Carleton University Physics Department
PHYS 3308 – Electromagnetism (Fall 2014)
Homework assignment #4

Handed out Tue Sept 30; due Thurs Oct 9, 2014, at the start of class. H. Logan

Problems are worth 5 points each unless noted otherwise.

1. Find the energy stored in a uniformly charged solid sphere of radius R and total charge q. Do it three different ways:

   (a) Use the expression for the electrostatic energy in terms of the potential and the charge density,

   $$W = \frac{1}{2}\int \rho V \, d\tau. \tag{1}$$

   You can use the result for the potential of a uniformly charged solid sphere that you found in problem 2 of Homework #3,

   $$V_{\text{outside}} = \frac{q}{4\pi\epsilon_0 r}, \qquad V_{\text{inside}} = \frac{q}{8\pi\epsilon_0 R}\left[3 - \frac{r^2}{R^2}\right]. \tag{2}$$

   (b) Use the expression for the electrostatic energy in terms of the square of the electric field,

   $$W = \frac{\epsilon_0}{2}\int_{\text{all space}} E^2 d\tau. \tag{3}$$

   Don't forget to integrate over *all space*.

   (c) Use the expression for the electrostatic energy integrating E^2 over a finite volume and including the surface term,

   $$W = \frac{\epsilon_0}{2}\left[\int_{\mathcal{V}} E^2 d\tau + \oint_{\mathcal{S}} V\vec{E}\cdot d\vec{a}\right]. \tag{4}$$

   Take the finite volume to be a sphere of radius a. Explain how the two terms in your solution reduce to the result of part (b) as you take $a \to \infty$.

2. Consider a conducting sphere of radius R carrying total charge Q. Compute the electrostatic pressure (force per unit area, $\vec{f}$) at the surface of the sphere in three ways:

   (a) Use the surface charge density,

   $$\vec{f} = \frac{\sigma^2}{2\epsilon_0}\hat{n}. \tag{5}$$

   (b) Use the electric field at the surface,

   $$\vec{f} = \frac{\epsilon_0}{2}E^2\hat{n}. \tag{6}$$

   (You can use Gauss's law to find $\vec{E}$ or take it from a previous example; we've calculated this a few times in class already.)

(c) Compute the stored electrostatic potential energy of the system. You learned in mechanics that the force on a particle is the gradient of its potential energy. Use this concept to find the electrostatic pressure at the surface of the sphere.

3. Two spherical cavities with radii a and b are hollowed out from the interior of an uncharged conducting sphere of radius R (i.e., there is no net charge on the metal itself). Then a point charge q_a is placed at the centre of cavity a and a point charge q_b is placed at the centre of cavity b.

   (a) Find the surface charge densities on the inner surfaces of cavities a and b and on the outside surface of the conducting sphere. Justify your answers using Gauss's law and symmetry.

   (b) What is the electric field outside the conducting sphere?

   (c) What is the electric field within each cavity?

   (d) What is the force on q_a and q_b?

   (e) Which of these answers would change if a third point charge, q_c, were brought near the conductor? (Just describe, don't solve.)

4. A very long solid metal rod of radius a is threaded down the centre of a very long metal pipe with inner radius b and outer radius c ($a < b < c$). The rod is held at potential V_0 and the pipe is grounded.

   (a) Find the electric field *everywhere* and the surface charge density on all three surfaces. *(Hint: you already know the potential at three key points (radii a, b and c). You can use Gauss's law to find the functional form of $\vec{E}$ between the rod and the inside of the pipe and integrate it to get the functional form of the potential. You can use Gauss's law or the expression for the discontinuity in the electric field to find the surface charge densities.)*

   (b) What is the capacitance per unit length of this system? Use the definition of capacitance in terms of the charge stored per unit voltage difference.

5. Two square plates of metal, each with large area A and small thickness ℓ, are arranged parallel to each other with their inner surfaces a small distance d apart. A total charge $Q_{\text{upper}} = \sigma_0 A$ is put on the upper plate and the lower plate is left uncharged.

   (a) Find the surface charge density on all four surfaces. You can think of this as four parallel planes of charge and ignore edge effects; you know what the *sum* of the surface charge densities are on the two upper and the two lower plates, and you know that the *total* electric field must be zero inside each of the two conductors.

   (b) Find the capacitance of this device starting from the definition of capacitance in terms of the charge stored per unit voltage difference and using the charge configuration of this problem. Check that your result agrees with the usual formula for the capacitance of a parallel-plate capacitor, $C = \epsilon_0 A/d$.