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Validation of a new spectral functional form

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Abstract

This report supplements the publication: Ali and Rogers, "Functional forms for photon spectra of clinical linacs", Phys. Med. Biol. **57**, 31 - 50, (2012), which is referred to henceforth as 'the paper'. In the paper, a new photon spectral functional form is proposed which offers improvements over existing ones. For space limitations, only representative example fits of the proposed function are shown in the paper. This report presents the fits of the proposed function to the entire benchmark set of 65 spectra, along with their fit parameters. For the version of the proposed function used in this report (function 13 in table 2 of the paper), the energy fluence values in each bin are reproduced, on average, with a normalized root-mean-square deviation of 1.5 %. The mean incident electron kinetic energy, maximum photon energy, most-probable energy and average energy are reproduced, on average, within 1.4 %, 4.3 %, 3.6 % and 0.6 % of their true values, respectively.

1 The functional form

The fits shown in this report are for function 13 in table 2 of the paper. Function 13 is the version of the proposed functional form which assumes a monoenergetic incident electron beam with a mean kinetic energy E_e, and which models the 511 keV annihilation peak. This version is given by

$$\psi(E) = \psi_{\mathrm{thin}}(E) \exp\left(-\mu_W(E)\, C_1^2 - \mu_{Al}(E)\, C_2^2\right),$$
$$\psi_{\mathrm{thin}}(E) = \left[1 + C_3 \frac{E}{E_e} + \left(\frac{E}{E_e}\right)^2\right] \left[\ln\left(\frac{E_e(E_e - E)}{E} + 1.65\right) - 0.5\right] + C_4 \frac{\delta(E - E_{511})}{dE_{511}}, \tag{1}$$

where E is the photon energy in MeV, $\psi(E)$ is the differential energy fluence at E, E_e is the mean incident electron kinetic energy in MeV, $\delta(E - E_{511})$ is the Dirac delta function at the center of the 511 keV energy bin, dE_{511} is the width of the 511 keV bin in MeV (user-defined), $\mu_W(E)$ and $\mu_{Al}(E)$ are, respectively, the mass attenuation coefficients in cm^2/g for tungsten and aluminum at energy E, and C_1, C_2, C_3, C_4 and E_e are free parameters. Note that C_4 is the *integral* energy fluence contribution of the 511 keV photons, and it is bin-size independent. The values used for $\mu_W(E)$ and $\mu_{Al}(E)$ are the XCOM values (http://physics.nist.gov/xcom), with linear interpolation in $\log E$ and $\log \mu$. If it is desired to have a self-contained function that does not require reading or interpolating attenuation data, $\mu_W(E)$ and $\mu_{Al}(E)$ can be represented by their parameterized versions given in table 3 of the paper. The fit parameters given in this report can be used with either representation of μ values, with negligible differences in the resulting spectra.

2 Spectral fits

Section 2.3 of the paper presents a detailed description of the diverse benchmark set of spectra, including how the spectra are generated and the rationale for including them. In this report, table 1 gives the parameters of fitting the proposed functional form (equation 1 above) to the energy fluence spectra of the benchmark set. The fits to the full benchmark set are graphically shown in figures 1 – 19. The observations based on the data in the table and graphs can be found in the paper.

Table 1: Parameters of fitting the proposed functional form (equation 1 in this report) to the energy fluence spectra of the benchmark set. In column 8, the tabulated values given for C_4 ($\times 10^2$) should be divided by 100 to give C_4, and they are bin-size independent. The column 'correct E_e' refers to the mean incident electron kinetic energy in the Monte Carlo simulation that generated the benchmark photon spectrum. The last column is the root-mean-square deviation between the original and the fit spectrum, normalized to the mean energy fluence value of the original spectrum and displayed as a per cent. The terms 'central', 'off-axis', 'WFF' and 'FFF' refer to, respectively, a central-axis spectrum, an off-axis spectrum, with flattening filter, and flattening-filter free. Except for E_e, the fit parameters should not be interpreted to represent actual physical quantities. The table continues in the following two pages.

#	linac	MV	additional description	C_1 $g^{1/2}cm^{-1}$	C_2 $g^{1/2}cm^{-1}$	C_3 –	C_4 ($\times 10^2$) MeV	fit E_e MeV	correct E_e MeV	%RMS dev. in ψ
1	Varian	4	WFF, central	3.824	3.522	–1.222	0.308	3.75	3.70	0.5
2	Clinac		off-axis	2.718	3.302	–1.245	0.330	3.71	3.70	0.5
3			FFF, central	1.225	2.708	–0.988	0.159	3.75	3.70	0.9
4			off-axis	1.199	2.771	–1.108	0.195	3.70	3.70	0.9
5		6	WFF, central	1.222	5.147	–1.186	0.881	5.76	5.70	0.5
6			off-axis	1.087	3.865	–1.203	0.790	5.66	5.70	0.8
7			FFF, central	1.071	3.170	–1.035	0.477	5.77	5.70	0.7
8			off-axis	1.015	3.249	–1.176	0.732	5.68	5.70	0.8
9		10	WFF, central	0.702	6.226	–1.285	3.891	10.46	10.50	0.7
10			off-axis	0.709	4.445	–1.295	2.535	10.03	10.50	1.9
11			FFF, central	0.460	3.091	–0.985	0.540	10.47	10.50	1.9
12			off-axis	0.430	3.127	–1.181	1.150	10.03	10.50	2.4
13		15	WFF, central	4.614	3.804	–1.060	14.034	14.58	14.50	0.5
14			off-axis	2.092	3.640	–1.185	3.867	14.16	14.50	1.3
15			FFF, central	0.980	3.517	–0.905	2.421	14.60	14.50	0.7
16			off-axis	0.859	3.480	–1.130	3.839	14.16	14.50	1.5
17		18	WFF, central	3.347	5.847	–1.228	43.160	18.33	18.30	0.8
18			off-axis	1.101	4.865	–1.294	6.754	17.74	18.30	1.7
19			FFF, central	0.877	3.817	–0.928	4.333	18.39	18.30	0.8
20			off-axis	0.715	3.808	–1.202	5.367	17.78	18.30	2.0

