3. Heavy Quarkonia

1. Spectroscopy
2. em decays
3. decays
2. The November Revolution:

Experimental Observation of a Heavy Particle $J^+$


Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

and

Y. Y. Lee

Brookhaven National Laboratory, Upton, New York 11973

(Received 12 November 1974)

We report the observation of a heavy particle $J$, with mass $m = 3.1$ GeV and width approximately zero. The observation was made from the reaction $p + B = e^+ + e^- + x$ by measuring the $e^+e^-$ mass spectrum with a precise pair spectrometer at the Brookhaven National Laboratory's 30-GeV alternating-gradient synchrotron.

Discovery of a Narrow Resonance in $e^+e^-$ Annihilation


Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

and


Lawrence Berkeley Laboratory and Department of Physics, University of California, Berkeley, California 94720

(Received 13 November 1974)

We have observed a very sharp peak in the cross section for $e^+e^- \rightarrow \text{hadrons}$, $e^+e^-$, and possibly $\mu^+\mu^-$ at a center-of-mass energy of $3.105 \pm 0.003$ GeV. The upper limit to the full width at half-maximum is 1.3 MeV.
Is Bound Charm Found?*

A. De Rújula

Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138

and

S. L. Glashow†

Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

(Received 27 November 1974)

We argue that the newly discovered narrow resonance at 3.1 GeV is a $^3S_1$ bound state of charmed quarks and we show the consistency of this interpretation with known meson systematics. The crucial test of this notion is the existence of charmed hadrons near 2 GeV.
Spectroscopy of the New Mesons*

Thomas Appelquist,† A. De Rujula, and H. David Politzer

Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138

and

S. L. Glashow

Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

Received 11 December 1974

The interpretation of the narrow boson resonances at 3.1 and 3.7 GeV as charmed quark-antiquark bound states implies the existence of other states. Some of these should be copiously produced in the radiative decays of the 3.7-GeV resonance. We estimate the masses and decay rates of these states and emphasize the importance of γ-ray spectroscopy.

Two earlier papers[1,2] present our case that the recently discovered[3] and confirmed[4] resonance at 3.105 GeV is the ground state of a charmed quark bound to its antiquark, by colored gauge gluons: orthocharmonium I. More recently, a second state at 3.465 GeV has been reported[6] with an estimated width of 0.5–2.7 MeV and a partial decay rate ~ 2 keV into e⁺e⁻. We interpret this state as an S-wave radially excited, orthocharmonium II, with J² = 1⁺ and J² = 0⁺. Here are three indications of the correctness of our interpretation: (1) Much of the time, orthocharmonium II decays into orthocharmonium I and two pions. This behavior suggests that orthocharmonium II is an excited state of orthocharmonium I. (2) The leptonic width of orthocharmonium II is about half that of orthocharmonium I, not unexpected for an excited state whose wave function at the origin is smaller. (3) Orthocharmonium II is not seen in the Brookhaven National Laboratory–Massachusetts Institute of Technology experiment.[7] In a thermodynamic model,[8] the production cross section of a hadron of 3.7 GeV is suppressed by ~10⁻⁹ relative to that of a hadron of 1.1 GeV. Moreover, the leptonic branching ratio of orthocharmonium II is smaller than that of orthocharmonium I by a factor of 10.

We predict the existence of other states of charmonium with masses less than 3.7 GeV, a

S. Godfrey, Carleton University
The Charmonium Spectrum

Observation of an $\eta_c$ Candidate State with Mass $2978 \pm 9$ MeV
Spectroscopy convinced us that quarks were real.

"New" Spectroscopy of Mesons

Charmonium family

$\psi(4S)$ or hybrid
$\psi(2D)\rightarrow \pi^+\pi^- J/\psi$
$\psi(3S)\rightarrow DD$
$\eta_c(3S)\rightarrow DD$
$\eta_c(2S)\rightarrow \psi(2S)$
$\eta_c(1S)\rightarrow \psi(1S)$
$\eta_c(1S)\rightarrow \eta_c(1P)$
$X_{c1}(2P)\rightarrow X(3872)$
$X_{c2}(2P)\rightarrow \omega(\gamma)$
$X_{c3}(2P)\rightarrow \omega(\gamma)$
$X_{c8}(1P)\rightarrow \gamma$
$X_{c8}(1P)\rightarrow \gamma$
$X_{c8}(1P)\rightarrow \gamma$
$X_{c8}(1P)\rightarrow \gamma$
$X_{c8}(1P)\rightarrow \gamma$
$X_{c8}(1P)\rightarrow \gamma$

Mass (GeV/c$^2$)

L = 0 1 1 1 1 1
PC = 0$^+$ 1$^-$ 1$^+$ 0$^+$ 1$^+$ 2$^+$
"New" Spectroscopy of Mesons
1. Potential Models:

- Spin independent potentials
- Relativistic corrections
- Spin dependent effects
- Coupled channel effects

Reviews:


Thomas as has recent review and maybe quigg?
Mesons are composed of a quark-antiquark pair

Combine u, d, s, c, b quark and antiquark to form various mesons:

Meson quantum numbers characterized by given $J^{PC}$

$$S = S_1 + S_2$$
$$J = L + S$$
$$P = (-1)^L + 1$$
$$C = (-1)^{L+S}$$

Allowed:
$$J^{PC} = 0^{--} 1^{--} 1^{+ +} 0^{++} 1^{++} 2^{++} \ldots$$

Not allowed: exotic combinations:
$$J^{PC} = 0^{--} 0^{+-} 1^{-+} 2^{+-} \ldots$$
4.1 The Spin-Independent Potential

Previously gave qualitative arguments why the spin-independent potential is linear + Coulomb

\[ V(r) = -\frac{4}{3} \frac{\alpha_s(r)}{r} + br \quad b \approx 0.18 \text{ GeV}^2 \]

We also saw how this potential is consistent with results from Lattice QCD

However, Historically this form was arrived at through trial and error (Although Appelquist and Politzer got it right in an early paper ~ 1975)

Emperically, the Schrodinger eqn was solved for a given potential which was modified until agreement was achieved between theory and experiment.
\[ M = m_1 + m_2 + E_{nl} \]
\[
\left[ \frac{p^2}{2\mu} + V(r) \right] \psi = E_{nl} \psi \quad (\mu = \frac{m_1 m_2}{m_1 + m_2})
\]
\[
\left[ \frac{\hbar^2}{2\mu} \nabla^2 + V(r) \right] \psi = E_{nl} \psi
\]
\[
\nabla^2 = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \phi^2}
\]
\[
\psi(r, \theta, \phi) = R(r)Y_{lm}(\theta, \phi) \quad U(r) \equiv r R(r)
\]
\[
\frac{\hbar^2}{2\mu} \frac{d^2 U}{dr^2} + \left[ V(r) + \frac{\hbar^2}{2\mu} \frac{l(l+1)}{r^2} \right] U = E_{nl} U
\]
\[ (U(0) = 0, \ U'(0) = R(0)) \]
\[
\Delta (\chi(3685) - \chi(2017)) \approx \Delta (\chi(10023) - \chi(19460))
\]

\[
\Rightarrow \begin{cases} 
V(r) = \lambda r^{\nu} & r \approx 0.1 \\
V(r) = c \ln(r/r_0) & c \approx 0.73
\end{cases}
\]

Also \( V(r) = -\frac{4}{3} \frac{\alpha_s}{r} + br \) for suitable \( \alpha_s, b \)
Lattice QCD gives qq potential:
Quark-antiquark Potential

For given spin and orbital angular momentum configurations & radial excitations generate our known spectrum of light quark mesons

\[ H_{ij}^{conf} = -\frac{4}{3} \frac{\alpha_s(r)}{r} + b r \]

\[ M = m_1 + m_2 + E_{nl} \]

\[ \left[ \frac{p^2}{2\mu} + V(r) \right] \psi = E_{nl} \psi \]

Solve Schrodinger eqn for meson masses
Quark potential models are strongly supported by empirical agreement with quarkonium spectroscopy and with lattice QCD.

From Buchmuller & Tye
PR D24, 132 (1981)

Figure 21: Various $Q\bar{Q}$ potentials. The potentials have been shifted to agree at $r=0.5$ fm. The numbers refer to the following references: 1: Martin [101], 2: Buchmüller, Grunberg and Tye [99], 3: Bhanot and Rudaz [102], 4: Cornell group [97].
Could also use position of P-waves

Spin averaged $^3P_J$ gives

$$\bar{M} = \frac{(5M_{3P_2} + 3M_{3P_1} + M_{3P_0})}{9}$$

For $c\bar{c}$ $\bar{M} = 3522$ MeV

$$\frac{M(2S) - M(1P)}{M(2S) - M(1S)} \begin{cases} 
1/2 & \text{H.O.}(\nu = 2) \\
\simeq 1/4 & \text{for } \nu = 0 \\
0 & \text{Coulomb}(\nu = -1) 
\end{cases}$$

c\bar{c} \Rightarrow \nu \simeq 0.15
Spin-dependent potentials:

Generally expect spin-dependent Interactions:

\[ \vec{S}_1 \cdot \vec{S}_2 \quad \vec{L} \cdot \vec{S} \quad S_{12} \]

Start by looking at spin-dependent interactions of QED in hydrogen atom

**Spin-Orbit:** electron sees the proton circling around

- The orbital motion creates a magnetic field at the centre:
  \[ B = \frac{ev}{cr^2} \]
  
  - In terms of \( L=mvr \)
    \[ \vec{B} = \frac{e}{mcr^3} \vec{L} \]

- The spinning electron constitutes a magnetic dipole
  \[ \vec{\mu} = -\frac{e}{mc} \vec{S} \]

- The interaction energy is
  \[ W = -\vec{\mu} \cdot \vec{B} \]
More rigorously (derived as a succession of infinitesimal Lorentz transformations) leads to the Thomas precession with a factor of 1/2

\[ \Delta H_{S.O.} = \frac{e^2}{2m^2c^2r^3} \vec{L} \cdot \vec{S} \]

\[ \vec{J}^2 = \vec{L}^2 + \vec{S}^2 + 2\vec{L} \cdot \vec{S} \]

\[ \Rightarrow \vec{L} \cdot \vec{S} = \frac{1}{2}[\vec{J}^2 - \vec{L}^2 - \vec{S}^2] \]

\[ = \frac{1}{2}[J(J + 1) - L(L + 1) - S(S + 1)] \]

For \( ^3P_2 \) \( \vec{L} \cdot \vec{S} = 1 \)

\( ^3P_1 \) \( \vec{L} \cdot \vec{S} = -1 \)

\( ^3P_0 \) \( \vec{L} \cdot \vec{S} = -2 \)
**Hyperfine:** Again in hydrogen, the proton has dipole moment:

\[ \vec{\mu}_P = \gamma_P \frac{e}{m_P c} \vec{S}_P \quad (\gamma_P = 2.73) \]

The magnetic dipole has a field:

\[ \vec{B}(\vec{r}) = \frac{1}{r^3} \left[ \frac{3(\vec{\mu} \cdot \vec{r})\vec{r}}{r^2} - \vec{\mu} \right] + \frac{8\pi}{3} \vec{\mu} \]

\[ r > a \]

\[ r < a \]

The energy of the electron in the presence of \( \mu_i \):

\[ \Delta H_{SS} = \frac{\gamma_P e^2}{mm_P c^2} \left\{ \frac{1}{r^3} [3(\vec{S}_P \cdot \hat{r})(\vec{S}_e \cdot \hat{r}) - \vec{S}_P \cdot \vec{S}_e] + \frac{8\pi}{3} \vec{S}_P \cdot \vec{S}_e \delta^3(\vec{r}) \right\} \]

