Spencer-Attix water to air mass collision stopping power ratios as a function of depth and beam quality, R_{50}

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Abstract

This brief report presents tabulated values of $\left(\frac{\overline{L}}{\rho}\right)^{\text{water}}_{\text{air}}$ as a function of depth, z, and beam quality expressed by the depth at which the dose falls to 50% of dose maximum, R_{50}. The values are those given by the general formula of Burns et al (Med. Phys. **23** (1996) 383–388). The report also presents a series of figures which compare the original database of stopping-power ratios which were fit to get the general formula.

Contents

Parts of this report have been upgraded and published as
"Accuracy of the Burns equation for stopping-power ratio as a function of depth and R_{50}",
Medical Physics **31** (2004) 2961 – 2963

1 The formula

Burns *et al.*[1] published a formula which gives the Spencer-Attix water-to-air mass collision stopping power ratio as a function of beam quality, R_{50}, and depth, Z (both in cm). R_{50} is the depth at which the dose in a broad electron beam drops to 50% of the maximum dose. The formula is:

$$\left(\frac{\overline{L}}{\rho}\right)^{\text{water}}_{\text{air}} (\mathrm{R}_{50}, z) = \frac{a + b(ln\mathrm{R}_{50}) + c(ln\mathrm{R}_{50})^2 + d(\mathrm{z}/\mathrm{R}_{50})}{1 + e(ln\mathrm{R}_{50}) + f(ln\mathrm{R}_{50})^2 + g(ln\mathrm{R}_{50})^3 + h(\mathrm{z}/\mathrm{R}_{50})} \tag{1}$$

The values for the 8 coefficients are:

```
a= 1.0752        b= -0.50867      c= 0.088670      d= -0.08402
e= -0.42806      f= 0.064627      g= 0.003085      h= -0.12460
```

A FORTRAN function routine is available at th a listing in the appendix of this report or directly on the NRC web site at:
http://www.irs.inms.nrc.ca/inms/irs/papers/SPRR50/node12.html

This fit was determined by allowing the Jandal Scientific program Table Curve to provide the best possible fit to a large number of stopping-power ratios calculated by Ding *et al.*[2] for realistic accelerator spectra from many different accelerators. These spectra were calculated using the code BEAM.[3]

The given coefficients result in an rms deviation of 0.4% and a maximum deviation of 1.0% for z/R_{50} ranging between 0.02 and 1.1. The maximum deviation is increases to 1.7% if z/R_{50} values up to 1.2 are considered. The values of R_{50} included in the fit ranged from 0.98 cm to 18.63 cm.

Table 1: Values of the Spencer-Attix water to air mass collision stopping power ratio ($\Delta = 10$ keV) as a function of R_{50} and depth/R_{50}. Values calculated by eqn 1 (from Burns *et al.*[1]).

