tilde{\Phi}_1 u_{Rj} - Y_{ij}^{\ell1} \bar{L}_{Li} \Phi_1 e_{Rj} + \text{h.c.}$$
$$-Y_{ij}^{d2} \bar{Q}_{Li} \Phi_2 d_{Rj} - Y_{ij}^{u2} \bar{Q}_{Li} \tilde{\Phi}_2 u_{Rj} - Y_{ij}^{\ell2} \bar{L}_{Li} \Phi_2 e_{Rj} + \text{h.c.}$$

Rotating to the fermion mass basis diagonalizes only the combinations $(Y^{u1}v_1 + Y^{u2}v_2)$, etc.; orthogonal combinations are not diagonal, source of FCNC and additional CPV.

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CPV leaks into 2HDM

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Sidestep the FCNC problem by imposing Natural Flavour Conservation (Glashow & Weinberg, 1977): Arrange for fermions of each different electric charge to couple to exactly one Higgs doublet.

Easy to impose using a Z_2 symmetry: $\Phi_1 \rightarrow -\Phi_1$, $\Phi_2 \rightarrow \Phi_2$

	u_R	d_R	e_R
Type I	+	+	+
Type II	+	_	_
Type X	+	+	_
Type Y	+	_	+

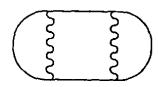
Also eliminates λ_6 , λ_7 , and m_{12}^2 ; can then absorb phase of λ_5 into unphysical rephasing of fields. No CPV in scalar potential!

Exact \mathbb{Z}_2 : trade m_{11}^2 and m_{22}^2 for Higgs vevs after EWSB; upper bound on all scalar masses $\sim \mathcal{O}(700 \text{ GeV})$. Types II, X, and Y fully excluded by global fit including LHC data (Chowdhury & Eberhardt, 2017)

Motivation

Fontes et al.'s argument:

- We know there is CP violation in the CKM matrix.
- CKM CPV can be transmitted to other operators via loop diagrams — e.g., contribution to Weinberg operator $f^{abc} \tilde{G}^a_{\alpha\beta} G^b_{\beta\mu} G^c_{\mu\alpha}$ in the SM has been computed at 3 loops (Pospelov 1994)



- No apparent reason why similar diagrams shouldn't generate imaginary parts for the operators multiplying m_{12}^2 and λ_5
- CKM phase is hard-breaking of CP, so no apparent reason why those generated imaginary parts shouldn't be divergent
- → need complex 2HDM from the beginning to have the necessary imaginary counterterms!

Motivation

Fontes et al.'s calculation:

Computed leading $(1/\epsilon)^3$ -divergent piece of A^0 tadpole diagram at 3 loops. (Most divergent piece \rightarrow 3-loop counterterm)

- Minimum number of loops required to get the Jarlskog invariant (4 powers of CKM matrix) $\sim \text{Im}(V_{\alpha i}V_{\beta j}V_{\alpha j}^*V_{\beta i}^*)$ [more on this later]
- At the very limit of modern Feynman-diagram computational technology
- Individual contributions are nonzero
- After summing over all 3 generations of up- and down-quark masses, the result is ZERO!?!

This talk \rightarrow (1) Why is it zero? (2) Can we dig deeper?

The Jarlskog invariant

Reparameterization-invariant measure of the CP violation in the CKM matrix, introduced by Cecilia Jarlskog in 1985

$$J = \left| \operatorname{Im}(V_{\alpha i} V_{\beta j} V_{\alpha j}^* V_{\beta i}^*) \right|, \qquad (\alpha \neq \beta, i \neq j)$$

- unaffected by moving phases around in V
- related to the area of the unitarity triangles in B-physics

Before EWSB, all the CPV in the SM CKM sector can be considered to live in the 3×3 Yukawa matrices Y_u , Y_d . Define the Hermitian combinations:

$$H_u = \frac{v^2}{2} Y_u Y_u^{\dagger} = U_{u_L} M_U^2 U_{u_L}^{\dagger}$$

$$H_d = \frac{v^2}{2} Y_d Y_d^{\dagger} = U_{d_L} M_D^2 U_{d_L}^{\dagger}$$

(CKM matrix is $V \equiv U_{u_L}^{\dagger} U_{d_L}$)