Gives rise to the hyperfine structure of hydrogen:

\[ \vec{S}_1 \cdot \vec{S}_2 = \frac{1}{2} [\vec{S}_1^2 - \vec{S}_1^2 - \vec{S}_2^2] = \frac{1}{2} [s(s + 1) - \frac{3}{2}] \]

21 cm line in hydrogen
One can take this over to 1-gluon interaction of QCD:

$$\Delta H_{ij}^{hyp} = \frac{\alpha_s(r)}{m_i m_j} \left\{ \frac{8\pi}{3} \vec{S}_i \cdot \vec{S}_j \delta^3(\vec{r}_{ij}) + \frac{1}{r_{ij}^3} [3(\vec{S}_i \cdot \hat{r}_{ij})(\vec{S}_j \cdot \hat{r}_{ij}) - \vec{S}_i \cdot \vec{S}_j] \right\} \vec{F}_i \cdot \vec{F}_j$$

$$\Delta H_{ij}^{S.O.(c.m.)} = -\frac{\alpha_s(r)}{r_{ij}^3} \left( \frac{1}{m_i} + \frac{1}{m_j} \right) \left( \frac{\vec{S}_i}{m_i} + \frac{\vec{S}_j}{m_j} \right) \cdot \vec{L} \vec{F}_i \cdot \vec{F}_j$$

$$\Delta H_{ij}^{S.O.(TP)} = -\frac{1}{2r_{ij}} \frac{\partial V(r)}{\partial r_{ij}} \left( \frac{\vec{S}_i}{m_i^2} + \frac{\vec{S}_j}{m_j^2} \right) \cdot \vec{L} \vec{F}_i \cdot \vec{F}_j$$

For mesons $\langle \vec{F}_i \cdot \vec{F}_j \rangle = -\frac{4}{3}$
Systematic treatment starts with Wilson loop

Eichten and Feinberg, PR D23, 2724 (1981)
Gromes, Yukon Advanced Study Inst.

• Expanding in 1/m_Q write spin-dependent Hamiltonian in terms of static potential and correlation functions of colour electric and magnetic fields
• With some assumptions one obtains:

\[
V_{\text{spin}}(r) = \frac{1}{m^2} \left( -\frac{k}{2r} + \frac{2\alpha_s}{3r^3} \right) \vec{L} \cdot \vec{S} \\
+ \frac{1}{m^2} \frac{4\alpha_s}{3r^3} S_{12} + \frac{1}{m^2} \frac{32\pi\alpha_s}{9} \delta^3(\vec{r}) \vec{S}_1 \cdot \vec{S}_2
\]

Which corresponds to short range vector and long range scalar exchange

Observation of \(^{1}P_1\) states is important test
Spin-dependent potentials:

• Need some sort of reduction to find spin dependent terms
• Depends on Lorentz nature of potential
  we find phenomenologically
  short range Lorentz Vector 1-gluon exchange
  + long range Lorentz scalar confining potential
• Use Breit-Fermi Hamiltonian
• Spin-dependent interactions are \((v/c)^2\) corrections

Spin-spin interactions:

\[
H_{ij}^{hyp} = \frac{4\alpha_s(r)}{3m_im_j} \left\{ \frac{8\pi}{3} \vec{S}_i \cdot \vec{S}_j \delta^3(\vec{r}_{ij}) + \frac{1}{r_{ij}^3} \left[ 3\vec{S}_i \cdot \vec{r}_{ij}\vec{S}_j \cdot \vec{r}_{ij} - \vec{S}_i \cdot \vec{S}_j \right] \right\}
\]

\[
\vec{S}_1 \cdot \vec{S}_2 = \frac{1}{2} \left[ S^2 - S_1^2 - S_2^2 \right] = \frac{1}{2} \left[ s(s+1) - \frac{3}{2} \right]
\]
Spin-orbit interactions:

\[ H_{ij}^{\text{s.o.}(cm)} = \frac{4\alpha_s(r)}{3r_{ij}^3} \left( \frac{1}{m_i} + \frac{1}{m_j} \right) \left( \frac{\vec{S}_i}{m_i} + \frac{\vec{S}_j}{m_j} \right) \cdot \vec{L} \]

\[ H_{ij}^{\text{s.o.}(ip)} = -\frac{1}{2r_{ij}} \frac{\partial V(r)}{\partial r_{ij}} \left( \frac{\vec{S}_i}{m_i^2} + \frac{\vec{S}_j}{m_j^2} \right) \cdot \vec{L} \]

\[
\begin{align*}
\chi_2(1^{3}P_2): & \quad \vec{L} \cdot \vec{S} = 1 \\
\chi_1(1^{3}P_1): & \quad \vec{L} \cdot \vec{S} = -1 \\
\chi_0(1^{3}P_0): & \quad \vec{L} \cdot \vec{S} = -2 
\end{align*}
\]
Let us examine the spin-dependent splittings in charmonium

- Using H.O. wavefunctions simplifies the calculations
- Fitting the oscillator parameter to the r.m.s. radii of exact solutions is a good approximation:

\[
\psi_{1S} = \frac{2}{\pi^{1/4}} \beta^{3/2} e^{-\beta^2 r^2/2} Y_{00}
\]

\[
\psi_{2S} = \sqrt{\frac{8}{3}} \frac{\beta^{3/2}}{\pi^{1/4}} \left( \frac{3}{2} - \beta^2 r^2 \right) e^{-\beta^2 r^2/2} Y_{00}
\]

\[
\psi_{1P} = \sqrt{\frac{8}{3}} \frac{\beta^{5/2} r}{\pi^{1/4}} e^{-\beta^2 r^2/2} Y_{1m}
\]

\[
\langle r^2 \rangle_{1S} = \frac{3}{2} \frac{1}{\beta^2} = 2.5 \Rightarrow \beta = 0.77
\]

\[
\langle r^2 \rangle_{2S} = \frac{7}{2} \frac{1}{\beta^2} = 11 \Rightarrow \beta = 0.564
\]

\[
\langle r^2 \rangle_{1P} = \frac{5}{2} \frac{1}{\beta^2} \approx 7 \Rightarrow \beta = 0.598
\]

\[
\langle 1/r \rangle_{1P} = \frac{4}{3} \frac{\beta}{\pi^{1/2}} = 0.45
\]

\[
\langle 1/r^3 \rangle_{1P} = \frac{4}{3} \frac{\beta^3}{\pi^{1/2}} = 0.16
\]
Hyperfine Effects:

\[ H_{ij}^{hyp} = \frac{32 \pi}{9} \frac{\alpha_s}{m^2} \vec{S}_1 \cdot \vec{S}_2 \, \delta^3(r_{ij}) \]

\[ \vec{S}_1 \cdot \vec{S}_2 = \frac{1}{2} [s(s + 1) - 3/2] \]

\[ \Rightarrow \begin{cases} 
\langle ^3S_1|\vec{S}_1 \cdot \vec{S}_2|^3S_1 \rangle = +1/4 \\
\langle ^1S_0|\vec{S}_1 \cdot \vec{S}_2|^1S_0 \rangle = -3/4 
\end{cases} \]

\[ \therefore M(^3S_1) - M(^1S_0) = \frac{32 \pi}{9} \frac{\alpha_s}{m^2} \langle \delta^3(r_{ij}) \rangle \]

\[ = \frac{32 \pi}{9} \frac{\alpha_s}{m^2} |\psi(0)|^2 \]

\[ = \frac{32 \pi}{9} \frac{\alpha_s}{m^2} \beta^3 \]

\[ = \frac{32 \pi}{9} \frac{\alpha_s}{m^2} \frac{\beta^3}{\pi^{3/2}} \]

\[ = 115 \text{ MeV} \text{ (where } \beta = 0.77 \text{ GeV, } \alpha_s = 0.32, \, m_c = 1.6 \text{ GeV)} \]

vs 115 MeV from experiment \( (1) \)

\[ M(2^3S_1) - M(2^1S_0) = 67 \text{ MeV} \]
Fine Structure:
We can write the $^3P_J$ Masses as:

$$M = M(1P) + a\langle \vec{L} \cdot \vec{S} \rangle + b\langle S_{12} \rangle$$

$$M(3P_2) = M(1P) + a - \frac{2}{5}b = 3556$$

$$M(3P_1) = M(1P) - a - 2b = 3511$$

$$M(3P_0) = M(1P) - 2 - 4b = 3415$$

$$M(1P) = 3525$$

Lorentz Vector 1-gluon exchange gives:

$$a = \frac{3}{2m^2} \frac{4\alpha_s}{3r^3} = 40\text{ MeV}$$

$$b = \frac{1}{4m^2} \frac{4\alpha_s}{3r^3} = 7\text{ MeV}$$
$^1P_1$ vs $^3P_{cog}$ mass - distinguish models

- Important to distinguish models

- In QM triplet-singlet splittings test
  - the Lorentz nature of the confining potential
  - Relativistic effects

- Important validation of
  - lattice QCD calculations
  - NRQCD calculations

**Observation of $^1P_1$ states is an important test of theory**
Wide variation of theoretical predictions:

TABLE I. Predictions for hyperfine splittings $M(n^3P_{cog}) - M(n^1P_1)$ for $c\bar{c}$ and $b\bar{b}$ levels.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Approach</th>
<th>$n = 1$ $c\bar{c}$ (MeV)</th>
<th>$n = 1$ $b\bar{b}$ (MeV)</th>
<th>$n = 2$ $b\bar{b}$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GI85 [14]</td>
<td>a</td>
<td>8</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>MR83 [15]</td>
<td>b</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>LPR92 [16]</td>
<td>c</td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>OS82 [17]</td>
<td>d</td>
<td>10</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>MB83 [18]</td>
<td>e</td>
<td>$-5$</td>
<td>$-2$</td>
<td>$-2$</td>
</tr>
<tr>
<td>GRR86 [19]</td>
<td>f</td>
<td>$-2$</td>
<td>$-1$</td>
<td>$-1$</td>
</tr>
<tr>
<td>IO87 [20]</td>
<td>g</td>
<td>$24.1 \pm 2.5$</td>
<td>$3.73 \pm 0.1$</td>
<td>$3.51 \pm 0.02$</td>
</tr>
<tr>
<td>GOS84 $\eta_s = 1$ [21]</td>
<td>h</td>
<td>6</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>GOS84 $\eta_s = 0$ [21]</td>
<td>h</td>
<td>17</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>PJF92 [22]</td>
<td>i</td>
<td>$-20.3 \pm 3.7$</td>
<td>$-2.5 \pm 1.6$</td>
<td>$-3.7 \pm 0.8$</td>
</tr>
<tr>
<td>HOOS92 [23]</td>
<td>j</td>
<td>$-0.7 \pm 0.2$</td>
<td>$-0.18 \pm 0.03$</td>
<td>$-0.15 \pm 0.03$</td>
</tr>
<tr>
<td>PTN86 [25]</td>
<td>j</td>
<td>$-3.6$</td>
<td>$-0.4$</td>
<td>$-0.3$</td>
</tr>
<tr>
<td>PT88 [26]</td>
<td>j</td>
<td>$-1.4$</td>
<td>$-0.5$</td>
<td>$-0.4$</td>
</tr>
<tr>
<td>SESAM98 [31]</td>
<td>k</td>
<td>–</td>
<td>$\sim -1$</td>
<td>–</td>
</tr>
<tr>
<td>CP-PACS00 [33]</td>
<td>l</td>
<td>$1.7 - 4.0$</td>
<td>$1.6 - 5.0$</td>
<td>–</td>
</tr>
</tbody>
</table>

EFG 0 -1 -1
Quark Potential Models with 1-gluon exchange:

\[
H_{q\bar{q}}^{hyp} = \frac{32\pi}{9} \frac{\alpha_s}{m_q m_{\bar{q}}} \vec{S}_q \cdot \vec{S}_{\bar{q}} \delta^3(r)
\]

\(\delta\) function is short range but smeared by relativistic effects modeled by a Gaussian.