	R_{50} (cm)									
z/ R_{50}	1.00	1.50	2.00	2.50	3.00	3.50	4.00	4.50	5.00	5.50
0.000	1.0752	1.0552	1.0406	1.0288	1.0189	1.0103	1.0027	0.9958	0.9896	0.9839
0.050	1.0777	1.0581	1.0437	1.0322	1.0224	1.0140	1.0065	0.9997	0.9936	0.9880
0.100	1.0802	1.0610	1.0469	1.0356	1.0261	1.0178	1.0104	1.0038	0.9978	0.9922
0.150	1.0828	1.0639	1.0501	1.0391	1.0297	1.0216	1.0144	1.0080	1.0020	0.9966
0.200	1.0854	1.0669	1.0534	1.0426	1.0335	1.0256	1.0186	1.0122	1.0064	1.0011
0.250	1.0881	1.0700	1.0568	1.0462	1.0374	1.0297	1.0228	1.0166	1.0109	1.0057
0.300	1.0907	1.0730	1.0602	1.0499	1.0413	1.0338	1.0271	1.0211	1.0155	1.0104
0.350	1.0935	1.0762	1.0637	1.0537	1.0453	1.0381	1.0316	1.0257	1.0203	1.0153
0.400	1.0962	1.0793	1.0672	1.0576	1.0495	1.0424	1.0361	1.0304	1.0252	1.0203
0.450	1.0990	1.0826	1.0708	1.0615	1.0537	1.0468	1.0408	1.0353	1.0302	1.0255
0.500	1.1018	1.0859	1.0745	1.0655	1.0580	1.0514	1.0456	1.0402	1.0353	1.0308
0.550	1.1047	1.0892	1.0782	1.0696	1.0624	1.0561	1.0505	1.0454	1.0407	1.0363
0.600	1.1076	1.0926	1.0820	1.0738	1.0669	1.0609	1.0555	1.0507	1.0461	1.0419
0.650	1.1105	1.0960	1.0859	1.0780	1.0715	1.0658	1.0607	1.0561	1.0518	1.0478
0.700	1.1135	1.0995	1.0899	1.0824	1.0762	1.0708	1.0660	1.0617	1.0576	1.0538
0.750	1.1165	1.1031	1.0939	1.0868	1.0810	1.0760	1.0715	1.0674	1.0636	1.0600
0.800	1.1196	1.1067	1.0980	1.0914	1.0860	1.0813	1.0772	1.0734	1.0698	1.0664
0.850	1.1227	1.1104	1.1022	1.0960	1.0910	1.0868	1.0830	1.0795	1.0762	1.0731
0.900	1.1258	1.1141	1.1065	1.1008	1.0962	1.0924	1.0889	1.0858	1.0828	1.0800
0.950	1.1290	1.1180	1.1108	1.1057	1.1016	1.0981	1.0951	1.0923	1.0896	1.0871
1.000	1.1322	1.1218	1.1153	1.1106	1.1070	1.1040	1.1014	1.0990	1.0967	1.0945
1.050	1.1355	1.1258	1.1198	1.1157	1.1126	1.1101	1.1079	1.1059	1.1040	1.1021
1.100	1.1389	1.1298	1.1245	1.1209	1.1184	1.1163	1.1146	1.1131	1.1115	1.1100
1.150	1.1422	1.1339	1.1292	1.1263	1.1243	1.1228	1.1216	1.1204	1.1194	1.1183
1.200	1.1457	1.1381	1.1341	1.1318	1.1303	1.1294	1.1287	1.1281	1.1275	1.1268
1.250	1.1491	1.1423	1.1390	1.1374	1.1366	1.1362	1.1361	1.1360	1.1359	1.1357
1.300	1.1527	1.1466	1.1440	1.1431	1.1430	1.1433	1.1437	1.1442	1.1446	1.1449
1.350	1.1563	1.1510	1.1492	1.1490	1.1496	1.1505	1.1516	1.1527	1.1537	1.1545
1.400	1.1599	1.1555	1.1545	1.1550	1.1563	1.1580	1.1598	1.1615	1.1631	1.1645

Table 2: Values of the Spencer-Attix water to air mass collision stopping power ratio ($\Delta = 10$ keV) as a function of R_{50} and depth/R_{50}. Values calculated by eqn 1 (from Burns *et al.*[1]).

