

Carleton University Physics Department
PHYS 4708 (Winter 2016, H. Logan)
Homework assignment #2

Handed out Wed. Jan. 20; due Mon. Feb. 1, 2016 at the start of class.

Problems are worth 5 points each unless noted otherwise.

1. [10 points] (similar to Gasiorowicz 3rd edition, problem 11-5) Proton charge radius.

The proton is not really a point charge; it is an extended object with its electric charge distributed throughout some volume. The effect of this can be detected in spectroscopy, allowing a measurement of the “proton charge radius”.

- (a) Assume that the charge of the proton is distributed uniformly throughout a spherical volume of radius R . This means that the Coulomb potential is modified for $r < R$. Treat the modification as a perturbation and compute the first-order shift in the energies of the $1s$ and $2p$ states. You can work to leading order in powers of the ratio R/a_0 , where $R \ll a_0$ and a_0 is the Bohr radius.
- (b) For $R = 1 \text{ fm} = 10^{-15} \text{ m}$, compute the fractional shift $\Delta\nu/\nu$ in the frequency of the Lyman alpha spectral line (from the $2p \rightarrow 1s$ electric dipole transition). How big is the shift if you instead use muonic hydrogen, in which the electron is replaced by a muon?
- (c) If you assume instead that the charge of the proton is distributed uniformly on a spherical *shell* of radius R , instead of throughout a spherical volume, does your answer to part (a) change at leading order in R/a_0 ?
2. Consider the perturbed harmonic oscillator in Gasiorowicz 3rd ed. Example 11-1 (we also did the example in class): $H_0 = p^2/2m + m\omega^2 x^2/2$, $\lambda H_1 = q\mathcal{E}x$.
- (a) Compute the first-order correction to the ground state wavefunction $|\phi_0\rangle$ (i.e., find $|\psi_0\rangle$ to order $q\mathcal{E}$). (*Hint: use the raising and lowering operators as in the example.*)
- (b) Calculate the expectation value of x in the perturbed ground state wavefunction that you found in part (a). How does your result compare to the location of the minimum of the perturbed potential, which you can find analytically?
3. [10 points] (similar to Gasiorowicz prob. 11-3) Consider a particle in an infinite potential well with width L and centred at $x = 0$ (i.e., symmetric). The system is perturbed by tilting the “floor” of the potential well:

$$\lambda H_1 = V(x) = V_0 \frac{x}{L}. \quad (1)$$

- (a) Obtain a summation formula for the second-order shift in the energy eigenvalue for the n th state. (The first-order shift in the energy eigenvalue for the n th level is zero by parity.) (*Hint: for the integral, try using the $\sin(A \pm B)$ identities to combine trig functions and then integrate by parts to get rid of the x .*)
- (b) Estimate the shift in units of $V_0^2/E_1^{(0)}$ for the lowest three levels. (In each case include only the contributions of the level’s immediate neighbours; the terms in the sum decrease quickly with increasing k .)