\cdot gives \(M(3P_{cog}) > M(1P_1)\)

Godfrey & Isgur, PR D32, 189 (1985)

wide variation in predictions indicates need for experimental data
Decays and Transitions

• To calculate Decays and Transitions we need to calculate hadronic matrix elements.
• Define a “Mock” meson which we equate with the wavefunction of the physical meson

\[ |M(\vec{K})| = \sqrt{2E_M} \int d^3p \Phi(\vec{p}) \chi_{s\bar{s}} \phi_{q\bar{q}} \phi_{\text{colour}} |q(\frac{m_q}{m_q + m_{\bar{q}}} \vec{K} + p, s)\bar{q}(\frac{m_{\bar{q}}}{m_q + m_{\bar{q}}} \vec{K} - p, \bar{s})| \]
There are two generic types of matrix elements:

\[ \langle 0|A|M_i \rangle \text{ like in } J/\psi \to e^+e^- \]
\[ \langle M_f|A|M_i \rangle \text{ like in } \chi_{c2} \to J/\psi + \gamma \]

A is some sort of transition operator like:

\[ j_{em}^\mu = \bar{q} \gamma^\mu q \]
Fig. 5. Inclusive photon spectrum at the $\psi'$ obtained by the Crystal Ball experiment. Note that the logarithmic energy scale yields bin sizes approximately proportional to photon energy resolution. The numbers over the spectrum correspond to the expected radiative transitions shown in the spectrum inset.
Fig. 4. - The $b\bar{b}$ level diagram showing transitions between states. We use the familiar spectroscopic notation $n^{2S+1}L_J$, where $n$ is the principal quantum number (with the convention that $n$ is one plus the number of nodes in the wavefunction), and $L$, $S$, and $J$ are the orbital angular momentum, total spin, and total angular momentum. The parity and $C$-parity are given by $P = (-)^{J+1}$ and $C = (-)^{J+3}$. Note that not all states and transitions shown have been observed and not all possible transitions are shown. -- $\gamma$-transitions, —— hadronic transitions.
Figure 11: ARGUS [74] $\gamma(2S) \rightarrow \gamma +$ hadrons with $\gamma \rightarrow e^+e^-$. 

Figure 12: CLEO [75] $T(2S) \rightarrow \gamma +$ hadrons with $\gamma \rightarrow e^+e^-$. The data do not re-line, and it is not included in the fit shown.
Phonon Transitions in $Y(2S)$ and $Y(3S)$ Decays

(CLEO Collaboration)

FIG. 2. Fit to the $Y(2S) \rightarrow \gamma \chi_{bJ} (1P) (J = 2, 1, 0)$ photon lines in the data. The points represent the data (top plot). Statistical errors on the data are smaller than the point size. The solid line represents the fit. The dashed line represents total fitted background. The background subtracted data (points with error bars) are shown at the bottom. The solid line represents the fitted photon lines together. The dashed lines show individual photon lines.

FIG. 3. Fit to the $Y(3S) \rightarrow \gamma \chi_{bJ} (2P) (J = 2, 1, 0)$ photon lines in the data. See caption of Fig. 2 for the description. Small solid line peaks in the bottom plot show the $\chi_{bJ} (2P) \rightarrow \gamma Y (1D)$ and $Y(2S) \rightarrow \gamma Y_{bJ} (1P)$ contributions.
Radiative (e.m.) Transitions

Same physics as in atomic and nuclear systems
An e.m. transition is described by:

For 2 body decay \( M_i \rightarrow M_f \gamma \)

\[
\frac{d\Gamma}{d\cos \theta} = \frac{|M_{if}|^2}{16\pi^2 M_i} (1 - M_f^2/M_i^2) = \frac{|M_{if}|^2}{8\pi^2 M_i^2} k_{\gamma}
\]

\[
\Gamma = \frac{1}{2\pi M^2} |M_{if}|^2 p
\]

where \( p = \frac{(M_i^2 - M_f^2)}{2M_i} \)

\[
d\Gamma = \frac{(2\pi)^4}{2M_i} \delta^4(P_f + p_\gamma - p_i) |M_{fi}|^2 \frac{d^3p_f}{(2\pi)^3 (2E_f)} \frac{d^3p_\gamma}{(2\pi)^3 (2E_\gamma)}
\]
Start with E1 Transitions:

\[ \frac{p^2}{2m} \rightarrow \frac{(\vec{p} - e\vec{A})^2}{2m} = \frac{p^2}{2m} - \frac{e\vec{p} \cdot \vec{A}}{2m} - \frac{e\vec{A} \cdot \vec{p}}{2m} + e^2 \frac{\vec{A}^2}{2m} \]

\[ p^2 / 2m \] is the original kinetic energy term.

Drop higher order \( e^2 \vec{A}^2 \) terms.

Interested in:

\[ H_I = -\frac{e}{2m} (\vec{A} \cdot \vec{p} + \vec{p} \cdot \vec{A}) \]

\[ \vec{A}(x) = \frac{1}{\sqrt{2\omega}} \vec{\epsilon}(\vec{k}) e^{i\vec{k} \cdot \vec{x}} \]

\[ e^{i\vec{k} \cdot \vec{x}} \simeq 1 + i\vec{k} \cdot \vec{x} + \ldots \]

In the long wavelength limit \( \frac{1}{k} \gg r \)

\[ \Rightarrow \vec{A}(x) \simeq \frac{1}{\sqrt{2\omega}} \vec{\epsilon}(\vec{k}) \]

\[ H_I = -\frac{e}{2m} (\vec{\epsilon} \cdot \vec{p} + \vec{p} \cdot \vec{\epsilon}) \]
To evaluate \( \langle A|\vec{p}|B\rangle \cdot \vec{e} \)

Start with \([p_i, r_j] = -i\delta_{ij}\)

\[
[p^2, r_j] = p_i[p_i, r_j] + [p_i, r_j]p_i = -2ip_j
\]

\[
\langle A|p_i|B\rangle = i\langle A|[\vec{p}^2/2, r_j]|B\rangle
\]

\[
= i\mu\langle A|[H, r_j]|B\rangle
\]

\( (H = p^2/2\mu + V(r) \text{ but } [V(r), r] = 0) \)

\[
= i\mu\langle A|Hr_j - r_jH|B\rangle
\]

\[
= i\mu(E_A - E_B)\langle A|r_j|B\rangle
\]

\[
= i\frac{m}{2}\omega\langle A|r_j|B\rangle
\]

\[
\langle A|H|B\rangle = -i\frac{e\mu\omega}{2m} \langle A|r_i|B\rangle \epsilon_i
\]

\[
= -\frac{i\epsilon\omega}{2} \langle A|r_i|B\rangle \epsilon_i
\]

\[
= -\frac{i\epsilon\omega}{2} \langle A|\vec{r}|B\rangle \cdot \vec{e}
\]
There are two methods for evaluating the matrix element

**Method 1:**

The sum over final polarizations is:

$$\sum_{pol} \bar{\epsilon}_i(k) \bar{\epsilon}^*(k) = \delta_{ij} - k_i k_j / \bar{k}^2$$

So:

$$\sum_{pol} |\langle B|H_I|A\rangle|^2 = \omega^2 e^2 Q^2 \left\{ |\langle B|\vec{r}|A\rangle|^2 - |\langle B|\vec{r} \cdot \hat{k}|A\rangle|^2 \right\}$$

Averaging over directions:

$$= \omega^2 e^2 Q^2 \frac{2}{3} |\langle B|\vec{r}|A\rangle|^2$$

**Start with** $^3P_J \rightarrow ^3S_1$

- The orbital angular momentum is zero in the final state
- We may choose any $J_Z$ since we averaged over the photon directions

  Convenient to choose $J_Z = J$
Start by writing down the meson wavefunction: \( |M\rangle = \sqrt{2M} \psi(r) \)
where \( \sqrt{2M} \) is introduced to normalize the wavefunction when integrating over relativistic phase space.

\( ^3 P_2(J_z = 2) : |J = J_Z = 2\rangle = |L = L_Z = 1\rangle \otimes |S = S_Z = 1\rangle = |Y_{11} \uparrow \uparrow\rangle \)
Only \( J'_Z = S'_Z = 1 \) contributes since \( H_I \) does not flip spin.

\[
\langle f | \vec{r} | i \rangle = \langle f | r | i \rangle \int \langle Y_{00} \uparrow \uparrow | \sqrt{\frac{4\pi}{3}} Y_{1-1} | Y_{11} \uparrow \uparrow \rangle d\Omega = \langle f | r | i \rangle \sqrt{\frac{1}{3}}
\]
where \( \langle f | r | i \rangle = \int r^2 dr R_f(r) r R_i(r) \sqrt{2M_i} \sqrt{2M_f} \)

\[
\Gamma(^3P_2 \rightarrow ^3 S_1) = \frac{1}{8\pi M^2} |M_{if}|^2 \omega
\]
\[
= \frac{\omega}{8\pi M^2} \omega^2 e^2 Q^2 |\langle f | r | i \rangle|^2 (s M_i)(2M_f) \times \frac{2}{3} \times \frac{1}{3}
\]
\[
= \frac{4\pi \alpha \omega^3 e_q^2}{8\pi} \frac{8}{9} |\langle f | r | i \rangle|^2 \left( \frac{M_i M_f}{M_i M_i} \right)
\]
\[
= \frac{4}{9} \alpha \omega^3 e_q^2 |\langle f | r | i \rangle|^2 \left( \frac{M_f}{M_i} \right)
\]
For $^3P_1 \rightarrow ^3S_1$

$$|J = J_Z = 1\rangle = \frac{1}{\sqrt{2}}|Y_{11}\rangle \frac{1}{\sqrt{2}}((\uparrow\downarrow + \downarrow\uparrow) - Y_{10} \uparrow\uparrow)$$

so that

$$\langle Y_{00}|\vec{r}|J = J_Z = 1\rangle = \frac{1}{\sqrt{2}} \langle Y_{00}|\vec{r}|Y_{11}\rangle - \frac{1}{\sqrt{2}} \langle Y_{00}|\vec{r}|Y_{10}\rangle$$

$$= \left[ \frac{1}{\sqrt{2}} \langle Y_{00}\frac{1}{\sqrt{3}}(-\hat{x} + i\hat{y})|Y_{11}\rangle - \frac{1}{\sqrt{2}} \langle Y_{00}\frac{1}{\sqrt{3}}\hat{z}|Y_{10}\rangle \right] \langle 1S|r|1P\rangle$$

$$\Rightarrow |\langle ^3S_1|\vec{r}|^3P_1\rangle|^2 = \left[ \frac{11}{2} - \frac{1}{3} + \frac{11}{2} \right]|\langle 1S|r|1P\rangle|^2$$