	R_{50} (cm)									
z/ R_{50}	6.00	6.50	7.00	7.50	8.00	8.50	9.00	9.50	10.00	10.50
0.000	0.9786	0.9738	0.9693	0.9651	0.9612	0.9575	0.9541	0.9508	0.9478	0.9450
0.050	0.9828	0.9780	0.9736	0.9694	0.9655	0.9619	0.9584	0.9552	0.9522	0.9493
0.100	0.9871	0.9824	0.9780	0.9738	0.9700	0.9664	0.9630	0.9597	0.9567	0.9539
0.150	0.9916	0.9869	0.9825	0.9784	0.9746	0.9710	0.9676	0.9644	0.9614	0.9585
0.200	0.9961	0.9915	0.9872	0.9832	0.9794	0.9758	0.9724	0.9693	0.9662	0.9634
0.250	1.0008	0.9963	0.9920	0.9881	0.9843	0.9808	0.9774	0.9742	0.9712	0.9684
0.300	1.0056	1.0012	0.9970	0.9931	0.9894	0.9859	0.9826	0.9794	0.9764	0.9736
0.350	1.0106	1.0063	1.0022	0.9983	0.9947	0.9912	0.9879	0.9848	0.9818	0.9789
0.400	1.0158	1.0115	1.0075	1.0037	1.0001	0.9967	0.9934	0.9903	0.9873	0.9845
0.450	1.0210	1.0169	1.0130	1.0092	1.0057	1.0023	0.9991	0.9960	0.9931	0.9903
0.500	1.0265	1.0225	1.0186	1.0150	1.0115	1.0082	1.0050	1.0020	0.9991	0.9963
0.550	1.0321	1.0282	1.0245	1.0210	1.0176	1.0143	1.0112	1.0082	1.0053	1.0025
0.600	1.0379	1.0342	1.0306	1.0271	1.0238	1.0206	1.0175	1.0146	1.0117	1.0090
0.650	1.0440	1.0403	1.0368	1.0335	1.0303	1.0272	1.0242	1.0213	1.0184	1.0157
0.700	1.0502	1.0467	1.0434	1.0401	1.0370	1.0340	1.0310	1.0282	1.0254	1.0227
0.750	1.0566	1.0533	1.0501	1.0470	1.0440	1.0411	1.0382	1.0354	1.0327	1.0300
0.800	1.0632	1.0601	1.0571	1.0541	1.0512	1.0484	1.0456	1.0429	1.0402	1.0376
0.850	1.0701	1.0672	1.0643	1.0616	1.0588	1.0561	1.0534	1.0507	1.0481	1.0455
0.900	1.0772	1.0745	1.0719	1.0693	1.0667	1.0641	1.0615	1.0589	1.0564	1.0538
0.950	1.0846	1.0822	1.0797	1.0773	1.0748	1.0724	1.0699	1.0675	1.0650	1.0625
1.000	1.0923	1.0901	1.0879	1.0856	1.0834	1.0811	1.0787	1.0764	1.0740	1.0716
1.050	1.1002	1.0983	1.0964	1.0943	1.0923	1.0901	1.0880	1.0857	1.0834	1.0811
1.100	1.1085	1.1069	1.1052	1.1034	1.1016	1.0996	1.0976	1.0955	1.0933	1.0911
1.150	1.1171	1.1158	1.1144	1.1129	1.1113	1.1096	1.1077	1.1058	1.1037	1.1016
1.200	1.1260	1.1251	1.1240	1.1228	1.1215	1.1200	1.1183	1.1165	1.1146	1.1126
1.250	1.1353	1.1348	1.1341	1.1332	1.1321	1.1309	1.1295	1.1279	1.1261	1.1242
1.300	1.1450	1.1449	1.1446	1.1441	1.1433	1.1423	1.1412	1.1398	1.1382	1.1365
1.350	1.1551	1.1555	1.1556	1.1555	1.1551	1.1544	1.1535	1.1524	1.1510	1.1494
1.400	1.1657	1.1666	1.1671	1.1674	1.1674	1.1671	1.1665	1.1656	1.1645	1.1631

Table 3: Values of the Spencer-Attix water to air mass collision stopping power ratio ($\Delta = 10$ keV) as a function of R_{50} and depth/R_{50}. Values calculated by eqn 1 (from Burns *et al.*[1]).