. . . continuation of table 1 from the previous page

#	linac	MV	additional description	C_1 $g^{1/2}cm^{-1}$	C_2 $g^{1/2}cm^{-1}$	C_3 –	C_4 ($\times 10^2$) MeV	fit E_e MeV	correct E_e MeV	%RMS dev. in ψ
21	Elekta	6	WFF, central	1.320	5.072	–1.109	2.526	6.49	6.30	2.4
22	SL		off-axis	1.172	3.751	–1.173	2.154	6.33	6.30	2.3
23			FFF, central	1.247	3.962	–1.084	1.686	6.49	6.30	2.0
24			off-axis	1.210	3.719	–1.176	2.251	6.33	6.30	2.3
25		25	WFF, central	0.000	7.504	–1.274	57.864	19.02	19.00	0.9
26			off-axis	0.453	6.020	–1.338	14.650	18.41	19.00	1.9
27			FFF, central	0.919	5.492	–0.963	8.277	19.07	19.00	0.7
28			off-axis	0.599	5.268	–1.243	12.290	18.39	19.00	2.1
29	Siemens	6	WFF, central	1.184	4.840	–1.161	1.416	6.83	6.80	1.9
30	KD		off-axis	1.041	3.626	–1.179	1.290	6.65	6.80	2.0
31			FFF, central	1.126	3.057	–1.032	0.925	6.83	6.80	1.9
32			off-axis	1.002	3.334	–1.163	1.302	6.65	6.80	2.2
33		18	WFF, central	1.213	6.142	–1.126	12.522	14.94	14.70	2.2
34			off-axis	1.832	4.610	–1.209	0.497	14.46	14.70	2.4
35			FFF, central	1.135	3.562	–0.837	4.290	14.96	14.70	2.2
36			off-axis	1.562	4.020	–1.157	–1.182	14.54	14.70	2.6
37	Vickers	15	Be target	0.240	3.582	–0.539	–1.163	15.03	15.18	4.0
38	NRC	10	Al target	0.145	3.206	–0.742	–0.126	10.01	10.09	2.2
39		15		0.065	3.650	–0.667	0.187	15.01	15.18	2.4
40		20		0.000	3.884	–0.623	0.532	20.07	20.28	2.5
41		25		0.000	4.297	–0.598	1.262	25.12	25.38	2.6
42		30		0.000	4.416	–0.574	1.734	30.14	30.45	2.8
43		10	Pb target	2.626	2.254	–0.711	3.284	10.18	10.09	0.7
44		15		3.011	2.311	–0.641	5.516	15.29	15.18	0.7
45		20		3.347	2.362	–0.608	7.496	20.43	20.28	0.8
46		25		3.308	2.458	–0.591	7.155	25.57	25.38	1.0
47		30		3.585	2.532	–0.604	8.508	30.67	30.45	1.1

. . . continuation of table 1 from the previous page

#	linac	MV	additional description	C_1 $g^{1/2}cm^{-1}$	C_2 $g^{1/2}cm^{-1}$	C_3 –	C_4 ($\times 10^2$) MeV	fit E_e MeV	correct E_e MeV	%RMS dev. in ψ
48	NPL linac	4	light filter	1.267	2.506	–0.638	0.194	4.11	4.00	1.7
49		6		2.166	2.627	–0.700	1.407	6.14	6.00	1.4
50		8		2.116	2.893	–0.731	2.196	8.16	8.00	1.3
51		10		2.063	3.088	–0.753	2.854	10.17	10.00	1.1
52		12		2.793	2.819	–0.752	2.995	12.18	12.00	1.0
53		16		3.326	2.963	–0.806	4.531	16.17	16.00	0.8
54		19		3.260	3.190	–0.862	–1.862	19.16	19.00	0.7
55		4	heavy filter	2.170	4.252	–0.681	0.612	4.10	4.00	2.1
56		6		2.089	4.510	–0.763	1.505	6.14	6.00	1.7
57		8		1.934	5.994	–0.807	2.241	8.15	8.00	1.6
58		10		1.594	6.993	–0.859	1.854	10.16	10.00	1.4
59		12		2.325	6.724	–0.882	0.411	12.17	12.00	1.2
60		16		2.800	6.809	–0.954	–1.031	16.15	16.00	0.8
61		19		2.794	6.854	–0.987	–4.606	19.15	19.00	0.6
62	Tomotherpay	6	treatment, central	1.861	3.896	–1.062	0.827	5.66	5.70	2.2
63			treatment, off-axis	1.866	3.689	–1.111	1.312	5.55	5.70	2.4
64		3.5	imaging, full field	2.730	1.701	–0.914	0.463	3.50	3.50	2.9
65	Cyberknife	6	central	2.081	3.612	–1.050	0.740	6.75	6.70	0.6