$^3P_0 \rightarrow ^3S_1$

$$|J = J_Z = 0\rangle = \sqrt{\frac{1}{3}}|Y_{11}\rangle \downarrow\downarrow - Y_{10} \sqrt{\frac{1}{2}}((\uparrow\downarrow + \downarrow\uparrow) + Y_{1-1} \uparrow\uparrow)$$

resulting in $|\langle ^3S_1|\vec{r}|^3P_0\rangle|^2 = \left[ \frac{11}{3} + \frac{11}{3} + \frac{11}{3} \right]|\langle 1S|r|1P\rangle|^2$
Summarizing all these results we obtain:

\[
\Gamma( ^3 P_2 \rightarrow ^3 S_1 \gamma) = \frac{\omega^3 e^2 Q^2}{3\pi} \frac{1}{3} \left| \langle 1S | r | 1P \rangle \right|^2
\]

\[
\Gamma( ^3 P_1 \rightarrow ^3 S_1 \gamma) = \frac{\omega^3 e^2 Q^2}{3\pi} \left\{ \frac{1}{2} \frac{1}{3} + \frac{1}{2} \frac{1}{3} \right\} \frac{1}{3} \left| \langle 1S | r | 1P \rangle \right|^2
\]

\[
\Gamma( ^3 P_0 \rightarrow ^3 S_1 \gamma) = \frac{\omega^3 e^2 Q^2}{3\pi} \left\{ \frac{1}{3} \frac{1}{3} + \frac{1}{3} \frac{1}{3} + \frac{1}{3} \frac{1}{3} \right\} \left| \langle 1S | r | 1P \rangle \right|^2
\]

Comparing these expressions we see that in all cases

\[
\Gamma( ^3 P_J \rightarrow ^3 S_1 \gamma) = \frac{4\alpha \omega^3 Q^2}{9} \left| \langle 1S | r | 1P \rangle \right|^2
\]

Similarly we obtain:

\[
\Gamma( ^3 S_1 \rightarrow ^3 P_J \gamma) = \frac{4\alpha \omega^3 Q^2(2J + 1)}{27} \left| \langle 1S | r | 1P \rangle \right|^2
\]
Let us return to our effective wavefunctions:

\[ \psi_{1S} = \frac{2}{\pi^{1/4}} \beta^{3/2} e^{-\beta^2 r^2/2} Y_{00} \quad \beta = 0.77 \text{ GeV} \]

\[ \psi_{1P} = \sqrt{\frac{8}{3}} \frac{\beta^{5/2} r}{\pi^{1/4}} e^{-\beta^2 r^2/2} Y_{1m} \quad \beta = 0.598 \text{ GeV} \]

This gives:

\[
\langle \psi_{1S} | r | \psi_{1P} \rangle = \frac{2}{\pi^{1/4}} \sqrt{\frac{8}{3}} \frac{1}{\pi^{1/4}} \beta_S^{3/2} \beta_P^{5/2} \int r^4 e^{-(\beta_S^2 + \beta_P^2) r^2/2} dr \\
= \sqrt{\frac{8}{3}} \frac{15}{15} \frac{\beta_S^{3/2} \beta_P^{5/2}}{(\beta_S^2 + \beta_P^2)^{5/2}} \\
= 5.2 \text{ GeV}^{-1}
\]

\[ \Rightarrow \Gamma(3P_2 \rightarrow 3S_1 \gamma) = 0.59 \text{ MeV} \quad \text{vs} \quad \Gamma^{exp} = 0.351^{+0.2}_{-0.14} \text{ MeV} \]

\[ \Gamma(3P_1 \rightarrow 3S_1 \gamma) = \text{MeV} \quad \text{vs} \quad \Gamma^{exp} < 0.355 \text{ MeV} \]
3. E1 transitions

E1 decays sensitive to nodes in wavefunction

radiative transitions
tests internal structure

\[ \Gamma = \frac{4}{3} e_q^2 \alpha C(J_i L_i J_f L_f S) \langle P | r | S \rangle \alpha^3 \]

\[ C(J_i L_i J_f L_f S) = \max(L_i, L_f)(2J_f + 1) \left[ \begin{array}{cc} L_f & J_f \\ J_i & L_i \end{array} \right] \left[ \begin{array}{c} S \\ 1 \end{array} \right]^2 \]

McClary and Byers, PR D28, 1692 (1983)
Including relativistic corrections corresponds to using eigenfunctions and eigenvalues of the Breit-Fermi Hamiltonian (Siegert's theorem)

|         | $<2P|r|3S>$ | $<1P|r|2S>$ | $<1P|r|3S>$ | $<1S|r|2P>$ | $<2S|r|2P>$ |
|---------|-------------|-------------|-------------|-------------|-------------|
|         | GeV$^{-1}$  | GeV$^{-1}$  | GeV$^{-1}$  |             |             |
| DATA    | 2.7±0.2     | 1.9±0.2     |             | 0.050±0.006 | 0.096±0.005 |
| World Average |             |             |             | This measurement |             |
| Model   | NR rel      | NR rel      | NR rel      | NR rel      | NR rel      |
| Kwong,Rosner [13] | 2.7 | 1.6 | 0.023 | 0.13 |
| Fulcher [14]     | 2.6 | 1.6 | 0.023 | 0.13 |
| Büchmuller et al.[15] | 2.7 | 1.6 | 0.010 | 0.12 |
| Moxhay,Rosner [16] | 2.7 | 2.7 | 1.6 | 1.6 | 0.024 | 0.044 | 0.13 | 0.15 |
| Gupta et al.[17] | 2.6 | 1.6 | 0.040 | 0.11 |
| Gupta et al.[18] | 2.6 | 1.6 | 0.010 | 0.12 |
| Fulcher [19]     | 2.6 | 1.6 | 0.018 | 0.11 |
| Danghighian et al.[20] | 2.8 | 2.5 | 1.7 | 1.3 | 0.024 | 0.037 | 0.13 | 0.10 |
| McClary,Byers [21] | 2.6 | 2.5 | 1.7 | 1.6 |             |             |          |
| Eichten et al.[22] | 2.6 | 1.7 | 0.110 | 0.15 |
| Grotch et al.[23] | 2.7 | 2.5 | 1.7 | 1.5 | 0.011 | 0.061 | 0.13 | 0.19 |

Tomasz Skwarnicki, Syracuse U. ICHEP, Amsterdam July,2002
Relativistic effects gives differences between E1 matrix elements:

\[ \langle 2P | r | 3S \rangle = 2.7 \pm 0.2 \text{ GeV}^{-1} \]

\[ \langle 2^3P_2 | r | 3^3S_1 \rangle \approx -2.4 \text{ GeV}^{-1} \]

\[ \langle 2^3P_1 | r | 3^3S_1 \rangle \approx -2.3 \text{ GeV}^{-1} \]

\[ \langle 2^3P_0 | r | 3^3S_1 \rangle \approx -2.2 \text{ GeV}^{-1} \]

\[ \langle 1P | r | 2S \rangle \pm 1.9 \pm 0.2 \text{ GeV}^{-1} \]

\[ \langle 1^3P_2 | r | 2^3S_1 \rangle \approx -1.5 \text{ GeV}^{-1} \]

\[ \langle 1^3P_1 | r | 2^3S_1 \rangle \approx -1.4 \text{ GeV}^{-1} \]

\[ \langle 1^3P_0 | r | 2^3S_1 \rangle \approx -1.3 \text{ GeV}^{-1} \]

see also McClary and Byers, PR D28, 1692 (1983)
Node in $3S$ wavefunction near maximum in $1P$ wavefunction so large cancellation very sensitive to details of the wavefunctions

\[
\begin{align*}
\langle 1^3 P_2 | r | 3^3 S_1 \rangle & \approx +0.096 \text{ GeV}^{-1} \\
\langle 1^3 P_1 | r | 3^3 S_1 \rangle & \approx +0.040 \text{ GeV}^{-1} \\
\langle 1^3 P_0 | r | 3^3 S_1 \rangle & \approx -0.026 \text{ GeV}^{-1}
\end{align*}
\]
Table I: Properties of $\psi(2S) \rightarrow \gamma \chi_{cJ}$ decays, using results from Refs. [54] and [66] as well as Eq. (6).

| $J$ | $k_\gamma$ (MeV) | $B$ [66] (%) | $\Gamma[\psi(2S) \rightarrow \gamma \chi_{cJ}]$ (keV) | $|\langle 1P | r | 2S \rangle|$ (GeV$^{-1}$) |
|-----|-----------------|--------------|----------------------------------|----------------------------------|
| 2   | 127.60±0.09     | 9.33±0.14±0.61 | 31.4±2.4                         | 2.51±0.10                       |
| 1   | 171.26±0.07     | 9.07±0.11±0.54 | 30.6±2.2                         | 2.05±0.08                       |
| 0   | 261.35±0.33     | 9.22±0.11±0.46 | 31.1±2.0                         | 1.90±0.06                       |
Table III: Properties of the transitions $\chi_{cJ} \rightarrow \gamma J/\psi$. (Ref. [54]; Eq. (6)).

| $J$ | $k_\gamma$ (MeV) | $\Gamma(\chi_{cJ} \rightarrow \gamma J/\psi)$ (keV) | $|\langle 1S|r|1P\rangle|$ (GeV$^{-1}$) |
|-----|------------------|---------------------------------|---------------------------------|
| 2   | 429.63±0.08      | 416±32                          | 1.91±0.07                       |
| 1   | 389.36±0.07      | 317±25                          | 1.93±0.08                       |
| 0   | 303.05±0.32      | 135±15                          | 1.84±0.10                       |
Production of the D-wave states

• By direct scans in $e^+e^-$ to produce $^3D_1$ ($J^P_C = 1--$)
• Use for $4\gamma E1$ cascade to search for $\gamma(1^3D_J)$

• Estimate the radiative widths and $BR$ using quark model
• In e.m. cascades: \( Y(3S) \rightarrow \gamma \chi'_b \rightarrow \gamma\gamma \, ^3D_J \)

\[
\Gamma = \frac{4}{3} e_Q^2 \alpha \mathcal{C}(J_i L_i J_f L_f S) \langle P| r| S \rangle \omega^3 \quad \mathcal{C}(J_i L_i J_f L_f S) = \max(J_i, L_f)(2J_f + 1) \left\{ \frac{L_f}{J_i} \frac{J_f}{L_i} \right\}^2
\]

• Some 4\( \gamma \) cascades with observable # of events/10\(^6\) \( Y(3S) \)'s:

<table>
<thead>
<tr>
<th>Cascade</th>
<th>Events</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 3^3S_1 \rightarrow 2^3P_2 \rightarrow 1^3D_3 \rightarrow 1^3P_2 \rightarrow 1^3S_1 )</td>
<td>7.8</td>
</tr>
<tr>
<td>( 3^3S_1 \rightarrow 2^3P_2 \rightarrow 1^3D_2 \rightarrow 1^3P_1 \rightarrow 1^3S_1 )</td>
<td>2.7</td>
</tr>
<tr>
<td>( 3^3S_1 \rightarrow 2^3P_1 \rightarrow 1^3D_2 \rightarrow 1^3P_1 \rightarrow 1^3S_1 )</td>
<td>20</td>
</tr>
<tr>
<td>( 3^3S_1 \rightarrow 2^3P_1 \rightarrow 1^3D_1 \rightarrow 1^3P_1 \rightarrow 1^3S_1 )</td>
<td>3.3</td>
</tr>
</tbody>
</table>

S.G + J. Rosner, Phys Rev D64, 097501 (2001)

Expect ~38 events /10\(^6\) \( Y(3S) \) via \(^3D_J\)

• The e\(^+\)e\(^-\) final states leads to less background
• \( \mu^+\mu^- \) final states also contribute if \( \mu \)'s are identified
CLEO finds:

\[ \mathcal{B}(\Upsilon(3S) \to \gamma \gamma \gamma) \to \gamma \gamma \gamma \to \gamma \gamma \gamma \ell^+ \ell^-) = (3.3 \pm 0.6 \pm 0.5) \times 10^{-5} \]

(vs GR prediction of 3.8 x 10^{-5})

