Formula sheet for final exam

This formula sheet, and copies of the front and back covers of Griffiths, will be provided with the exam.

Electric field due to a source charge q at the origin:

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}} \tag{1}$$

Force on test charge Q at $\vec{\mathbf{r}}$ due to source charge q at the origin:

$$\vec{\mathbf{F}} = Q\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r^2}\hat{\mathbf{r}} \tag{2}$$

Electric potential:

$$V(\vec{r}) = -\int_{\mathcal{O}}^{\vec{\mathbf{r}}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}, \qquad \vec{\mathbf{E}} = -\vec{\nabla} V \tag{3}$$

Choosing $\mathcal{O}$ at infinity, potential of a source charge q at the origin:

$$V(\vec{\mathbf{r}}) = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \tag{4}$$

Gauss's law:

$$\oint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{a}} = \frac{Q_{\text{encl}}}{\epsilon_0}, \qquad \vec{\nabla} \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0} \tag{5}$$

Poisson's equation (reduces to Laplace's equation for $\rho = 0$):

$$\nabla^2 V = -\frac{\rho}{\epsilon_0} \tag{6}$$

Capacitance, definition of:

$$C = \frac{Q}{\Delta V}, \qquad C = \frac{A\epsilon_0}{d} \text{ for parallel plate} \tag{7}$$

Electrostatic energy for point charges and continuous charge distributions:

$$W = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\sum_{j>i}^{n}\frac{q_i q_j}{|\vec{r}_i - \vec{r}_j|}, \qquad W = \frac{1}{2}\int \rho V \, d\tau = \frac{\epsilon_0}{2}\int_{\text{all space}} E^2 d\tau \tag{8}$$

$$W = \frac{1}{2}C(\Delta V)^2 \text{ stored in a capacitor} \tag{9}$$

Discontinuities across a surface charge:

$$\vec{\mathbf{E}}_{\text{above}} - \vec{\mathbf{E}}_{\text{below}} = \frac{\sigma}{\epsilon_0}\hat{\mathbf{n}}, \tag{10}$$

$$V_{\text{above}} = V_{\text{below}}, \qquad \frac{\partial V_{\text{above}}}{\partial n} - \frac{\partial V_{\text{below}}}{\partial n} = -\frac{\sigma}{\epsilon_0} \tag{11}$$

Electrostatic pressure (on surface of conductor):

$$\vec{f} = \frac{\sigma^2}{2\epsilon_0}\hat{n} = \frac{\epsilon_0}{2}E^2\hat{n} \tag{12}$$

Method of Images for a grounded conducting sphere with radius R and a point charge q a distance a from the centre of the sphere:

$$q' = -\frac{R}{a}q, \qquad b = \frac{R^2}{a} \tag{13}$$

where q' is a distance b from the centre of the sphere. For a conducting sphere at potential V_0, add a second image charge q'' at the centre of the sphere such that

$$V_0 = \frac{1}{4\pi\epsilon_0}\frac{q''}{R} \tag{14}$$

Solutions of Laplace's equation:

$$V(r,\theta) = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta) \tag{15}$$

$$V(s,\phi) = A_0 \ln s + B_0 + \sum_{m=1}^{\infty}\left(A_m \cos m\phi + B_m \sin m\phi\right)\left(s^m + C_m s^{-m}\right) \tag{16}$$

Legendre polynomials:

$P_0(x) = 1$	$P_3(x) = (5x^3 - 3x)/2$
$P_1(x) = x$	$P_4(x) = (35x^4 - 30x^2 + 3)/8$
$P_2(x) = (3x^2 - 1)/2$	$P_5(x) = (63x^5 - 70x^3 + 15x)/8$

Potential due to a dipole at the origin:

$$V(\vec{\mathbf{r}}) = \frac{1}{4\pi\epsilon_0}\frac{\hat{\mathbf{r}}\cdot\vec{\mathbf{p}}}{r^2} \tag{17}$$

Bound charge in polarized materials:

$$\rho_b = -\vec{\nabla}\cdot\vec{\mathbf{P}} \qquad \sigma_b = \vec{\mathbf{P}}\cdot\hat{\mathbf{n}} \tag{18}$$

$\vec{\mathbf{D}}$ field:

$$\vec{\mathbf{D}} \equiv \epsilon_0\vec{\mathbf{E}} + \vec{\mathbf{P}} \qquad \vec{\nabla}\cdot\vec{\mathbf{D}} = \rho_f \qquad \oint \vec{\mathbf{D}}\cdot d\vec{\mathbf{a}} = Q_{f,\text{encl}} \tag{19}$$

Linear dielectrics: electric susceptibility χ_e

$$\vec{\mathbf{P}} = \epsilon_0\chi_e\vec{\mathbf{E}} \qquad \vec{\mathbf{D}} = \epsilon_0(1+\chi_e)\vec{\mathbf{E}} \equiv \epsilon_0\epsilon_r\vec{\mathbf{E}} \equiv \epsilon\vec{\mathbf{E}} \tag{20}$$

Energy

$$W = \frac{1}{2}\int \vec{\mathbf{D}}\cdot\vec{\mathbf{E}}\,d\tau \qquad \text{In vacuum, } W = \frac{\epsilon_0}{2}\int E^2\,d\tau \tag{21}$$