	R_{50} (cm)									
z/ R_{50}	11.00	12.00	13.00	14.00	15.00	16.00	17.00	18.00	19.00	20.00
0.000	0.9423	0.9373	0.9329	0.9290	0.9254	0.9222	0.9193	0.9167	0.9143	0.9121
0.050	0.9466	0.9417	0.9373	0.9333	0.9297	0.9264	0.9235	0.9208	0.9183	0.9161
0.100	0.9512	0.9462	0.9417	0.9377	0.9340	0.9307	0.9277	0.9250	0.9225	0.9202
0.150	0.9558	0.9508	0.9463	0.9423	0.9386	0.9352	0.9321	0.9293	0.9268	0.9244
0.200	0.9607	0.9557	0.9511	0.9470	0.9433	0.9398	0.9367	0.9338	0.9312	0.9288
0.250	0.9657	0.9606	0.9561	0.9519	0.9481	0.9446	0.9415	0.9385	0.9358	0.9333
0.300	0.9709	0.9658	0.9612	0.9570	0.9532	0.9496	0.9464	0.9434	0.9406	0.9380
0.350	0.9762	0.9712	0.9665	0.9623	0.9584	0.9548	0.9515	0.9484	0.9455	0.9429
0.400	0.9818	0.9767	0.9721	0.9678	0.9638	0.9601	0.9568	0.9536	0.9507	0.9480
0.450	0.9876	0.9825	0.9778	0.9735	0.9694	0.9657	0.9623	0.9590	0.9560	0.9532
0.500	0.9936	0.9885	0.9838	0.9794	0.9753	0.9715	0.9680	0.9647	0.9616	0.9587
0.550	0.9998	0.9947	0.9900	0.9855	0.9814	0.9775	0.9739	0.9705	0.9673	0.9644
0.600	1.0063	1.0012	0.9964	0.9920	0.9878	0.9838	0.9801	0.9766	0.9734	0.9703
0.650	1.0130	1.0079	1.0032	0.9986	0.9944	0.9904	0.9866	0.9830	0.9796	0.9764
0.700	1.0201	1.0150	1.0102	1.0056	1.0013	0.9972	0.9933	0.9896	0.9861	0.9828
0.750	1.0274	1.0223	1.0175	1.0129	1.0085	1.0043	1.0003	0.9966	0.9930	0.9895
0.800	1.0350	1.0300	1.0251	1.0205	1.0160	1.0118	1.0077	1.0038	1.0001	0.9965
0.850	1.0430	1.0380	1.0331	1.0284	1.0239	1.0196	1.0154	1.0114	1.0075	1.0038
0.900	1.0513	1.0464	1.0415	1.0368	1.0322	1.0277	1.0234	1.0193	1.0153	1.0115
0.950	1.0600	1.0551	1.0503	1.0455	1.0408	1.0363	1.0319	1.0276	1.0235	1.0195
1.000	1.0692	1.0643	1.0595	1.0547	1.0499	1.0453	1.0407	1.0363	1.0320	1.0279
1.050	1.0788	1.0740	1.0691	1.0643	1.0595	1.0547	1.0500	1.0455	1.0410	1.0367
1.100	1.0888	1.0841	1.0793	1.0744	1.0695	1.0646	1.0598	1.0551	1.0505	1.0460
1.150	1.0994	1.0948	1.0900	1.0851	1.0801	1.0751	1.0701	1.0652	1.0604	1.0558
1.200	1.1105	1.1060	1.1013	1.0963	1.0912	1.0861	1.0810	1.0759	1.0709	1.0660
1.250	1.1222	1.1179	1.1132	1.1082	1.1030	1.0978	1.0925	1.0872	1.0820	1.0769
1.300	1.1346	1.1304	1.1257	1.1208	1.1155	1.1101	1.1047	1.0992	1.0938	1.0884
1.350	1.1477	1.1436	1.1391	1.1341	1.1288	1.1232	1.1176	1.1119	1.1062	1.1005
1.400	1.1615	1.1577	1.1532	1.1482	1.1428	1.1371	1.1313	1.1253	1.1193	1.1134

2 Comparisons with original data

The following figures present comparisons of the stopping-power ratios generated by the general formula and the original database of stopping-power ratios calculated by Ding et al.[3]

The quality of the fit breaks down rapidly after the depth is greater than R_{50}. On the otherhand, the dose is decreasing rapidly as well.

