Carleton University Physics Department
PHYS 4708 (Winter 2016, H. Logan)
Homework assignment #1

Handed out Mon. Jan. 11; due Wed. Jan. 20, 2016 at the start of class.
Problems are worth 5 points each unless noted otherwise.

1. Show that
$$C_-(\ell, m) = \hbar\sqrt{(\ell+m)(\ell-m+1)} \tag{1}$$
[Eq. (7-24) in Gasiorowicz 3rd edition], where $L_-|\ell, m\rangle = C_-(\ell, m)|\ell, m-1\rangle$, by following the same steps as done in lecture or Gasiorowicz sec. 7-2 for $C_+(\ell, m)$.

2. (Gasiorowicz 3rd edition, problem 10-14) A particle of spin 1 moves in a central potential of the form
$$V(r) = V_1(r) + \frac{\vec{S}\cdot\vec{L}}{\hbar^2}V_2(r) + \frac{(\vec{S}\cdot\vec{L})^2}{\hbar^4}V_3(r). \tag{2}$$
What are the values of $V(r)$ in the states $j = \ell+1$, ℓ, and $\ell-1$? Give your answer in terms of $V_1(r)$, $V_2(r)$, $V_3(r)$, and ℓ.

3. Construct the eigenbasis of states $|j, m_j\rangle$ in terms of the eigenbasis $|m_\ell, m_s\rangle$ for $\ell = 1$, $s = 1$. Check that your results agree with the "1×1" section of the table of Clebsch-Gordan coefficients handed out in class.[1] Verify the j value of one state in each multiplet of given j using the J^2 eigenvalue equation. *(Hint: Use repeated application of the J_- operator as done in class. When writing down the orthogonal state of a given m_j, compare to the table of Clebsch-Gordan coefficients to help you choose the sign conventions.)*

4. A particle with orbital angular momentum $\ell = 1$ and spin $s = 1$ is in the state $|\Psi\rangle$, which can be expressed in the eigenbasis $|m_\ell, m_s\rangle$ as
$$|\Psi\rangle = \frac{1}{\sqrt{2}}|{+1}, -1\rangle + \frac{1}{\sqrt{2}}|0, 0\rangle. \tag{3}$$
   (a) Re-express $|\Psi\rangle$ in the $|j, m_j\rangle$ eigenbasis (use the results of problem 3 and/or a table of Clebsch-Gordan coefficients).
   (b) Using in each case the most sensible choice of basis, compute the expectation values of L_z, S_z, J^2, and J_z in the state Ψ.

continued....

[1] This table can be downloaded from the Particle Data Group website at http://pdg.lbl.gov > Reviews, Tables, Plots > Mathematical Tools > Clebsch-Gordan coeff., sph. harmonics, and d functions.

5. Consider again a particle with orbital angular momentum $\ell = 1$ and spin $s = 1$ in the state $|\Psi\rangle$, which can be expressed in the eigenbasis $|m_\ell, m_s\rangle$ as in problem 4 as

$$|\Psi\rangle = \frac{1}{\sqrt{2}}|+1, -1\rangle + \frac{1}{\sqrt{2}}|0, 0\rangle. \tag{4}$$

   (a) If you were to measure L_z in this state, what would be the possible outcomes and their probabilities?

   (b) If you were to measure J^2 in this state, what would be the possible outcomes and their probabilities?

   (c) Now imagine that you make the measurement of J^2 in this state and get the value $6\hbar^2$. What is the new state of the particle after the measurement? Express this new state in both the $|j, m_j\rangle$ and the $|m_\ell, m_s\rangle$ eigenbases.

   (d) If you were to measure L_z in this new state, what would be the possible outcomes and their probabilities?