- Mass averaged over different fits: \(10162.2 \pm 1.6 \text{ MeV}\)
- Inconsistent with the \(\Upsilon(1D_3)\)
- Could be the \(\Upsilon(1D_2)\) or \(\Upsilon(1D_1)\)
- The theory predicts the rate ratio: \(\Upsilon(1D_2)/\Upsilon(1D_1) = 6\)
- Thus, the \(\Upsilon(1D_2)\) is the most likely interpretation
Because quarks have spin they may emit a photon via a spin flip
- The magnetic dipole transition

To obtain the interaction Hamiltonian we perform a non-relativistic reduction of

\[ H_I = e \int dx j_{em}^\mu(x) A_\mu(x) \]

where \( j_{em}^\mu(x) = \bar{q}(x) Q \gamma^\mu q(x) \)

We expand the Dirac spinors to lowest order in \( p/m \)
Denoting the large and small components by \( q_1 \) and \( q_2 \)

\[ q_2(x) = -\frac{i \vec{\sigma} \cdot \vec{\nabla}}{2m} q_1(x) \]
\[ \vec{j}_{em}(x) = \frac{-i}{2m} [q_1^\dagger Q(\nabla q_1) - (\nabla q_1^\dagger)Qq_1 + i\nabla \times q_1^\dagger Q\vec{\sigma}q_1] \]

So the interaction Hamiltonian is given by:

\[ H_I = \frac{-eQ}{2m} [\vec{A}(\vec{r}) \cdot \vec{p} + \vec{p} \cdot \vec{A}(\vec{r}) + \vec{\sigma} \cdot [\vec{\nabla} \times \vec{A}(\vec{r})] \]

So:

\[ \langle 0 | H_I | \gamma(\vec{k}, \epsilon) \rangle = -\frac{1}{(2\pi)^{3/2}} \frac{1}{(2\omega)^{1/2}} eQ \frac{1}{2m} [e^{i\vec{k} \cdot \vec{r}} \vec{\epsilon} \cdot \vec{p} + \vec{\epsilon} \cdot \vec{p} e^{i\vec{k} \cdot \vec{r}} + i\vec{\sigma} \cdot (\vec{k} \times \vec{\epsilon}) e^{i\vec{k} \cdot \vec{r}}] \]

(For antiquarks change the sign of the charge)
\[ \mu = \frac{e}{2m_1} \]

Is the magnetic dipole moment of the quark

For magnetic dipole transitions:

\[ M_{if} = i\mu \langle f | \vec{\sigma} | i \rangle \cdot \vec{k} \times \vec{\epsilon}^* \]

\[ \vec{\epsilon} = \frac{1}{\sqrt{2}} (1, \pm i, 0) \]

\[
\begin{vmatrix}
\sigma_x & \sigma_y & \sigma_z \\
k_x & k_y & k_z \\
1 & i & 0 \\
\end{vmatrix}
= i\sigma_z (k_x + ik_y) - ik_z \sigma_x + k_z \sigma_y
\]

Choosing \( z \) as the \( \gamma \) direction

\[ M_{if} = -\frac{i e_q}{2m} k_\gamma \langle f | \sigma_x - i\sigma_y | i \rangle \text{ where } \sigma_x - i\sigma_y = \sigma_+ \]

if instead take \( \vec{k} = k_y \)

\[ M_{if} = -\frac{i e_q}{2m} k_\gamma \langle f | \sigma_z | i \rangle \]

\[ = k_\gamma \sqrt{2M_i} \sqrt{2M_f} \int d^3r \psi_f^*(r) \psi_i(r) \times \langle f | \sum \mu_i \sigma_i | i \rangle \]
e.g. $J/\psi \rightarrow \eta_c \gamma (^3S_1 \rightarrow ^1S_0 \gamma)$

\[
A(^3S_1 \rightarrow ^1S_0 \gamma) = -i\kappa \gamma \sqrt{2M_i} \sqrt{2M_f} \langle f|i \rangle \\
\times \langle \sqrt{\frac{1}{2}}(\uparrow \downarrow - \downarrow \uparrow)(\sigma_x - i\sigma_y)_{\bar{q}} \frac{e_q}{2m_q} \frac{(\sigma_x - i\sigma_y)_{\bar{q}}}{\sqrt{2}} + \frac{\mu_{\bar{q}}}{\sqrt{2}} \frac{(\sigma_x - i\sigma_y)_{\bar{q}}}{\sqrt{2}} \mid \uparrow \uparrow \rangle \\
= -i\kappa \gamma \sqrt{2M_i} \sqrt{2M_f} \langle f|i \rangle \left[ \frac{-e_q}{2m_q} + \frac{e_{\bar{q}}}{2m_{\bar{q}}} \right] \\
= -i\kappa \gamma \sqrt{2M_i} \sqrt{2M_f} \langle f|i \rangle \frac{e e_q}{m_c}
\]

\[
\Rightarrow \frac{d\Gamma}{d\Omega} = k_{\gamma} \frac{4\pi \alpha}{8\pi^2} k_{\gamma}^2 |\langle f|i \rangle|^2 \frac{e_c^2}{m_c^2}
\]

averaging over angles gives the total width

\[
\Gamma = \frac{k_{\gamma}^3}{3\pi} |\langle f|i \rangle|^2 \frac{e_c^2}{m_c^2}
\]

Take $\langle f|i \rangle = 1 \quad \omega = 115$

so $\Gamma = 0.19$ MeV vs 0.88 keV (expt)

What about? $^2^3S_1 \rightarrow ^1^1S_0$

$\langle f|i \rangle = 0$ since $2S \perp 1S$
The decay $\psi(2S) \rightarrow \gamma \eta_c(1S)$ is a forbidden magnetic dipole (M1) transition. The photon energy is 638 MeV, leading to a non-zero matrix element $\langle 1S|j_0(kr/2)|2S \rangle$.

$$\Gamma[\psi(2S) \rightarrow \gamma \eta_c(1S)] = (1.00 \pm 0.16)$$

$$|\langle 1S|j_0(kr/2)|2S \rangle| = 0.045 \pm 0.004.$$

The $h_c(1^1P_1)$ matrix element $\Gamma[h_c(1^1P_1) \rightarrow \eta_c(1^1S_0) + \gamma] = \frac{4}{9} \alpha e^2 \left|\langle 1^3S_0|l^0P_1 \rangle\right|^2 \omega^3 = 37$ keV.
M1 transitions: production of $\eta_b(nS)$ states

Proceeds via magnetic dipole (M1) transitions:

$$\Upsilon(nS) \rightarrow \eta(n'S) + \gamma$$

$$\Gamma(3S_1 \rightarrow 1S_0 + \gamma) = \frac{4}{3} \alpha \frac{e_Q^2}{m_Q^2} \left| \langle f | j_0(kr/2) | i \rangle \right|^2 \omega^3$$

- Hindered transitions have large phase space
- Relativistic corrections resulting in differences in $^3S_1$ and $^1S_0$ wavefunctions due to hyperfine interaction
<table>
<thead>
<tr>
<th></th>
<th>Transition</th>
<th>BR (10^{-4})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y(3S)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Γ_{tot}=52.5 keV)</td>
<td>→ $2^1S_0$</td>
<td>4.7</td>
</tr>
<tr>
<td></td>
<td>→ $1^1S_0$</td>
<td>25</td>
</tr>
<tr>
<td>Y(2S)</td>
<td>→ $2^1S_0$</td>
<td>0.21</td>
</tr>
<tr>
<td>(Γ_{tot}=44 keV)</td>
<td>→ $1^1S_0$</td>
<td>13</td>
</tr>
<tr>
<td>Y(1S)</td>
<td>→ $1^1S_0$</td>
<td>2.2</td>
</tr>
<tr>
<td>(Γ_{tot}=26.3 keV)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Expect substantial rate to produce $\eta_b$'s
- Also $Y(3S) \rightarrow h_b(^1P_1) \pi\pi \rightarrow \eta_b + \gamma + \pi\pi$

BR=0.1-1%  \hspace{1cm} BR = 50%

[Kuang & Yan PRD24, 2874 (1981); Voloshin Yad Fiz 43, 1571 (1986)]
Decays:

\[ J/\psi \rightarrow e^+ e^- \]

\[ (^3S_1 \rightarrow e^+ e^-) \]

\[
A(V_i \rightarrow e^+ e^-) \equiv \langle e^+ e^- | M | V_i \rangle
\]

\[
= \frac{4\pi\alpha e_q}{M^2} \langle e^+ e^- | j_k^{(em)} | 0 \rangle \langle 0 | j_k | V_i \rangle
\]

\[
= \frac{4\pi\alpha e_q}{M^2} \bar{U}_e(-p_+)\gamma_k U(p_-) \langle 0 | j_k | V_i \rangle
\]

\[
\langle 0 | j_k | V_i \rangle = \sqrt{3} \times \frac{2M}{2} \int d^3p \phi_s(p) Y_{00} \langle 0 | j^\mu_{em} | c\bar{c} \rangle
\]

where

\[
\sum_{\text{colour}} \sqrt{\frac{1}{3}} (r\bar{r} + b\bar{b} + g\bar{g}) = \frac{3}{\sqrt{3}} = \sqrt{3}
\]

Typically express the matrix element in the form:

\[
\langle 0 | j^\mu_{em}(0) | \psi(k, \lambda) \rangle = \frac{\epsilon^\mu(k, \lambda)}{(2\pi)^{3/2}} f_\psi
\]
In non-relativistic limit

\[ \Rightarrow \langle 0 | j^\mu_{em}(0) | V(\uparrow) \rangle = \sqrt{12M} \, \epsilon^\mu(\uparrow) \, \psi_S(0) \]
\[ \equiv \epsilon^\mu(k, \lambda) \, f_V \]
\[ f_V = \sqrt{12M} \, \psi_S(0) \]
\[\Gamma = \frac{1}{2M} \int |M|^2 \frac{m_e}{E_{e+}} \frac{m_e}{E_{e-}} \frac{d^3p_+}{(2\pi)^3} \frac{d^3p_-}{(2\pi)^3} (2\pi)^4 \delta^4(P - p_+ - p_-)\]

\[= \frac{e_Q^2 e^4}{12\pi M^3} (12M (2\pi)^3 |\psi(0)|^2)\]

\[= \frac{16\pi^2 \alpha^2 e_Q^2}{\pi M^3} M |\psi(0)|^2\]

\[= \frac{16\pi \alpha^2 e_Q^2}{M^2} |\psi(0)|^2\]

\[\psi_S(0) = \frac{1}{\sqrt{4\pi}} R(0)\]
What about $\psi''(3770)$? $e^+e^- \rightarrow \psi''(3770)$

$^3D_1$ state so expect $\Gamma=0$ since $\psi_D(0)=0$ but not so

$$|V(\uparrow)\rangle = \sqrt{6M} \int d^3p \, \phi_D(p) \{ \sqrt{3/5} Y_{2+2}(\theta, \phi) |q(\downarrow) \bar{q}(\downarrow) \rangle$$

$$- \sqrt{3/10} Y_{2+1}(\theta, \phi) |q(\uparrow) \bar{q}(\downarrow) \rangle + \sqrt{1/10} Y_{20}(\theta, \phi) |q(\uparrow) \bar{q}(\uparrow) \rangle \}$$