The plots also show that the errors can be significant right at the surface, especially for close-to-monoenergetic incident beams (such as those from the Therac and MM50 machines).

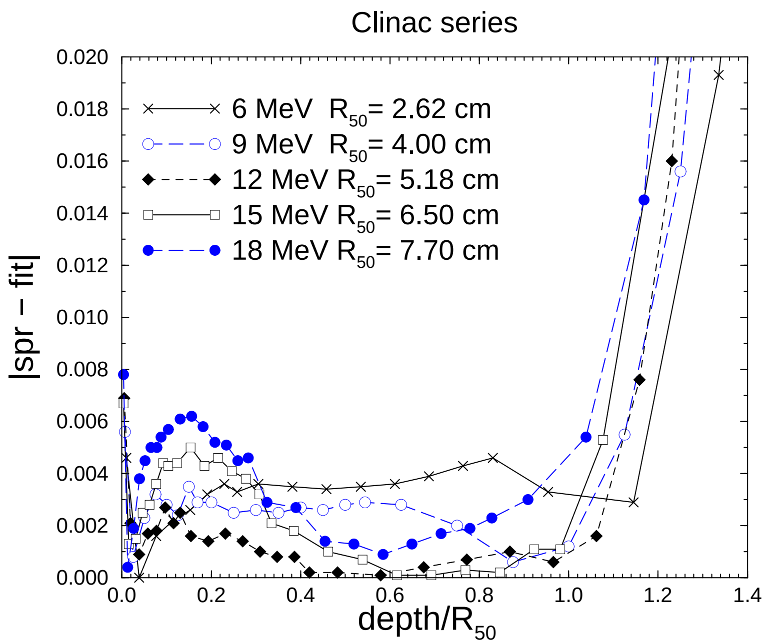

Figure 1: Differences between the generalized fit and the individual values for the Clinac 2100C beams as calculated by Ding *et al.*[2]

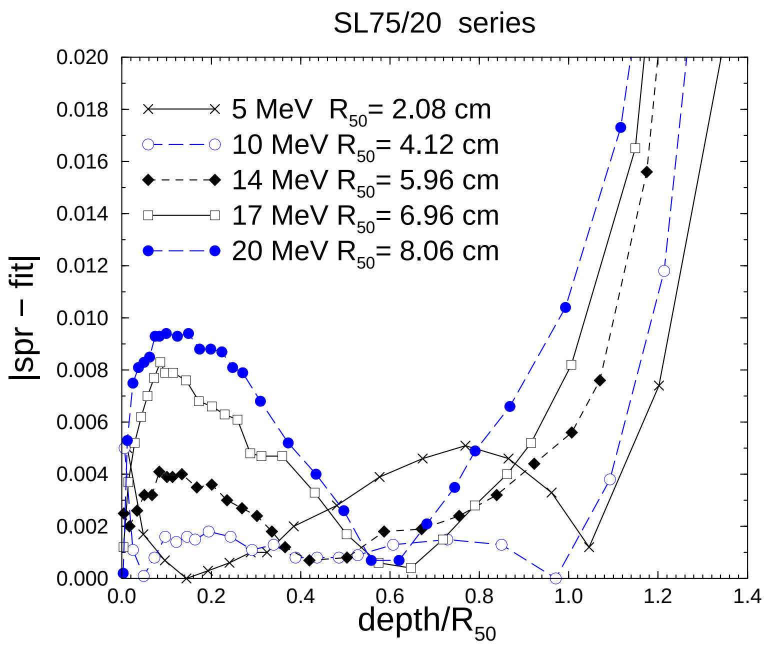

Figure 2: Differences between the generalized fit and the individual values for the SL75/20 beams as calculated by Ding *et al.*[2]