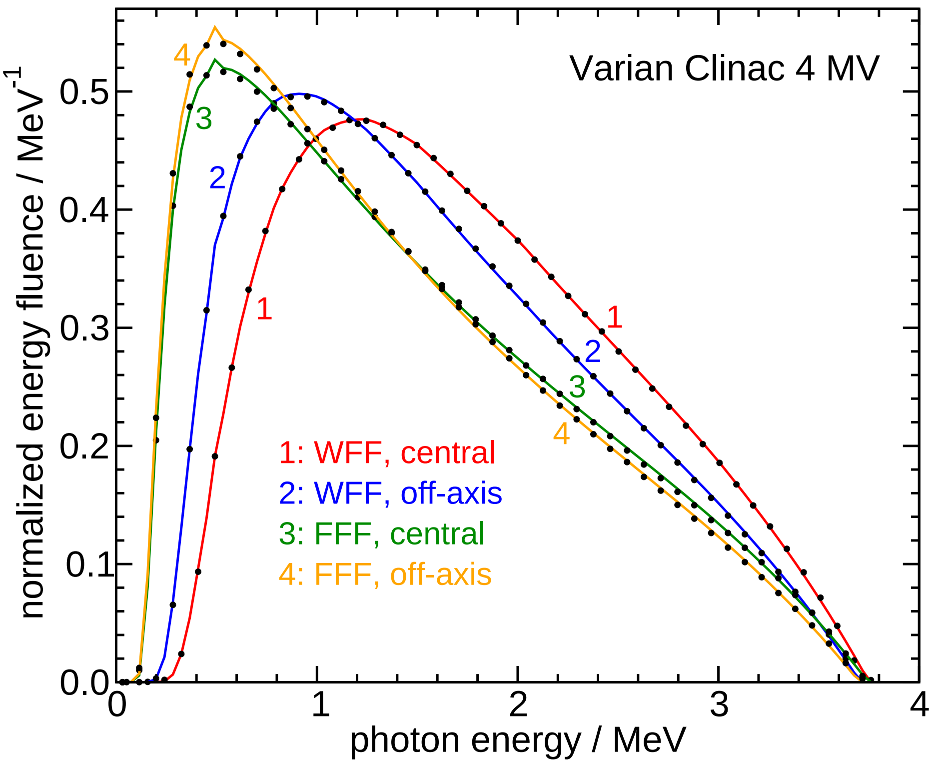

Figure 1: For the energy fluence spectra of the Varian Clinac 4 MV beam, the figure shows the fits of the proposed functional form (equation 1 in this report) to the spectra of the benchmark set. Fits are in solid lines and the original spectra are in dots. For graph clarity, only every other original spectral point is shown. The terms 'central', 'off-axis', 'WFF' and 'FFF' refer to, respectively, a central-axis spectrum, an off-axis spectrum, with flattening filter, and flattening-filter free. Spectra are normalized to unit energy fluence.

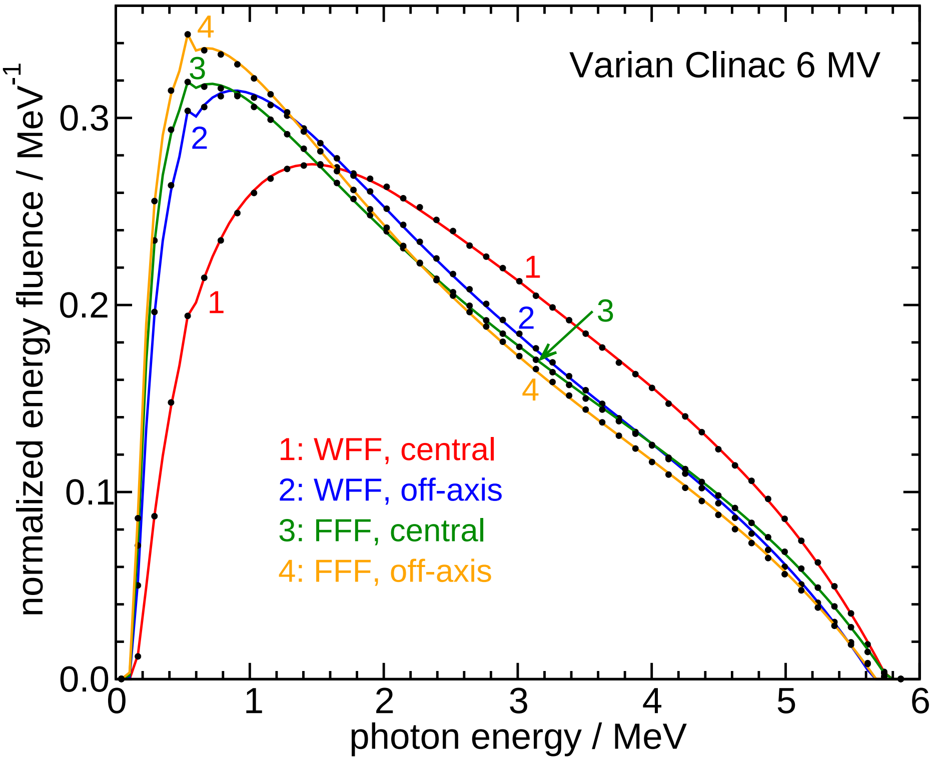

Figure 2: . . . same as figure 1 but for the spectra of the Varian Clinac 6 MV beam.

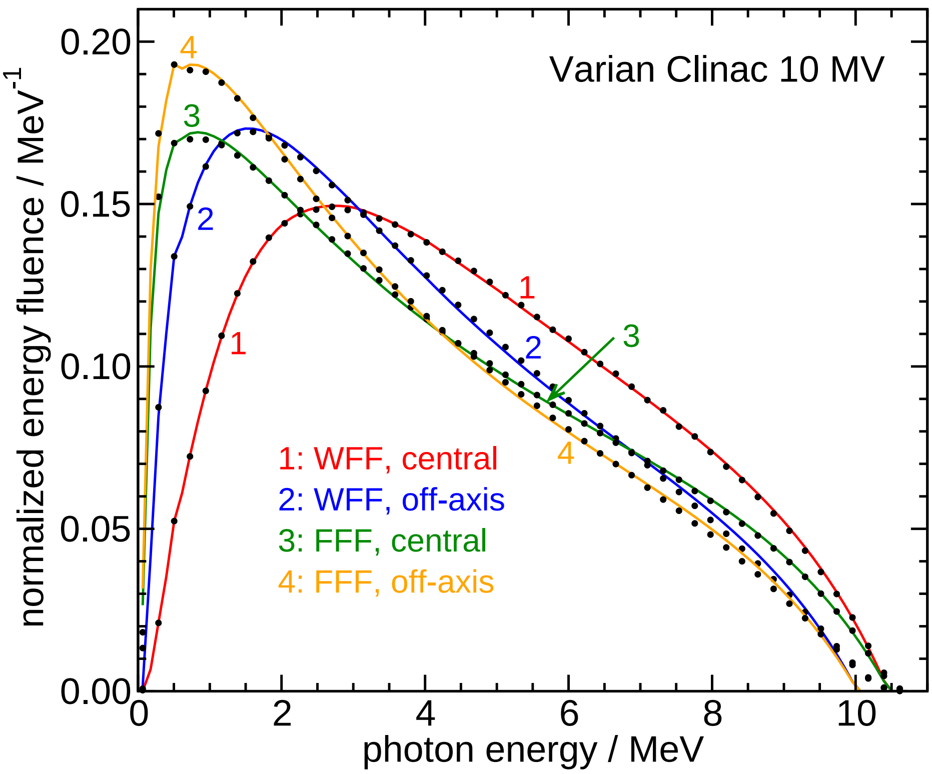

Figure 3: . . . same as figure 1 but for the spectra of the Varian Clinac 10 MV beam.