The Jarlskog invariant

Can then define another Jarlskog quantity, (Botella & Silva, 1995)

$$\bar{J} = \operatorname{Im} \left\{ \operatorname{Tr} \left(H_u H_d H_u^2 H_d^2 \right) \right\}$$

$$= \operatorname{Im} \left\{ \operatorname{Tr} \left(V^{\dagger} M_U^2 V M_D^2 V^{\dagger} M_U^4 V M_D^4 \right) \right\}$$

$$= T(M_U^2) B(M_D^2) J,$$

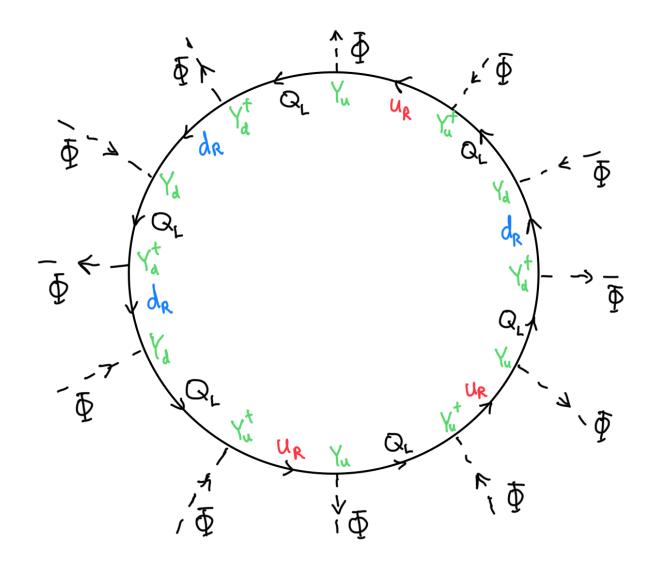
where

$$T(M_U^2) = (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2),$$

$$B(M_D^2) = (m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2).$$

Things to notice:

- H_u , H_d are Hermitian: $Tr(H_uH_dH_uH_d)$ would be real because of cyclic property of the trace. Need a different exponent on the 1st and 2nd H_u 's, and likewise H_d 's, to get an imaginary part.
- J always comes with (at least) 6 powers of up-quark masses and 6 powers of down-quark masses (i.e., 12 Yukawa insertions).



Type I:

6 incoming Φ_2 's 6 outgoing Φ_2 's

Type II:

3 incoming Φ_1 's 3 outgoing Φ_1 's 3 incoming Φ_2 's 3 outgoing Φ_2 's

 \mathcal{O}_5 is $(\Phi_1^{\dagger}\Phi_2)^2$: need to convert e.g. two outgoing Φ_2 's into Φ_1 's!

Can do this by inserting a λ_5 vertex. Novel ingredient!

Consider again the quark Yukawa couplings after imposing Natural Flavour Conservation:

$$\mathcal{L}_{Yuk} = -Y_{ij}^d \bar{Q}_{Li} \Phi_1 d_{Rj} - Y_{ij}^u \bar{Q}_{Li} \tilde{\Phi}_2 u_{Rj} + \text{h.c.}$$

(for Type II; replace Φ_1 with Φ_2 for Type I.)

We normally enforce this by imposing a \mathbb{Z}_2 symmetry.

But we could equally well have achieved this form for the Yukawa couplings by imposing a global U(1) symmetry, e.g.:

$$\Phi_1 \to e^{-i\theta} \Phi_1, \qquad \Phi_2 \to e^{i\theta} \Phi_2$$

with Q_L invariant and

$$u_R o e^{i heta} u_R, \qquad d_R o e^{-i heta} d_R \qquad ext{(Type I)} \ u_R o e^{i heta} u_R, \qquad d_R o e^{i heta} d_R \qquad ext{(Type II)}$$

(For Type II, this is equivalent to the Peccei-Quinn U(1).)