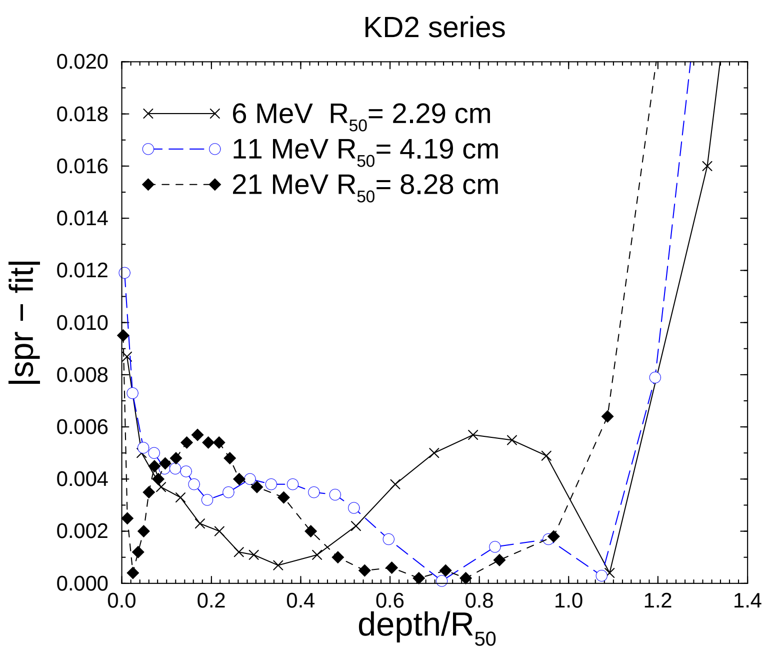

Figure 3: Differences between the generalized fit and the individual values for the KD 2 beams as calculated by Ding *et al.*[2]

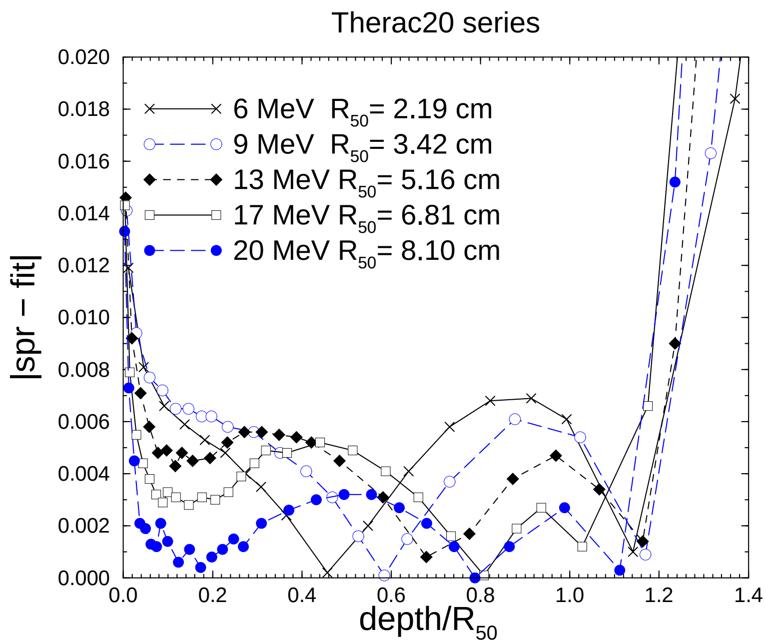

Figure 4: Differences between the generalized fit and the individual values for the Therac 20 beams as calculated by Ding *et al.*[2] These beams are very close to monoenergetic.