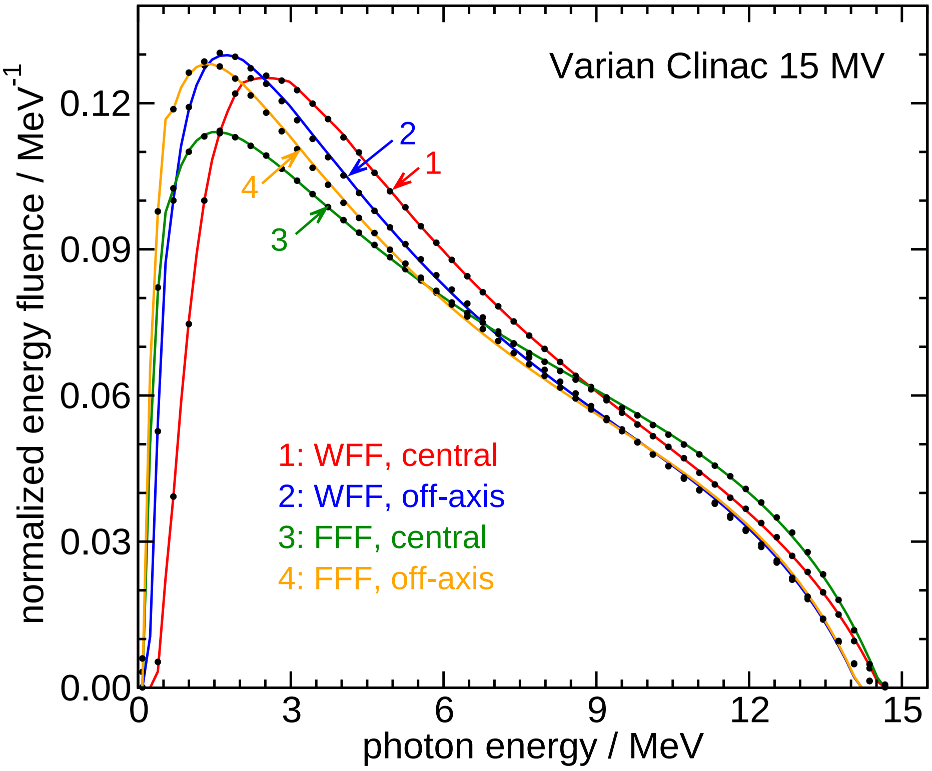

Figure 4: . . . same as figure 1 but for the spectra of the Varian Clinac 15 MV beam.

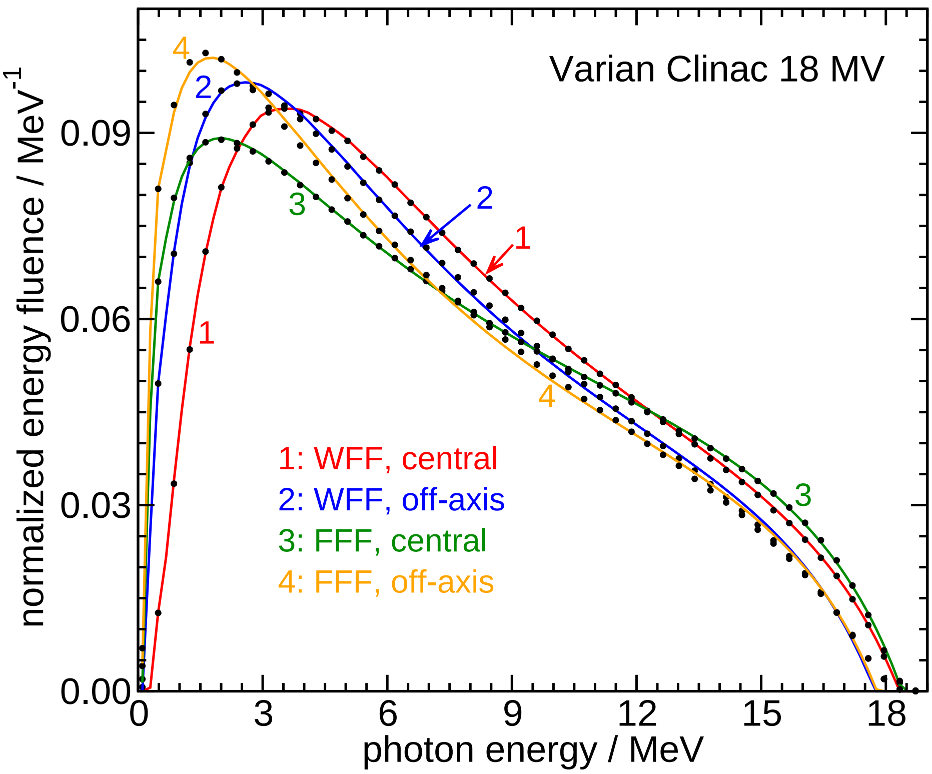

Figure 5: . . . same as figure 1 but for the spectra of the Varian Clinac 18 MV beam.

