Carleton University Physics Department

PHYS 4708 (Winter 2016, H. Logan)

Homework assignment #7

Handed out Mon. Mar. 28; due Wed. Apr. 6, 2016 at the start of class.
Problems are worth 5 points each unless noted otherwise.

1. (Gasiorowicz 3rd ed. problem 19-1) Show that for a central potential $V(\vec{r}) = V(r)$, the matrix element $M_{fi} = \langle \phi_f | V | \phi_i \rangle$ that appears in the Born approximation (see Gasiorowicz Eq. (19-77)) can be written in the form

$$M_{fi} = \frac{4\pi}{V} \int_0^\infty r V(r) \frac{\sin r\Delta}{\Delta} dr. \tag{1}$$

Note that this is a function of

$$\Delta \equiv |\vec{\Delta}| = \frac{1}{\hbar} |\vec{p}_f - \vec{p}_i|, \tag{2}$$

and hence is invariant under $\vec{\Delta} \to -\vec{\Delta}$.

2. Consider the scattering of two identical spin-1 particles. Following Gasiorowicz section 19-4, determine the appropriate form of the differential cross section for the states with total spin 0, 1, and 2. In each case, work out the differential cross section at $\theta = \pi/2$ in terms of $f(\pi/2)$, including the interference term. What is the differential cross section for unpolarized scattering, including the interference?

3. (Gasiorowicz 3rd ed. problem 19-3) Consider the potential

$$V(r) = V_0 \frac{a}{r} e^{-r/a}. \tag{3}$$

If the range parameter is $a = 1.2$ fm $= 1.2 \times 10^{-15}$ m and $|V_0| = 100$ MeV in magnitude, what is the total cross section for proton-proton scattering at 100 MeV centre-of-mass energy, calculated in the Born approximation? Pay attention to the fact that the two protons are identical fermions. *(This potential models scattering via the exchange of pions. It does not include the contribution from Coulomb scattering, which is a small effect anyway for the high centre-of-mass energy that we consider.)*

When calculating the total cross section, it's convenient to write

$$\hbar^2 \Delta^2 = |\vec{p}_f - \vec{p}_i|^2 = 2p^2(1 - \cos\theta) \tag{4}$$

so that

$$\int d\Omega = 2\pi \int d\cos\theta = \frac{\hbar^2 \pi}{p^2} \int d(\Delta^2). \tag{5}$$

continued....

4. (Gasiorowicz 3rd ed. problem 19-4) Suppose the scattering amplitude for neutron-proton scattering is given by the form

$$f(\theta) = \xi_f^\dagger (A + B\vec{\sigma}_p \cdot \vec{\sigma}_n)\xi_i, \tag{6}$$

where ξ_i and ξ_f are the initial and final spin states of the neutron-proton system. The possible states are

$$\xi_i = \begin{matrix} \chi_\uparrow^{(p)}\chi_\uparrow^{(n)} \\ \chi_\uparrow^{(p)}\chi_\downarrow^{(n)} \\ \chi_\downarrow^{(p)}\chi_\uparrow^{(n)} \\ \chi_\downarrow^{(p)}\chi_\downarrow^{(n)} \end{matrix} \qquad \xi_f = \begin{matrix} \chi_\uparrow^{(p)}\chi_\uparrow^{(n)} \\ \chi_\uparrow^{(p)}\chi_\downarrow^{(n)} \\ \chi_\downarrow^{(p)}\chi_\uparrow^{(n)} \\ \chi_\downarrow^{(p)}\chi_\downarrow^{(n)} \end{matrix} \tag{7}$$

and the scalar product of the Pauli spin matrices can be written as

$$\vec{\sigma}_p \cdot \vec{\sigma}_n = \sigma_z^{(p)}\sigma_z^{(n)} + 2\left(\sigma_+^{(p)}\sigma_-^{(n)} + \sigma_-^{(p)}\sigma_+^{(n)}\right). \tag{8}$$

Calculate the scattering amplitudes and corresponding total cross sections for all 16 initial and final spin combinations and make a table of your results.

You can work in the representation where

$$\sigma_+ = \frac{\sigma_x + i\sigma_y}{2} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \qquad \sigma_- = \frac{\sigma_x - i\sigma_y}{2} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{9}$$

and

$$\chi_\uparrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad \chi_\downarrow = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{10}$$

5. (Gasiorowicz 3rd ed. problem 19-6) Neutron-proton scattering, continued.

   (a) Use the table of scattering amplitudes computed in the previous problem to calculate the amplitudes and total cross sections for total-spin triplet → triplet and singlet → singlet scattering. Show that the triplet → singlet scattering vanishes. *(Note: for each total-spin state, add the amplitudes before squaring! The triplet comprises three states: for the total triplet → triplet cross section, you'll need to average over the initial and sum over the final triplet states.)*

   (b) Check your results by recomputing the scattering amplitudes and cross sections in the $|j, m_j\rangle$ basis (where $\vec{J} = \vec{S}_p + \vec{S}_n$), by noting that

$$\vec{J} = \frac{\hbar}{2}\vec{\sigma}_p + \frac{\hbar}{2}\vec{\sigma}_n, \tag{11}$$

   so that

$$\vec{\sigma}_p \cdot \vec{\sigma}_n = \frac{2J^2}{\hbar^2} - 3 = \begin{cases} 1 & \text{acting on triplet state,} \\ -3 & \text{acting on singlet state.} \end{cases} \tag{12}$$

   (Remember that there are three m_j states in the triplet, each contributing equally to the cross section.)