After much work get:

$$\langle 0| j^{\mu}_{em}(0)|V(\uparrow)\rangle = \frac{\sqrt{12M}}{(2\pi)^3} e^{\mu}(\uparrow) \int d^3p \, \frac{\phi_D(p)}{\sqrt{32\pi}} \frac{4}{3} \frac{p^2}{E(E+m)}$$

$$\lim_{x \rightarrow 0} \int d^3p \, \phi_D(p) \frac{p^2}{2m^2} \frac{e^{i\vec{p} \cdot \vec{x}}}{(2\pi)^{3/2}} = -\frac{1}{2m^2} \lim_{x \rightarrow 0} \frac{\partial^2}{\partial x_i^2} \int d^3p \, \frac{e^{i\vec{p} \cdot \vec{x}}}{(2\pi)^{3/2}} \phi_D(p)$$

$$= -\frac{1}{2m^2} \frac{\partial^2 R_D(0)}{\partial r^2} = -\frac{1}{2m^2} R''_D(0)$$

In general, for state of angular momentum $L$ get $R^{(L)}(0)$
More carefully get:

\[
\langle 0 | j_{\mu}^{em}(0) | V(\uparrow) \rangle = \frac{\sqrt{12M}}{(2\pi)^{3/2}} \frac{5}{4} \frac{R''(0)}{m^2 \sqrt{2\pi}} e^\mu(\uparrow)
\]

and

\[
\Gamma = \frac{\alpha^2 (e_q/e)^2}{M_V^2} \frac{25}{2} \frac{|R''_D(0)|^2}{m_q^2}
\]
Also have decays to hadronic final states:

\[ \eta_c \rightarrow 2\gamma \]

\[ \tau \rightarrow \eta \gamma \]

\[ \eta \rightarrow 3\gamma \]

\[ \phi \rightarrow \eta \gamma \]

\[ \eta' \rightarrow \gamma \gamma \]

\[ \chi_c \rightarrow \eta \gamma \]

\[ \chi_{bc} \rightarrow \gamma \gamma \]

\[ \chi_{bc} \rightarrow \eta \gamma \gamma \]

\[ \chi_{bc} \rightarrow \eta' \gamma \gamma \]

Start with annihilation rates for positronium:

\[ \Gamma(^1S_0 \rightarrow 2\gamma) = \frac{4\pi\alpha^2}{m^2} |\psi_S(0)|^2 = \frac{4\alpha^2}{m^2} |R_S(0)|^2 \]

\[ \Gamma(^3P_0 \rightarrow 2\gamma) = \frac{256 \alpha^2}{3 m^4} |R'_P(0)|^2 \]

\[ \Gamma(^3P_2 \rightarrow 2\gamma) = \frac{4}{15} \Gamma(^3P_0 \rightarrow \gamma\gamma) \left( \frac{M_0}{M_2} \right) \]

\[ \Gamma(^3S_1 \rightarrow 3\gamma) = \frac{16}{9\pi} (\pi^2 - 9) \frac{\alpha^3}{m^2} |R_S(0)|^2 \]

To relate to hadron decays include quark charges.

For decays to gluons must include \( \alpha_S \) and \( \lambda \)'s for each gluon.
For 3 gluons/photons:

\[
\frac{M(3g)}{M(3\gamma)} = \frac{\alpha_s^{3/2}}{e_q^3 \alpha^{3/2}} \frac{\delta_{j}^{i} \delta_{k}^{j} \delta_{i}^{k}}{\delta_{i}^{j} \delta_{j}^{k} \delta_{k}^{i}} = \frac{\alpha_s^{3/2}}{e_q^3 \alpha^{3/2}} \frac{1}{2} \text{Tr}(\{\lambda_a/2, \lambda_b/2\} \lambda_c/2)
\]

\[
\Rightarrow \frac{\Gamma(2g)}{\Gamma(2\gamma)} = \frac{5}{54} \frac{\alpha_s^{3}}{\alpha^{3} e_q^{6}} \text{ where } \sum_{a,b,c} (d_{abc})^2 = 40/3
\]

where \(\text{Tr}(\lambda_a/2 \lambda_b/2) = \frac{1}{2} \delta_{ab}\)
\[
\Gamma(\eta_c \to 2\gamma) = 12\alpha^2 e_q^4 \frac{|R_S(0)|^2}{M^2}
\]

\[
\Gamma(\eta_c \to 2g) = \frac{8}{3} \alpha_s \frac{|R_S(0)|^2}{M^2}
\]

\[
\Gamma(J/\psi \to 3\gamma) = \frac{16(\pi^2 - 9)\alpha^3}{3} e_q^6 \frac{|R_S(0)|^2}{M^2}
\]

\[
\Gamma(J/\psi \to 3g) = \frac{40}{81\pi} (\pi^2 - 9)\alpha_s^3 \frac{|R_S(0)|^2}{M^2}
\]

\[
\Gamma(J/\psi \to 2g\gamma) = \frac{32}{9\pi} (\pi^2 - 9)\alpha_s^2 \alpha e_1^2 \frac{|R_S(0)|^2}{M^2}
\]

For Completeness:

\[
\Gamma(\chi_0 \to 2g) = 96\alpha_s^2 \frac{|R'_{\chi_0}(0)|^2}{M_{\chi_0}^4}
\]

\[
\Gamma(\chi_1 \to q\bar{q}g) = \frac{n_f}{3} \frac{128}{3\pi} \alpha_s^3 \frac{|R'_{\chi_1}(0)|^2}{M_{\chi_0}^4} \ln \left( \frac{4m_c^2}{4m_c^2 - M_X^2} \right)
\]

\[
\Gamma(\chi_2 \to 2g) = \frac{128}{5} \alpha_s^2 \frac{|R'_{\chi_2}(0)|^2}{M_{\chi_0}^4}
\]

\[
\Gamma(h_c \to q\bar{q}g) = \frac{320}{9\pi} \alpha_s^3 \frac{|R'_{h_c}(0)|^2}{M_{h_c}^4} \ln \left( \frac{4m_c^2}{4m_c^2 - M_{h_c}^2} \right)
\]
4. **What about mesons with light quarks?**

Historically, it was the successes of the quark model that led many physicists to believe that the quark model has something to do with reality.
Essential features are the same, except:

- Relative importance of relativistic effects
- Hyperfine splittings are comparable in size to orbital splittings

Conclude

- Potential models approximately valid
Hadron masses in a gauge theory*

A. De Rújula, Howard Georgi,† and S. L. Glashow

Lyman Laboratory, Department of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 24 February 1975)

We explore the implications for hadron spectroscopy of the "standard" gauge model of weak, electromagnetic, and strong interactions. The model involves four types of fractionally charged quarks, each in three colors, coupling to massless gauge gluons. The quarks are confined within colorless hadrons by a long-range spin-independent force realizing infrared slavery. We use the asymptotic freedom of the model to argue that for the calculation of hadron masses, the short-range quark-quark interaction may be taken to be Coulomb-like. We rederive many successful quark-model mass relations for the low-lying hadrons. Because a specific interaction and symmetry-breaking mechanism are forced on us by the underlying renormalizable gauge field theory, we also obtain new mass relations. They are well satisfied. We develop a qualitative understanding of many features of the hadron mass spectrum, such as the origin and sign of the Σ-Λ mass splitting. Interpreting the newly discovered narrow boson resonances as states of charmonium, we use the model to predict the masses of charmed mesons and baryons.
Flavour content:

\[
\begin{align*}
|\rho^+\rangle, |\pi^+\rangle &= -|ud\rangle \\
|\rho^0\rangle, |\pi^0\rangle &= \frac{1}{\sqrt{2}}|u\bar{u} - d\bar{d}\rangle \\
|\omega\rangle &= \frac{1}{\sqrt{2}}|u\bar{u} + d\bar{d}\rangle \\
|\eta\rangle &= \frac{1}{\sqrt{6}}|u\bar{u} + d\bar{d} - 2s\bar{s}\rangle \\
|\eta'\rangle &= \frac{1}{\sqrt{3}}|u\bar{u} + d\bar{d} + s\bar{s}\rangle \\
|\phi\rangle &= |s\bar{s}\rangle \\
|K^+\rangle &= |u\bar{s}\rangle \\
|K^0\rangle &= |d\bar{s}\rangle \\
|\bar{K}^0\rangle &= -|s\bar{d}\rangle \\
|K^-\rangle &= |s\bar{u}\rangle
\end{align*}
\]
In heavy quarkonium we used:

\[
M = m_1 + m_2 + E_{nl} \\
\left[ \frac{p^2}{2\mu} + V(r) \right] \psi = E_{nl} \psi
\]

This is a non-relativistic formula \((v/c) = \) \(bb\) 0.26
\(cc\) 0.45
\(ss\) 0.78
\(uu\) 0.9

What do we do?
• Use it anyway and see what happens. Taking this approach the general features are OK
• Try to relativize it.
Spin dependent interactions:

\[ \Delta[M(^3S_1) - M(^1S_0)] = \frac{3\pi\alpha_s}{9m_1m_2} |\psi(0)|^2 \]

Approximate \(^3S_1\) and \(^1S_0\) masses by:

\[ M(^3S_1) = M(S) + \frac{1}{4} \frac{a}{m_q m_{\bar{q}}} \]

\[ M(^1S_0) = M(S) - \frac{3}{4} \frac{a}{m_q m_{\bar{q}}} \]

If \(a\) is approximately constant:

\[ \frac{M(\rho) - M(\pi)}{M(K^*) - M(K)} \approx \frac{m_u m_s}{m_u m_u} \approx \frac{m_s}{m_u} \approx \frac{500}{300} \approx 1.7 \]

\[ \begin{align*}
770 - 140 & \approx 630 \\
892 - 495 & \approx 400
\end{align*} \approx 1.7 \]

Similarly:

\[ \frac{M(K^*) - M(K)}{M(D^*) - M(D)} \approx \frac{m_u m_c}{m_u m_u} \approx \frac{m_c}{m_u} \approx \frac{1.6}{0.55} \approx 2.9 \]

\[ \begin{align*}
892 - 494 & \approx 400 \\
2010 - 1870 & \approx 140
\end{align*} \approx 2.9 \]

So splittings reasonably well described.

Because \(^3P_{cog} - ^1P_1\) splitting is small supports short range contact interaction
Electromagnetic transitions:

As before:

\[ \Gamma_{M1} = \frac{k^3}{3\pi} \left| \langle f | i \rangle \right|^2 \left| \sum \mu_i \sigma_{zi} \right|^2 \]

For example:

\[ K^{*-} \rightarrow K^+ \gamma \]

\[ \langle u\bar{s} \frac{1}{\sqrt{2}} (\uparrow \downarrow - \downarrow \uparrow) | \frac{e_i}{2m_i} \sigma_z | u\bar{s} \frac{1}{\sqrt{2}} (\uparrow \downarrow - \downarrow \uparrow) \rangle \]

\[ = \frac{1}{2} \langle u\bar{s} | \frac{e_u}{2m_u} + \frac{e_q}{2m_q} - \frac{e_q}{2m_q} - \frac{e_{\bar{q}}}{2m_{\bar{q}}} | u\bar{s} \rangle \]

\[ = \frac{1}{2} \left[ \frac{e_u}{m_u} - \frac{e_s}{m_s} \right] = \frac{1}{2} \left[ \frac{2}{3} \frac{1}{m_u} - \frac{1}{3} \frac{1}{m_s} \right] \]
Strong (Zweig allowed) Decays:

A number of models to calculate strong decays.

Give good qualitative agreement with experiment with only 1 free parameter (using QM wavefunctions)

Important input to disentangle hadron spectrum
Relativistic effects:

Clearly light quark hadrons are relativistic

Various attempts to "relativize" QM

Generally improves agreement

But much is missing. Major battles about what is correct approach.

BUT QM seems to get the physics right.