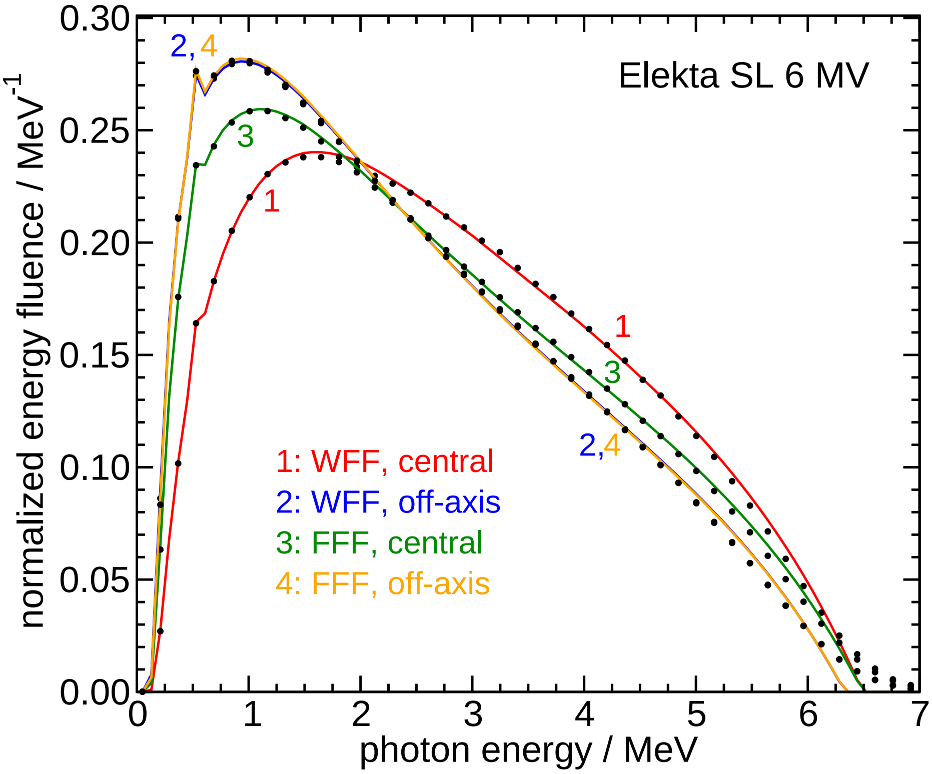

Figure 6: . . . same as figure 1 but for the spectra of the Elekta SL 6 MV beam.

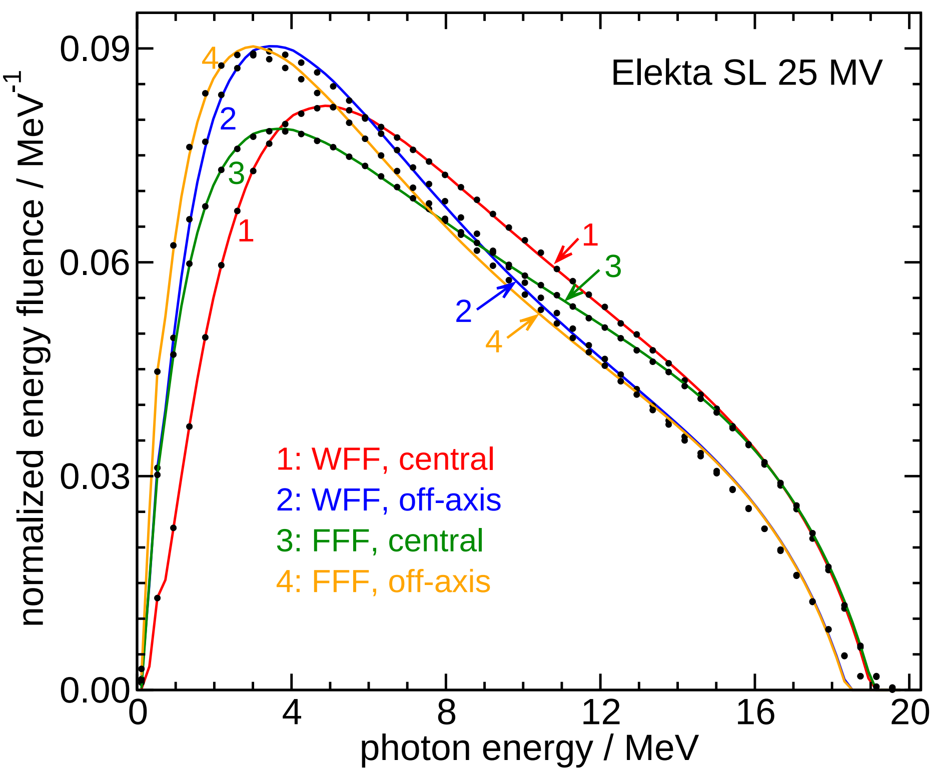

Figure 7: . . . same as figure 1 but for the spectra of the Elekta SL 25 MV beam.

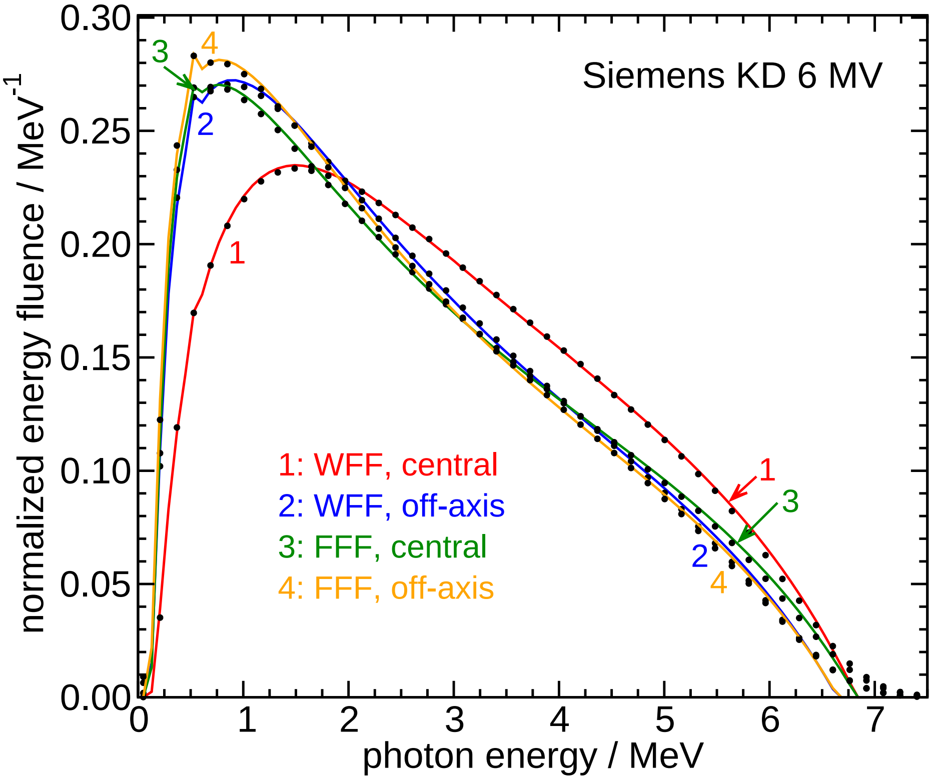

Figure 8: . . . same as figure 1 but for the spectra of the Siemens KD 6 MV beam.

