PHYS 5802 Assignment 1

Given: Wednesday, September 10, 2008

Due: Wednesday, September 24, 2008

1. Find the multiplication law for a group with 3 elements (that is, order 3) and prove that it is unique.

2. Show that the transformations produced by the matrices $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$, where $0 \le \theta < 2\pi$, constitute a group and leave invariant the real quadratic form $x_1^2 + x_2^2$. What group is this?

3. Generate a group from two elements A and B subject to the constraints $A^2 = B^k = (AB)^2 = E$, where k is a finite integer greater than 1. Find the order of the group. This is the dihedral group D_k. Use the constraints above rather than your geometric understanding of this group to do this problem.

4. The symmetry group of the proper covering operations of a square, D_4, has 8 elements.
    a. What are the group elements?
    b. Work out the group multiplication table.
    c. Divide the group into classes. Do this using the nature of the operations. If in doubt, check with the multiplication table.
    d. Write down the subgroups and identify the invariant subgroups.
    e. Work out the cosets of the invariant subgroups.
    f. Work out the group multiplication tables of the factor groups corresponding to the nontrivial invariant subgroups.

5. In this question, A and B are operators but I'm leaving off the "hats". An operator relation that is frequently used is the power series expansion $e^A = \sum_{n=0}^{\infty} \frac{A^n}{n!}$.

    Consider the function $f(\lambda) = e^{\lambda A} B e^{-\lambda A}$, where λ is a real number. Make a Taylor series expansion in λ to prove the identity

    $$e^{\lambda A} B e^{-\lambda A} = B + \frac{\lambda}{1!}[A, B] + \frac{\lambda^2}{2!}[A,[A,B]] + \frac{\lambda^3}{3!}[A,[A,[A,B]]] + \dots$$

    In doing so, show that

    $$\frac{d}{d\lambda} e^{\lambda A} \equiv \lim_{h \to 0} \frac{1}{h}\left(e^{(\lambda+h)A} - e^{\lambda A}\right) = A e^{\lambda A}$$

    and use this result.