Most general scalar potential:

$$\begin{split} V &= m_{11}^2 \Phi_1^{\dagger} \Phi_1 + m_{22}^2 \Phi_2^{\dagger} \Phi_2 - \left[m_{12}^2 \Phi_1^{\dagger} \Phi_2 + \text{h.c.} \right] \\ &+ \frac{1}{2} \lambda_1 \left(\Phi_1^{\dagger} \Phi_1 \right)^2 + \frac{1}{2} \lambda_2 \left(\Phi_2^{\dagger} \Phi_2 \right)^2 + \lambda_3 \left(\Phi_1^{\dagger} \Phi_1 \right) \left(\Phi_2^{\dagger} \Phi_2 \right) + \lambda_4 \left| \Phi_1^{\dagger} \Phi_2 \right|^2 \\ &+ \left\{ \frac{1}{2} \lambda_5 \left(\Phi_1^{\dagger} \Phi_2 \right)^2 + \left[\lambda_6 \left(\Phi_1^{\dagger} \Phi_1 \right) + \lambda_7 \left(\Phi_2^{\dagger} \Phi_2 \right) \right] \left(\Phi_1^{\dagger} \Phi_2 \right) + \text{h.c.} \right\}. \end{split}$$

Imposing $U(1)_{PQ}$ kills off m_{12}^2 , λ_6 , λ_7 , and λ_5 !

 $U(1)_{PQ}$ can't be exact or A^0 is massless (physical Goldstone boson of the spontaneous breaking of the extra U(1)).

Softly break $U(1)_{PQ}$: reinstate m_{12}^2 . Complex, but its phase can be trivially rotated away using the $U(1)_{PQ}$.

Then the scalar potential has no possible CPV terms.

Protected by a softly-broken symmetry: radiative corrections cannot generate a *divergent* $Im(\lambda_5)$ (or even $Re(\lambda_5)$). (Finite & calculable radiatively-generated $Im(\lambda_5)$ is ok.)

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CPV leaks into 2HDM

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Corollary 1: any diagrams in the softly-broken- \mathbb{Z}_2 2HDM that could generate a divergent $Im(\lambda_5)$ must know about $\lambda_5 \neq 0$, or they will be equivalent to the corresponding diagrams in the softly-broken- $U(1)_{PO}$ 2HDM and the divergent parts will sum to zero.

 \rightarrow Require a λ_5 insertion in the diagrams!

Unbroken phase: convert two outgoing Φ_2 's into Φ_1 's. Minimum of 6 loops!

Broken phase: must show up via triple- or quartic-Higgs couplings that still depend on λ_5 after all other quartic couplings are re-expressed in terms of masses and mixing angles.

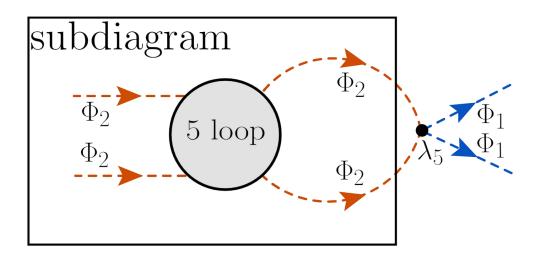
8 Lagrangian parameters: 7 + 1 physical parameters: m_{11}^2 , m_{22}^2 , m_{12}^2 , and 5 λ 's m_h , m_H , m_A , m_{H^+} , v, α , β , λ_5

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Corollary 2: If one wants a real 2HDM that is guaranteed in an obvious way to be safe from CPV "leaks" (and hence theoretically consistent), use the softly-broken- $U(1)_{PQ}$ 2HDM.

- Freedom of scalar masses and mixing angles is identical to that in softly-broken- \mathbb{Z}_2 model. (Still fully viable phenomenologically.)
- One coupling degree of freedom is removed from triple- and quartic-scalar couplings: $U(1)_{PQ}$ model is more predictive (less general) than Z_2 version, but the differences are experimentally rather subtle.

 λ_5 freedom shows up in $h^0H^+H^-$ coupling: $U(1)_{PQ}$ restricts the charged Higgs contribution to $h^0 \to \gamma \gamma$.



 \Rightarrow Creates a 6th loop.

But is this enough?

Have to also consider the contribution of the complex conjugate diagram: replace \mathcal{J} with \mathcal{J}^* , reverse the direction of all Φ_2 arrows, and move the λ_5 insertion to the left-hand pair of Φ_2 legs.

Type I:

only Φ_2 couples to quarks:

6 incoming Φ_2 's 6 outgoing Φ_2 's

But want $(\Phi_1^{\dagger}\Phi_2)^2$: need to convert two outgoing Φ_2 's into Φ_1 's!

Can do this by inserting a λ_5 vertex. Second ingredient!

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