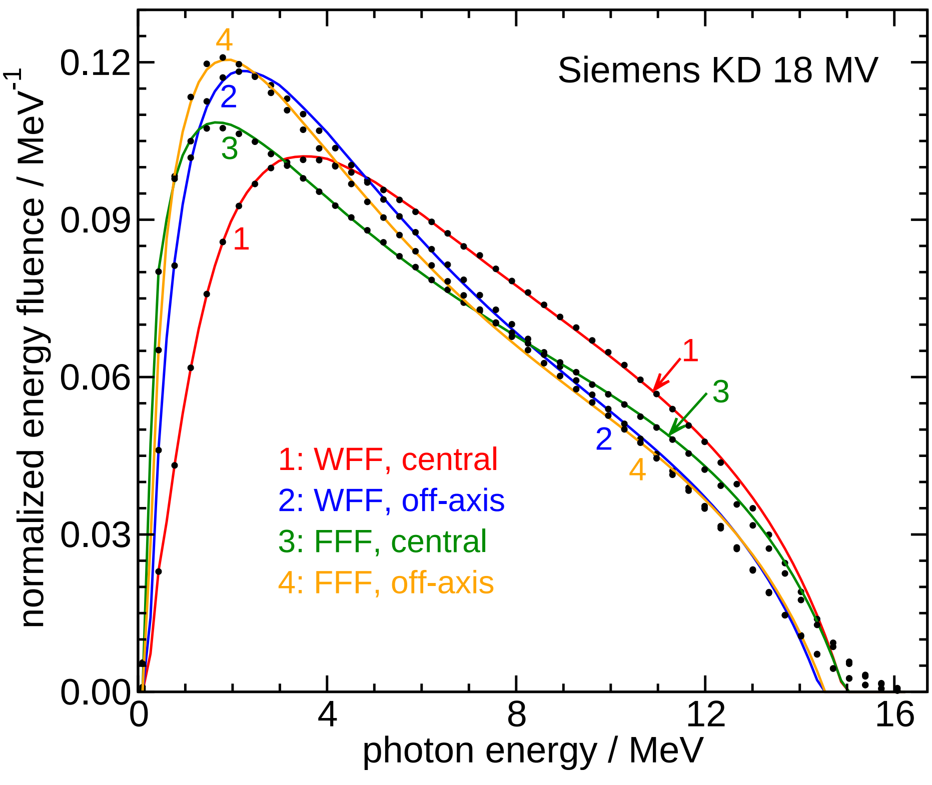

Figure 9: . . . same as figure 1 but for the spectra of the Siemens KD 18 MV beam.

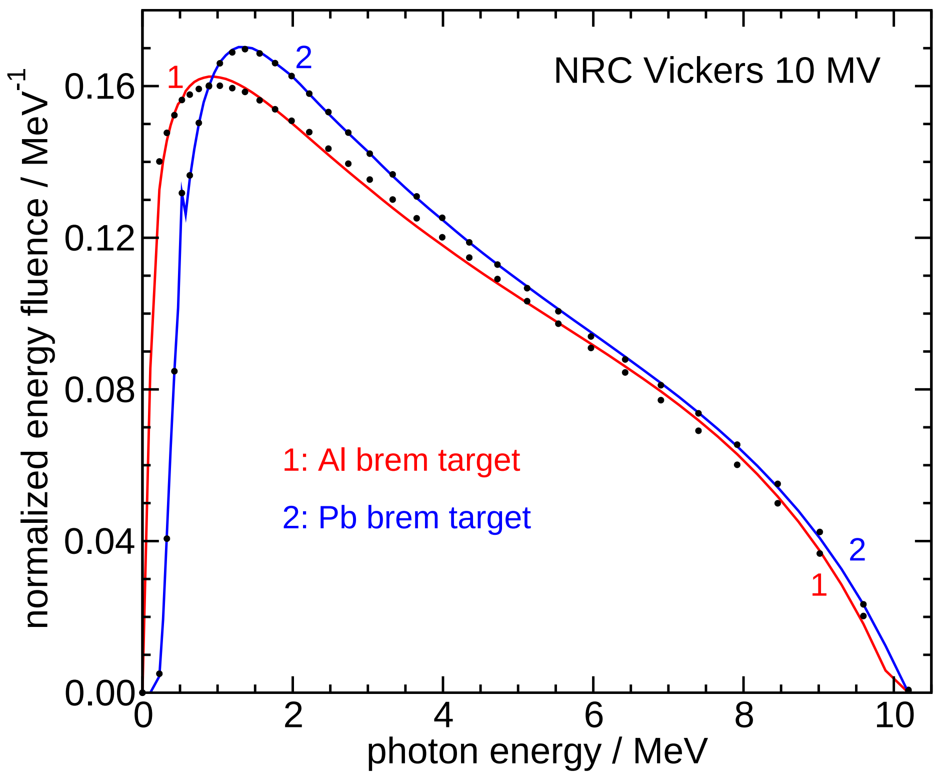

Figure 10: . . . same as figure 1 but for the spectra of the NRC Vickers 10 MV beam.