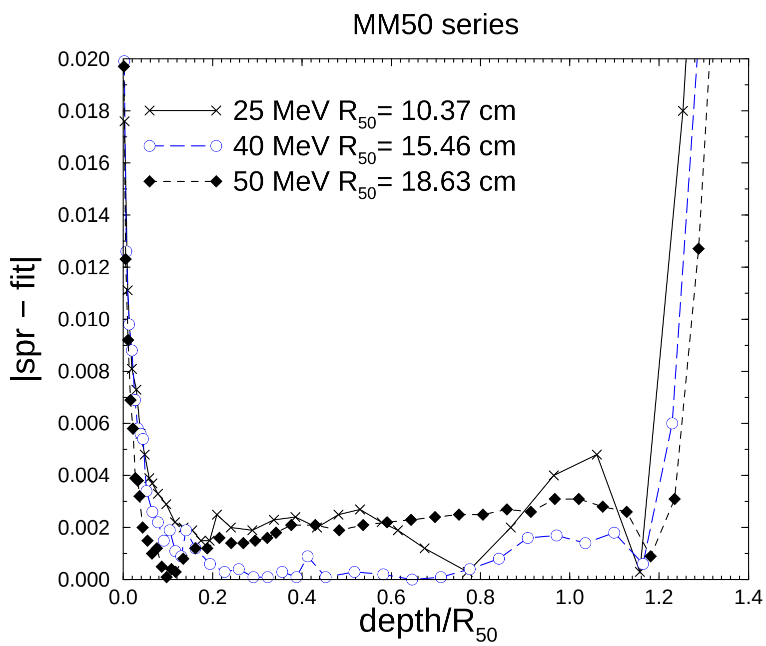

Figure 5: Differences between the generalized fit and the individual values for the Racetrack microtron (MM-50) beams as calculated by Ding *et al.*[2] These beams are very close to monoenergetic.

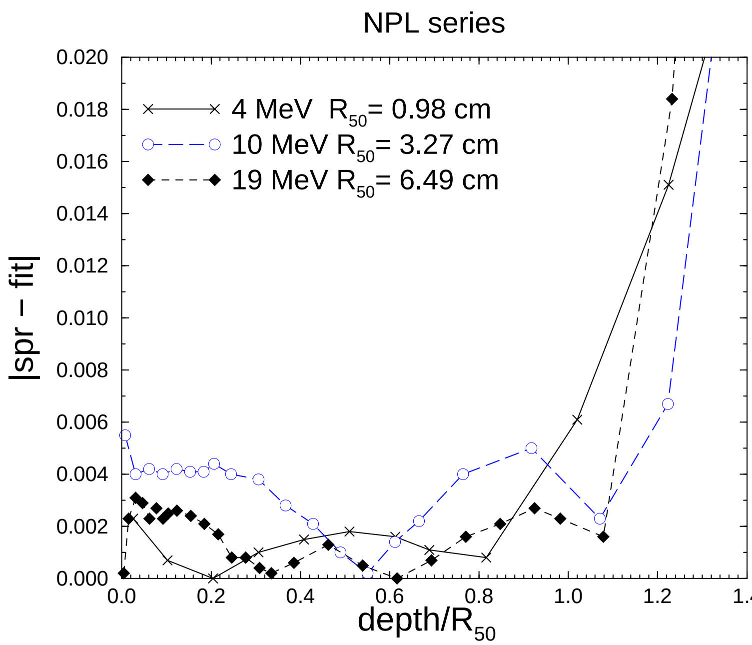

Figure 6: Differences between the generalized fit and the individual values for the NPL beams as calculated by Ding and Rogers.[4] These beams are very close to monoenergetic.

