Carleton University Physics Department

PHYS 3308 – Electromagnetism (Fall 2014)

Homework assignment #2

Handed out Thu Sept 11; due Tue Sept 23, 2014, at the start of class. H. Logan

Problems are worth 5 points each unless noted otherwise.

1. Starting from Maxwell's equations (given in the back cover of the textbook) and using the appropriate vector calculus theorems, derive the integral forms of:

   (a) Gauss's law

   (b) Ampère's law

   (c) Faraday's law.

   For full marks, define all quantities and explain your reasoning.

2. Vector derivatives in spherical coordinates. Find the derivatives of the unit vectors $\hat{r}$, $\hat{\theta}$, and $\hat{\phi}$ with respect to r, θ, and ϕ. *(Hint: one way to do this is to express the unit vectors in terms of $\hat{x}$, $\hat{y}$, and $\hat{z}$, which do not vary with position.)* Using these and the expression for the gradient operator in spherical coordinates,

$$\vec{\nabla} = \hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}, \tag{1}$$

   derive the formulas for $\vec{\nabla} \cdot \vec{A}$ and $\vec{\nabla} \times \vec{A}$, where $\vec{A} = A_r\hat{r} + A_\theta\hat{\theta} + A_\phi\hat{\phi}$.

3. *(10 points)* Using Gauss's Law, find the electric field everywhere (i.e., both outside and inside the charged objects) for each of the three following charge configurations. *(In each case, define a coordinate system and describe your choice of Gaussian surface using a diagram. When evaluating the surface integral, make sure you include all parts of your Gaussian surface!)*

   (a) A uniformly-charged solid sphere with radius a and total charge Q.

   (b) An infinitely-long solid rod with radius b and uniform volume charge density ρ.

   (c) An infinite slab of charge with thickness c and charge per unit area σ (the charge is distributed uniformly through the thickness of the slab).

4. An infinitely long solid rod of charge with uniform charge density ρ and radius b is oriented along the z axis. A spherical hole of radius a (with $a < b$) is hollowed out around the origin. Find the electric field everywhere. *(Hint: superposition!)*