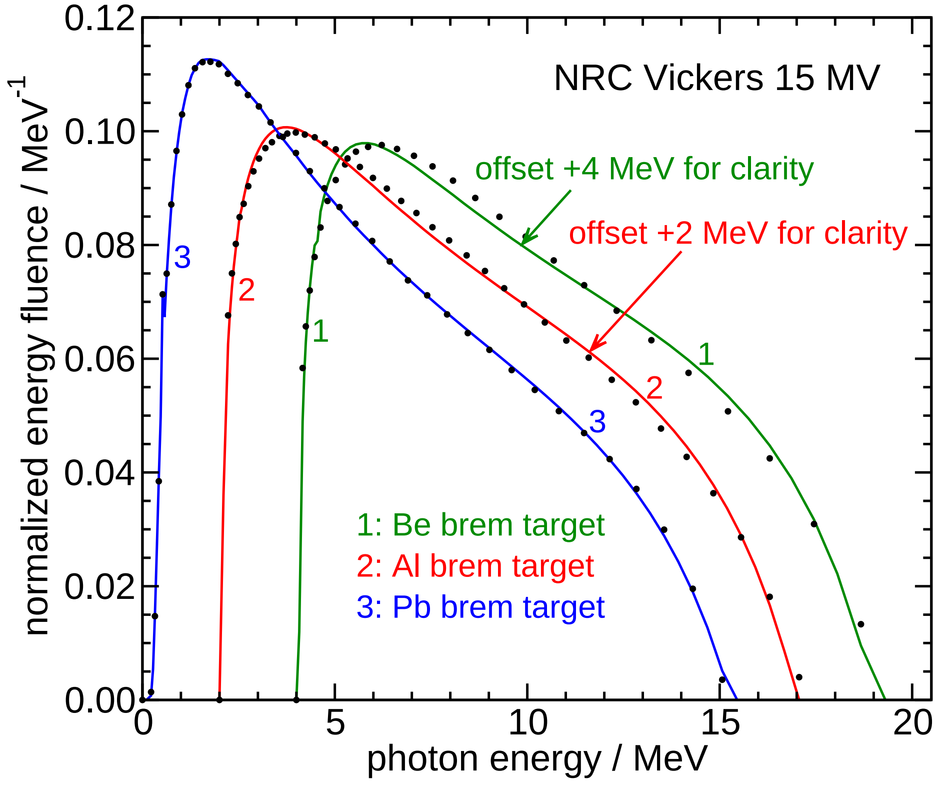

Figure 11: . . . same as figure 1 but for the spectra of the NRC Vickers 15 MV beam.

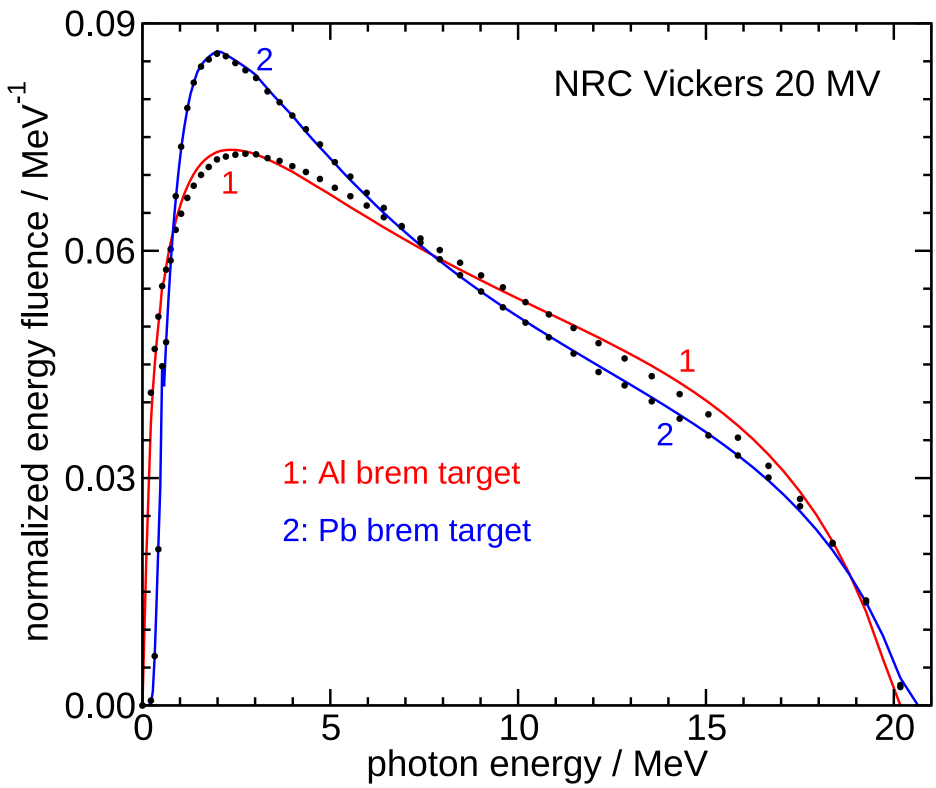

Figure 12: ...same as figure 1 but for the spectra of the NRC Vickers 20 MV beams.

