Carleton University Physics Department
PHYS 3308 – Electromagnetism (Fall 2014)
Homework assignment #10

Handed out Thurs Nov 27; due Fri Dec 5, 2014 by 5:00 p.m. (at my office, 2450 Herzberg).
Problems are worth 5 points each unless noted otherwise. H. Logan

1. Magnetostatics treats the "source current" (the one that sets up the field) and the "recipient current" (the one that experiences the force) so asymmetrically that it is by no means obvious that the magnetic force between two current loops is consistent with Newton's third law. Show, starting with the Biot-Savart law and the Lorentz force law, that the force on loop 2 due to loop 1 (see Griffiths Figure 5.61 on page 260) can be written as (using $\vec{\mathcal{R}}$ in place of Griffiths's cursive r, which does not seem to exist in LaTeX):

$$\vec{F}_2 = -\frac{\mu_0}{4\pi} I_1 I_2 \oint \oint \frac{\hat{\mathcal{R}}}{\mathcal{R}^2} d\vec{l}_1 \cdot d\vec{l}_2. \qquad (1)$$

   In this form it is clear that $\vec{F}_2 = -\vec{F}_1$, since $\hat{\mathcal{R}}$ changes direction when the roles of 1 and 2 are interchanged. (If you seem to be getting an extra term, it will help to note that $d\vec{l}_2 \cdot \hat{\mathcal{R}} = d\mathcal{R}$.)

2. A long solenoid with radius a and n turns per unit length is encircled by a loop of wire with total resistance R.

   (a) If the current in the solenoid is increasing at a constant rate $dI/dt = k$, what is the induced current in the loop? Draw a diagram to indicate the direction of the current.

   (b) If the current I in the solenoid is held constant but the solenoid is pulled out of the loop, turned around, and reinserted, what total charge flows past a given point on the loop?

3. Consider an ideal solenoid with radius a and n turns per unit length, carrying current I. Calculate the magnetic energy per unit length stored in this solenoid in two different ways:

   (a) Use the expression for the stored energy in terms of the magnetic field (you will need to express this "per unit length").

   (b) Calculate the self-inductance per unit length of the solenoid, then find the stored energy per unit length in terms of the self-inductance per unit length.

4. Electrons undergoing cyclotron motion can be sped up by increasing the magnetic field; the induced electric field will cause a tangential acceleration. This is the principle of the *betatron*. One would like to keep the radius of the orbit constant during the acceleration process. Show that this can be achieved by designing a magnet such that the average field over the area of the orbit is twice the field at the circumference. Assume that the apparatus is cylindrically symmetric about the centre of the orbit. (Assume also that the electron's velocity remains nonrelativistic.) *(Hint: differentiate the relation between v and B for cyclotron motion with respect to time, then use $F = ma = qE$ and find E in terms of the emf from the changing B field.)*

5. (Griffiths problem 9.27)

   (a) Derive Griffiths Eqs. (9.179) (page 426) by plugging $\tilde{\mathbf{E}}$ and $\tilde{\mathbf{B}}$ in component form into the Maxwell's equations involving curls, and from these obtain Eqs. (9.180).

   (b) Put Griffiths Eqs. (9.180) into the Maxwell's equations involving divergences and obtain Eqs. (9.181).