“Better to get the right degrees of freedom”
Generally, good agreement for confirmed states
Many unconfirmed states:
\( f_1(1530), h_1(1380) \)

Many puzzles:
\( \eta(1440), f_1(1420), f_0(1500), f_J(1710), f_J(2200) \)
Scientists find mystery particle

By Dr David Whitehouse
BBC News Online science editor

Scientists have found a sub-atomic particle they cannot explain using current theories of energy and matter.

The discovery was made by researchers based at the High Energy Accelerator Research Organisation in Tsukuba.

Classified as X(3872), the particle was seen fleetingly in an atom smasher and has been dubbed the "mystery meson".

The Japanese team says understanding its existence may require a change to the Standard Model, the accepted theory of the way the Universe is constructed.

An eternity

X(3872) was found among the decay products of so-called beauty mesons - sub-atomic particles that are produced in large numbers at the Tsukuba "meson factory".

It weighs about the same as a single atom of helium and exists for only about one billionth of a trillionth of a second before it decays into other longer-lived, more familiar
Possible new C= (+) cc states at these masses!

2P or not 2P that is the question!
\( X(3940) \)

Seen by Belle recoiling against \( J/\psi \) in \( e^+e^- \) collisions

\[ M = 3943 \pm 6 \pm 6 \text{ MeV} \]
\[ \Gamma < 52 \text{ MeV} \]

\[ \text{BR}(X \rightarrow DD^*) = 96^{+45}_{-32} \pm 22\% \]

\[ \text{BR}(X \rightarrow DD) < 41\% \ (90\% \ CL) \]

Suggests unnatural parity state

\[ \text{BR}(X \rightarrow \omega J/\psi) < 26\% \ (90\% \ CL) \]

• Decay to \( DD^* \) but not \( DD \) suggests unnatural parity state
Belle speculates that X is $3^1S_0$ given the $3^3S_1$ $\psi(4040)$
  - Mass is roughly correct
  - $\eta_c$ and $\eta_c'$ are also produced in double charm production

Predicted width for $3^1S_0$ with $M=3943 \sim 50$ MeV
  - close to $\Gamma(X(3943))$ upper bound

Identification of $\psi(4040)$ as $3^3S_1$ state implies hyperfine splitting 88 MeV with X(3943)

Larger than the 2S splitting and larger than predicted in potential models

Discrepancy could be due to:
  - Difficulty in fitting true pole position of $3^3S_1$ state
  - Nearby thresholds with s-wave + p-wave charm mesons so possibly stronger threshold effects

Test of $3^1S_0\eta_c$ assignment is search for this state in $\gamma\gamma \rightarrow DD^*$

See also Eichten Lane Quigg PRD73 014014(2006)
$Y(3940)$

See in $\omega J/\psi$ subsystem of the decay $B \rightarrow K \pi \pi \pi J/\psi$


$M=3943 \pm 11 \pm 13$ MeV
$\Gamma = 87 \pm 22 \pm 26$ MeV
Not seen in $Y \rightarrow DD$ or $DD^*$

Mass and width suggest radially excited $P$-wave charmonium

But $\omega J/\psi$ decay mode is peculiar:
$BR(B \rightarrow KY) \cdot BR(Y \rightarrow \omega J/\psi) = 7.1 \pm 1.3 \pm 3.1 \cdot 10^{-5}$
where one expects $BR(B \rightarrow K\chi'_{cJ}) < BR(B \rightarrow K\chi_{cJ}) = 4 \cdot 10^{-4}$

Implies $BR(Y \rightarrow \omega J/\psi) > 12\%$ which is unusual for state above open charm threshold
Possibility is $2^3P_1$ cc state: identifies $Y(3943)$ as $2P \chi'_c$.

- $DD^*$ is the dominant decay mode.
- Width consistent with $Y(3943)$: $\Gamma = 135 \text{ MeV}$.
- $\chi'_c$ is seen in B decays.

$1^{++} \rightarrow \omega J/\psi$ is unusual.

- but corresponding $\chi'_b_{1,2} \rightarrow \omega Y(1S)$ also seen.
- Maybe rescattering: $1^{++} \rightarrow DD^* \rightarrow \omega J/\psi$.
- Maybe due to mixing with $1^{++}$ molecular state $X(3872)$?

Important to - look for DD and $DD^*$.
- study angular distributions to DD and $DD^*$.
Observed by Belle in $\gamma\gamma \rightarrow DD$

$M = 3929 \pm 5 \pm 2$ MeV

$\Gamma = 29 \pm 10 \pm 2$ MeV

Two photon width:

$\Gamma_{\gamma\gamma} \cdot B_{DD} = 0.18 \pm 0.05 \pm 0.03$ keV

DD angular distribution consistent with $J=2$

Below $D^* D^*$ threshold

• Obvious candidate for \( \chi'_c \) (the \( \chi'_c \) cannot decay to DD)

• Predicted \( \chi'_c \) mass is 3972
  \[ \Gamma(\chi'_c \rightarrow DD) = 21.5 \text{ MeV} \]
  \[ \Gamma(\chi'_c \rightarrow DD^*) = 7.1 \text{ MeV} \]
  \[ \Gamma = 47 \text{ MeV assuming } M(\chi'_c) = 3931 \]

• In reasonable agreement with experiment

• Predicted \( BR(\chi'_c \rightarrow DD) = 70\% \Rightarrow \Gamma_{\gamma\gamma} \times B_{DD} = 0.47 \text{ keV} \)
  (\( \Gamma_{\gamma\gamma} \) from T. Barnes, IXth Intl. Conf. on \( \gamma\gamma \) Collisions, La Jolla, 1992.)

• Observed two-photon width about 1/2 predicted value for \( \chi'_c \)
Could further study $2^3P_J$ states via radiative transitions:

*Can find* all three $^32P_J$ cc states using

$\psi(4040)$ and $\psi(4160) \rightarrow \gamma DD, \gamma DD^*$

All three E1 rad BFs of the $\psi(4040)$ are $\sim 0.5 \times 10^{-3}$.

These would further test whether the $Z,X,Y (3.9)$ are 2P cc
New state 1\textsuperscript{st} observed by Belle: X(3871)


Confirmed by: CDF

DO

BABAR

\( M = 3872.0 \pm 0.6 \pm 0.5 \text{ MeV} \quad \Gamma < 2.3 \text{ MeV at 90\% C.L.} \)

width consistent with detector resolution.

1. D\( ^0 \)D\( ^*0 \) molecule
2. A charmonium hybrid
3. \( 2^3S_J \) \( 1^3D_2 \) state?
4. Glueball?
Charmonium Options for the X(3872)

Barnes, Godfrey & Swanson, in preparation

New state 1st observed by Belle: X(3871)  hep-ex/0309032

Observation of a new narrow charmonium state in exclusive
\[ B^\pm \rightarrow K^\pm \pi^+ \pi^- J/\psi \] decays

\[ M = 3872.0 \pm 0.6 \pm 0.5 \text{ MeV} \quad \Gamma < 2.3 \text{ MeV at 90\% C.L.} \]
width consistent with detector resolution.

1. D^0 D^{*0} molecule
2. A charmonium hybrid
3. 1^3D_2 state?
The mass of the state is right at the \( D^0D^{*0} \) threshold!

This suggests a loosely bound \( D^0D^{*0} \) molecule, right below the dissociation energy

“Molecular Charmonium” discussed in literature since 1975

<table>
<thead>
<tr>
<th>Quantity</th>
<th>( \text{MeV} )</th>
<th>( M_X - M_{\text{threshold}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_X )</td>
<td>3871.8±0.7±0.4</td>
<td></td>
</tr>
<tr>
<td>( M_{D^0} + M_{D^{*0}} )</td>
<td>3871.5±0.7</td>
<td>+0.3±1.1</td>
</tr>
<tr>
<td>( M_{D^+} + M_{D^{*+}} )</td>
<td>3879.5±0.7</td>
<td>−7.7±1.1</td>
</tr>
</tbody>
</table>
Charmonium Options for the X(3872)

• Consider all 1D and 2P cc possibilities
• Assume $M=3872$ MeV
  • calculate radiative widths and
  • strong decay widths
Strong Decays:

1. Zweig-allowed open-charm decays (DD)

   expect $1^3D_2$ and $1^1D_2$ but $1^3D_3$ also narrow because of angular momentum barrier

2. Annihilation type decays

   summarized in Ref.[50]. Expressions for decay widths relevant to the 1D and 2P $\bar{c}c$ states in particular are:

\[
\Gamma(3^D_J \to ggg) = \frac{10\alpha_s^3}{9\pi} C_J \left| \frac{R^{\bar{c}c}_J(0)}{m_Q^6} \right|^2 \ln(4m_Q<r>) \quad (7)
\]

\[
\Gamma(1^D_2 \to gc) = \frac{2\alpha_s^2}{3} \left| \frac{R^{\bar{c}c}_J(0)}{m_Q^6} \right|^2 \quad (8)
\]

\[
\Gamma(3^P_2 \to ggg) = \frac{8\alpha_s^2}{5} \left| \frac{R^{\bar{c}c}_P(0)}{m_Q^4} \right|^2 \quad (9)
\]

\[
\Gamma(3^P_1 \to g\bar{q}g) = \frac{8n_f\alpha_s^3}{9\pi} \frac{|R^{\bar{c}c}_P(0)|^2}{m_Q^4} \ln(m_Q<r>) \quad (10)
\]

\[
\Gamma(1^P_1 \to ggg) = \frac{20\alpha_s^3}{9\pi} \frac{|R^{\bar{c}c}_P(0)|^2}{m_Q^4} \ln(m_Q<r>) \quad (11)
\]

\[
\Gamma(1^P_1 \to gg\gamma) = \frac{36}{5} e_q^2 \frac{\alpha_s}{\alpha_s} \Gamma(1^P_1 \to ggg) \quad (12)
\]

\[
\Gamma(3^P_0 \to ggg) = \frac{6\alpha_s^2}{5} \frac{|R^{\bar{c}c}_P(0)|^2}{m_Q^4} \quad (13)
\]

3. Hadronic transitions
Radiative transitions:

\[
\Gamma(n^{2S+1}L_J \to n'^{2S'+1}L_{J'} + \gamma) = \frac{4}{3} e_c^2 \alpha \omega^3 C_{fi} \delta_{SS'} \left| \langle n^{2S+1}L_J | r | n^{2S'+1}L_{J'} \rangle \right|^2 ,
\]

\[
C_{fi} = \max(L, L')(2J' + 1) \left\{ \begin{array}{c} L' J' S \\ J L 1 \end{array} \right\}^2 .
\]

TABLE II: Radiative transitions in scenario 1: Predictions for the E1 transitions 1D→1P, 2P→2S, 2P→1S and 2P→1D, assuming in all cases that the initial c\bar{c} state has a mass of 3872 MeV. The matrix elements were obtained using the wavefunctions of the Godfrey-Isgur model, Ref.[17]. Unless otherwise stated, the widths are given in keV and the final c\bar{c} masses are PDG values [38].

| Initial state X(3872) | Final state \(\chi_{c2}(1^3P_2)\) | \(\chi_{c1}(1^3P_1)\) | \(\gamma\) | \(M_f\) (MeV) | \(\omega\) (MeV) | \(\langle f|r|i\rangle\) (GeV\(^{-1}\)) | \(C_{fi}\) | Width (keV) |
|---------------------|-------------------------------|-----------------|-----------------|--------------|--------------|----------------------------|---------|-----------|
| \(1^3D_3\)          | \(\chi_{c2}(1^3P_2)\)         | \(\chi_{c1}(1^3P_1)\) | \(\gamma\)      | 3556.2       | 303          | 2.762                              | \(\frac{2}{5}\) | 367       |
| \(1^3D_2\)          | \(\chi_{c2}(1^3P_2)\)         | \(\chi_{c1}(1^3P_1)\) | \(\gamma\)      | 3556.2       | 303          | 2.769                              | \(\frac{1}{10}\) | 92        |
|                     |                               |                 |                 | 3510.5       | 345          | 2.588                              | \(\frac{3}{10}\) | 356       |
| \(1^3D_1\)          | \(\chi_{c2}(1^3P_2)\)         | \(\chi_{c1}(1^3P_1)\) | \(\gamma\)      | 3556.2       | 303          | 2.769                              | \(\frac{1}{80}\) | 10.2      |
|                     |                               |                 |                 | 3510.5       | 345          | 2.598                              | \(\frac{1}{6}\)  | 199       |
|                     |                               |                 |                 | 3415         | 430          | 2.390                              | \(\frac{2}{9}\)  | 437       |
| \(1^1D_2\)          | \(h_{c}(1^1P_1)\) \(\gamma\)   |                 |                 | 3517\(^a\)  | 339          | 2.627                              | \(\frac{2}{5}\)  | 464       |
TABLE IV: Partial widths and branching fractions for strong and electromagnetic transitions in scenario 1: We assume in all cases that the initial $c\bar{c}$ state has a mass of 3872 MeV. Details of the calculations are given in the text.