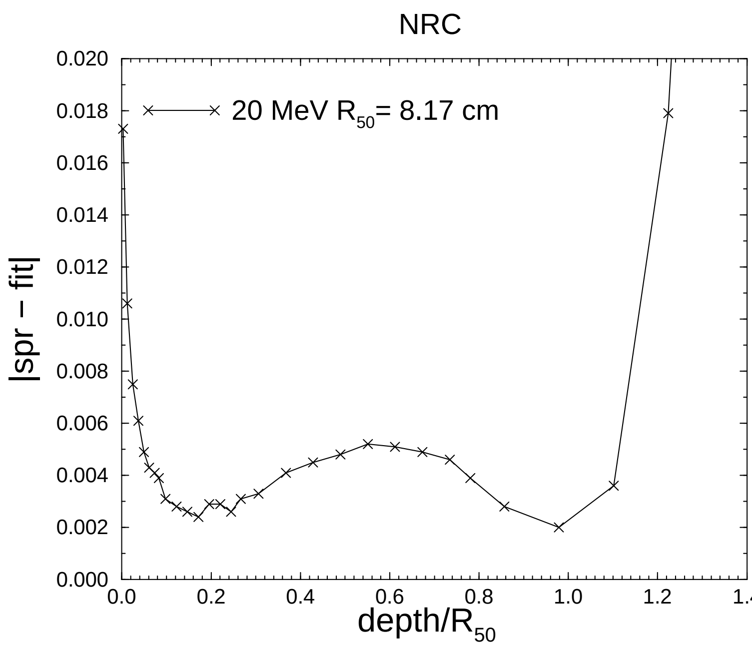

Figure 7: Differences between the generalized fit and the individual values for the NRC 20 MeV beam as calculated by Ding *et al.*[2] These beams are very close to monoenergetic.

3 Appendix: Fortran listing of the function

```
C       spr_3D -3D fit to sprs vs R50 & z/R50  (0.02 to 1.2 z/R50)
C       COPYRIGHT NRC 1995
C  This software and data are provided "as is" without any warranty or
C  guarantee of any kind, either expressed or implied, as to its accuracy
C  or ability to perform particular calculations.
C*********************************************************************
        real function spr_3D(z, R50)
C
C       This returns a value of the Spencer-Attix water to air stopping
C       power ratio as a function of depth z (in cm)  and R50(cm).
C
C       The fit is based on data in:
C       Calculation of stopping-power ratios using realistic clinical
C       electron beams, G.X. Ding et al. Med. Phys. 22(1995)489-501
C       and this fit is presented in:
C       R_50 as a beam quality specifier for selecting stopping-power
C       ratios and reference depths for electron dosimetry
C       D.T. Burns et al    Med. Phys.  23 (1996) 383-388
C       This fit is the best available via Jandel Scientific's 3D Tablecurve
C       Code put together by D.W.O. Rogers  NRC March 1995
C       Test results:   If working correctly the code returns the following
C       z       R50     spr_3D
C       1       3       1.0440
C       4       3       1.1473
C       1       20      0.9161
C       10      20      0.9587
C       20      20      1.0279
C*********************************************************************
      IMPLICIT NONE
      DOUBLE PRECISION x,y,z1,z3
      REAL z,R50
      x = R50
      x = DLOG(x)
      y = z/R50
      z1=1.075177841118226+x*(-0.5086698872574685+
     1x*(0.08867097624170420)) -0.08401847878029861*y
      z3=1.000000000000000+x*(-0.4280646009182138+
     1x*(0.06462718784963824+x*(0.003084672219886146)))
     2-0.1246007344450053*y
      spr_3D=z1/z3
      RETURN
      END
```

References

[1] D. T. Burns, G. X. Ding, and D. W. O. Rogers, R_{50} as a beam quality specifier for selecting stopping-power ratios and reference depths for electron dosimetry, Med. Phys. **23**, 383 – 388 (1996).

[2] G. X. Ding, D. W. O. Rogers, and T. R. Mackie, Calculation of stopping-power ratios using realistic clinical electron beams, Med. Phys. **22**, 489 – 501 (1995).

[3] D. W. O. Rogers, B. A. Faddegon, G. X. Ding, C.-M. Ma, J. Wei, and T. R. Mackie, BEAM: A Monte Carlo code to simulate radiotherapy treatment units, Med. Phys. **22**, 503 – 524 (1995).

[4] G. X. Ding and D. W. O. Rogers, Monte Carlo simulation of NPL linac and calculation of dose distributions and water/air stopping-power ratios, National Research Council of Canada Report PIRS 0399 (1993).