Lorentz force on point charge or current

$$\vec{\mathbf{F}} = q\,\vec{\mathbf{v}}\times\vec{\mathbf{B}} \qquad \vec{\mathbf{F}} = \int(\vec{\mathbf{J}}\times\vec{\mathbf{B}})\,d\tau \tag{22}$$

Biot-Savart law

$$\vec{\mathbf{B}}(\vec{\mathbf{r}}) = \frac{\mu_0}{4\pi}\int\frac{\vec{\mathbf{J}}(\vec{\mathbf{r}}')\times(\vec{\mathbf{r}}-\vec{\mathbf{r}}')}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|^3}d\tau' \qquad \vec{\mathbf{B}}(\vec{\mathbf{r}}) = \frac{\mu_0 I}{4\pi}\int\frac{d\vec{\mathbf{l}}'\times(\vec{\mathbf{r}}-\vec{\mathbf{r}}')}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|^3} \tag{23}$$

Ampère's law (magnetostatics)

$$\vec{\nabla}\times\vec{\mathbf{B}} = \mu_0\vec{\mathbf{J}} \qquad \oint\vec{\mathbf{B}}\cdot d\vec{\mathbf{l}} = \mu_0 I_{\text{encl}} \tag{24}$$

Vector potential

$$\vec{\mathbf{B}} = \vec{\nabla} \times \vec{\mathbf{A}} \qquad \text{If } \vec{\nabla} \cdot \vec{\mathbf{A}} = 0, \text{ then } \nabla^2 \vec{\mathbf{A}} = -\mu_0 \vec{\mathbf{J}} \text{ and } \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \frac{\mu_0}{4\pi} \int \frac{\vec{\mathbf{J}}(\vec{\mathbf{r}'})}{|\vec{\mathbf{r}} - \vec{\mathbf{r}}'|} d\tau' \tag{25}$$

Boundary conditions

$$\vec{\mathbf{B}}_{\text{above}} - \vec{\mathbf{B}}_{\text{below}} = \mu_0 (\vec{\mathbf{K}} \times \hat{\mathbf{n}}) \tag{26}$$

Torque and force on a magnetic dipole

$$\vec{\tau} = \vec{\mathbf{m}} \times \vec{\mathbf{B}} \qquad \vec{\mathbf{F}} = \vec{\nabla}(\vec{\mathbf{m}} \cdot \vec{\mathbf{B}}) \tag{27}$$

Vector potential due to a magnetic dipole at the origin

$$\vec{\mathbf{A}}(\vec{\mathbf{r}}) = \frac{\mu_0}{4\pi} \frac{\vec{\mathbf{m}} \times \hat{\mathbf{r}}}{r^2} \tag{28}$$

Bound current in magnetized materials

$$\vec{\mathbf{J}}_b = \vec{\nabla} \times \vec{\mathbf{M}} \qquad \vec{\mathbf{K}}_b = \vec{\mathbf{M}} \times \hat{\mathbf{n}} \tag{29}$$

$\vec{\mathbf{H}}$ field

$$\vec{\mathbf{H}} \equiv \frac{1}{\mu_0} \vec{\mathbf{B}} - \vec{\mathbf{M}} \qquad \vec{\nabla} \times \vec{\mathbf{H}} = \vec{\mathbf{J}}_f \qquad \oint \vec{\mathbf{H}} \cdot d\vec{\mathbf{l}} = I_{f,\text{encl}} \tag{30}$$

Linear magnetic media: magnetic susceptibility χ_m

$$\vec{\mathbf{M}} = \chi_m \vec{\mathbf{H}} \qquad \vec{\mathbf{B}} = \mu_0 (1 + \chi_m) \vec{\mathbf{H}} \equiv \mu \vec{\mathbf{H}} \tag{31}$$

Ohm's law and emf (here $\sigma \equiv$ conductivity)

$$\vec{\mathbf{J}} = \sigma \vec{\mathbf{f}} = \sigma (\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}) \qquad V = IR \qquad \mathcal{E} \equiv \oint \vec{\mathbf{f}} \cdot d\vec{\mathbf{l}} \tag{32}$$

Power

$$P = I \Delta V \tag{33}$$

Motional emf, induction, and Faraday's law

$$\mathcal{E} = -\frac{d\Phi_m}{dt} \qquad \vec{\nabla} \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t} \qquad \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = -\int \frac{\partial \vec{\mathbf{B}}}{\partial t} \cdot d\vec{\mathbf{a}} \tag{34}$$

Lenz's law: "Nature abhors a change in flux."

Mutual inductance and self-inductance

$$\Phi_2 = M I_1 \qquad \Phi_1 = M I_2 \qquad \Phi = LI \qquad \mathcal{E} = -L \frac{dI}{dt} \tag{35}$$

Energy

$$W = \frac{1}{2\mu_0} \int B^2 \, d\tau \tag{36}$$

Continuity equation and Maxwell's fix to Ampère's law

$$\vec{\nabla} \cdot \vec{\mathbf{J}} = -\frac{\partial \rho}{\partial t} \qquad \vec{\nabla} \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{J}} + \mu_0 \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t} \tag{37}$$