<table>
<thead>
<tr>
<th>Initial state</th>
<th>Final state</th>
<th>Width (MeV)</th>
<th>B.F. (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1^3D_3$ DD</td>
<td>4.04</td>
<td>84.2</td>
<td></td>
</tr>
<tr>
<td>$ggg$</td>
<td>0.18</td>
<td>3.8</td>
<td></td>
</tr>
<tr>
<td>$J/\psi \pi\pi$</td>
<td>0.21 ± 0.11</td>
<td>4.4</td>
<td></td>
</tr>
<tr>
<td>$\chi_{c1}(1^3P_2)\gamma$</td>
<td>0.37</td>
<td>7.7</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>4.80</strong></td>
<td><strong>100</strong></td>
<td></td>
</tr>
<tr>
<td>$1^3D_2$ $ggg$</td>
<td>0.08</td>
<td>10.8</td>
<td></td>
</tr>
<tr>
<td>$J/\psi \pi\pi$</td>
<td>0.21 ± 0.11</td>
<td>28.4</td>
<td></td>
</tr>
<tr>
<td>$\chi_{c2}(1^3P_2)\gamma$</td>
<td>0.09</td>
<td>12.2</td>
<td></td>
</tr>
<tr>
<td>$\chi_{c1}(1^3P_1)\gamma$</td>
<td>0.36</td>
<td>48.6</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>0.74</strong></td>
<td><strong>100</strong></td>
<td></td>
</tr>
<tr>
<td>$1^3D_1$ DD</td>
<td>184</td>
<td>98.9</td>
<td></td>
</tr>
<tr>
<td>$ggg$</td>
<td>1.15</td>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td>$J/\psi \pi\pi$</td>
<td>0.21 ± 0.11</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>$\chi_{c1}(1^3P_1)\gamma$</td>
<td>0.20</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>$\chi_{c0}(1^3P_0)\gamma$</td>
<td>0.44</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>186</strong></td>
<td><strong>100</strong></td>
<td></td>
</tr>
<tr>
<td>$1^3D_2$ $gg$</td>
<td>0.19</td>
<td>22.1</td>
<td></td>
</tr>
<tr>
<td>$\eta_\pi\pi$</td>
<td>0.21 ± 0.11</td>
<td>24.4</td>
<td></td>
</tr>
<tr>
<td>$h_\pi(1^3P_1)\gamma$</td>
<td>0.46</td>
<td>53.5</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>0.86</strong></td>
<td><strong>100</strong></td>
<td></td>
</tr>
</tbody>
</table>

| $2^3P_2$ DD | 21.1 | 82.4 |
| $gg$ | 4.4 | 17.2 |
| $\psi(2^3S_1)\gamma$ | 0.06 | 0.2 |
| $J/\psi(1^3S_1)\gamma$ | 0.04 | 0.2 |
| **Total** | **25.6** | **100** |
| $2^3P_1$ $q\bar{q}g$ | 1.65 | 95.9 |
| $\psi(2^3S_1)\gamma$ | 0.06 | 3.5 |
| $J/\psi(1^3S_1)\gamma$ | 0.01 | 0.6 |
| **Total** | **1.72** | **100** |
| $2^3P_0$ DD | 13.7 (see text) | 24.6 |
| $gg$ | 42. | 75.3 |
| $\psi(2^3S_1)\gamma$ | 0.07 | 0.1 |
| $\psi(1^3D_1)\gamma$ | $4 \times 10^{-2}$ | 4.4 |
| **Total** | **55.8** | **100** |
| $2^1P_1$ $ggg$ | 1.29 | 81.6 |
| $gg\gamma$ | 0.13 | 8.2 |
| $\eta_{c}(2^1S_0)\gamma$ | 0.09 | 5.7 |
| $\eta_{c}(1^1S_0)\gamma$ | 0.07 | 4.4 |
| **Total** | **1.58** | **100** |
### $^{13}\text{D}_2$ and $^{11}\text{D}_2$ and $^{13}\text{D}_3$

<table>
<thead>
<tr>
<th></th>
<th>ggg</th>
<th>$J/\psi\pi\pi$</th>
<th>$\chi_{e2}(^{13}\text{P}_2)\gamma$</th>
<th>$\chi_{e1}(^{13}\text{P}_1)\gamma$</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{13}\text{D}_2$</td>
<td>0.08</td>
<td>0.21 ± 0.11</td>
<td>0.09</td>
<td>0.36</td>
<td>0.74</td>
</tr>
<tr>
<td>$^{11}\text{D}_2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{13}\text{D}_3$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### $^{23}\text{P}_1$ and $^{21}\text{P}_1$

<table>
<thead>
<tr>
<th></th>
<th>ggg</th>
<th>$\psi(2^3\text{S}_1)\gamma$</th>
<th>$J/\psi(1^3\text{S}_1)\gamma$</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{23}\text{P}_1$</td>
<td>1.65</td>
<td>0.06</td>
<td>0.01</td>
<td>1.72</td>
</tr>
<tr>
<td>$^{21}\text{P}_1$</td>
<td>1.29</td>
<td>0.13</td>
<td>0.09</td>
<td>1.58</td>
</tr>
</tbody>
</table>

The problem here is that the BR to $\gamma$ and $\pi\pi$ is quite small and not the final states being looked for.
Consider the charmonium possibilities:

1D and 2P multiplets only states nearby in mass

\[ 1^{1}D_2, \ 2^3P_0, \ 2^3P_1, \ 2^3P_2 \] have \( C=+ \)

But \( X(3872) \rightarrow \gamma J/\psi \) implies \( C=+ \)

Angular distributions favour \( J^{PC}=1^{++} \)

The unique surviving charmonium candidate is \( 2^3P_1 \)

BUT identification of \( Z(3931) \) with \( 2^3P_2 \) implies 2P mass \( \sim 3940 \) MeV

\( D^0D^{*0} \) molecule or “tetraquark”
is a popular/likely explanation: see Voloshin
**Y(4260)**

Discovered by Babar as enhancement in $\pi\pi J/\psi$ subsystem in $e^+e^- \rightarrow \gamma_{\text{ISR}} \psi\pi\pi$  

PRL 95, 142001(2005) [hep-ex/0506081]

$M = 4259 \pm 8 \pm 4$ MeV

$\Gamma = 88 \pm 23 \pm 5$ MeV

$\Gamma_{ee}\times BR(Y \rightarrow \pi^+\pi^- J/\psi) = 5.5 \pm 1.0 \pm 0.8$ eV

ISR production tells us $J^{PC} = 1^{--}$

Further evidence in $B \rightarrow K(\pi^+\pi^- J/\psi)$ PR D73, 011101(2006)

Confirmed by CLEO  

hep-ex/0602034
• The first unaccounted $1^-\text{ state is the } \psi(3D)$

• Quark models estimate $M(\psi(3D)) \sim 4500 \text{ MeV}$ much too heavy for the $Y(4260)$

$Y(4260)$ represents an overpopulation of expected $1^-\text{ states}$

Absence of open charm production also against conventional $cc$ state

Other explanations are:

• $\psi(4S)$
  Phys Rev D72, 031503 (2005)

• Tetraquark
  Phys Rev D72, 031502 (2005)

• cc hybrid
  Phys Lett B625, 212 (2005);
**Y(4260): Hybrid?**

- Flux tube model predicts lowest cc hybrid at 4200 MeV
- LGT expects lowest cc hybrid at 4200 MeV [Phys Lett B401, 308 (1997)]
- Models of hybrids say $\Psi(0)=0$ so would have small $e^+e^-$ width

-LGT found $bb$ hybrids have large couplings to closed flavour modes
  - Similar to BaBar observation of $Y \to \pi^+\pi^-J/\psi$:
    - $\text{BR}(Y \to \pi^+\pi^-J/\psi)>8.8\%$
    - $\Gamma(Y \to \pi^+\pi^-J/\psi)>7.7\pm 2.1$ MeV

- Much larger than typical charmonium transitions: $\Gamma(\psi(3770) \to \pi^+\pi^-J/\psi)\sim 80$ keV
- $Y$ is seen while $\psi(4040)$, $\psi(4160)$ $\psi(4415)$ are not
How to test Y(4260) hybrid assignment:

Decays:

• LGT study suggest searching for other closed charm modes with $J^{PC}=1^{-}$: $J/\psi\eta$, $J/\psi\eta'$, $\chi_{J}\omega$ ... 

• Models predict the dominant hybrid charmonium open-charm decay modes will be a meson pair with $S$-wave ($D, D^*, D_s, D_s^*$) + $P$-wave ($D_J, D_{sJ}$) 

• The dominant decay mode expected to be $D+D_1(2430)$,
  $D_1(2420)$ has width $\sim 300$ MeV and decays to $D^*\pi$

• Suggests search for $Y(4260)$ in $DD^*\pi$

• Evidence of large $DD_1(2430)$ signal would be strong evidence for hybrid 

• But models of hybrids are untested so to be cautious

• If seen in other modes like $DD^*$, $D_sD_s^*$ comparable to $\pi^+\pi^-J/\psi$ maybe still hybrid but decay model not accurate
Search for Partner States: (fill in the multiplet)

• Mass ca. 4.0 - 4.5 GeV, with LGT preferring the higher range. (e.g.: X.Liao and T.Manke, hep-lat/0210030)

• Confirm that no cc states with the same $J^{PC}$ are expected at this mass.

• Identify $J^{PC}$ partners of the hybrid candidate nearby in mass.

• The most convincing evidence:
  • partners, especially $J^{PC}$ exotics.

• The f-t model expects:
  $0^{+-}, 1^{--}, 2^{++}, 0^{--}, 1^{++}, 2^{--}, 1^{++}, 1^{--}$
### Summary

Many new results, considerable progress!

| **X(3872)** | Molecule? - see Voloshin |
| DsJ(2317) | Most likely 0+(cς) |
| DsJ(2460) | Most likely 1+(cς) |
| DsJ(2632) | Needs confirmation |
| X(3943) | η''_c (3^1S_0) - look for γγ → DD* |
| Y(3943) | χ''_c1 (2^3P_1) - look for DD & DD* |
| Z(3930) | χ''_c2 (2^3P_2) - confirm by DD* |
| Y(4260) | Hybrid? |

- Much more to learn; ie search for 1^3D_3 1^3D_2 1^1D_2 1^3F_2 1^3F_4