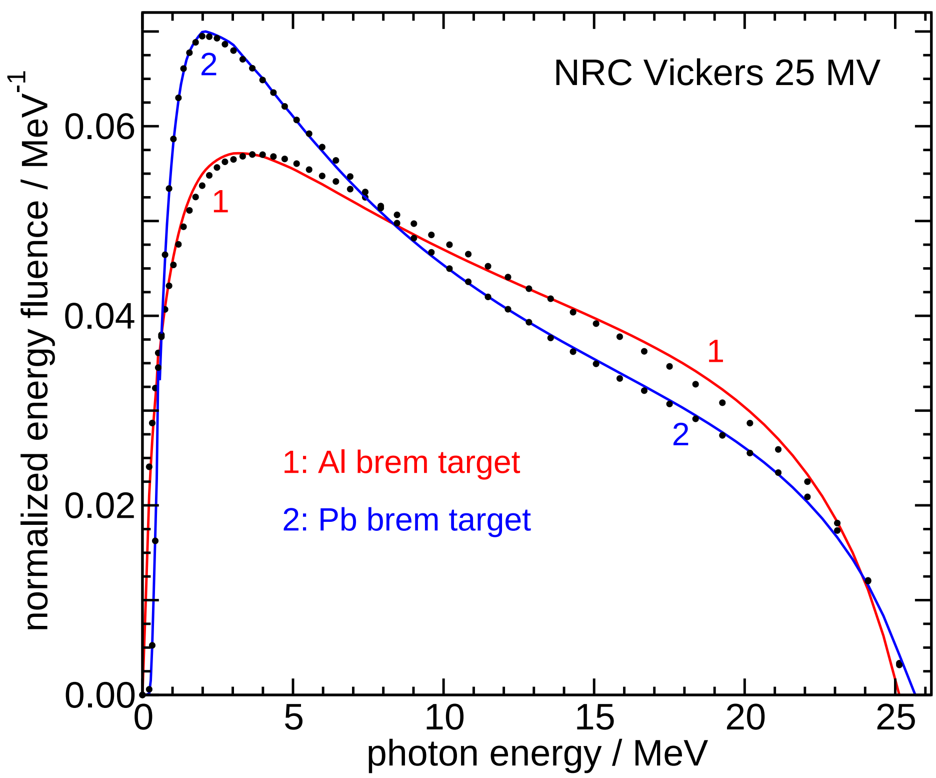

Figure 13: ...same as figure 1 but for the spectra of the NRC Vickers 25 MV beam.

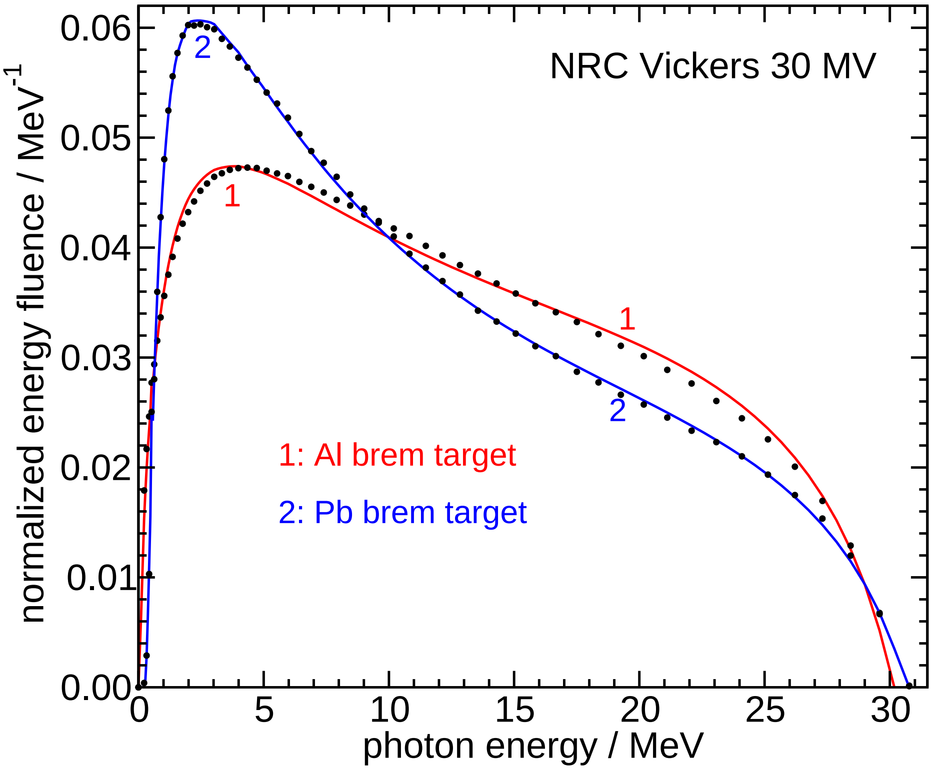

Figure 14: . . . same as figure 1 but for the spectra of the NRC Vickers 30 MV beams.

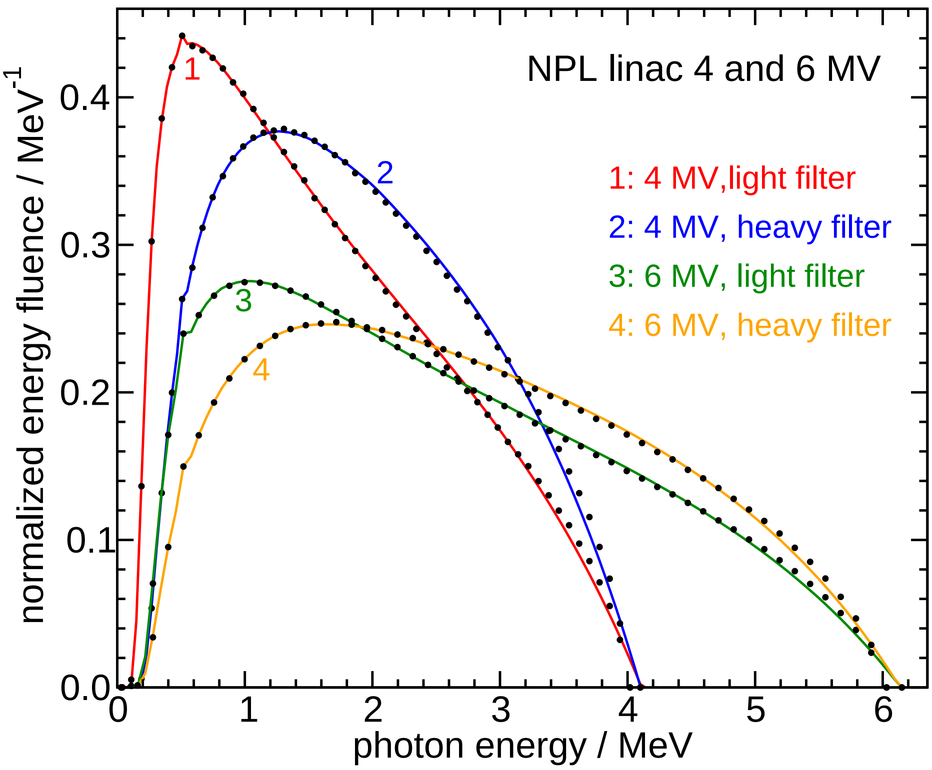

Figure 15: . . . same as figure 1 but for the spectra of the NPL 4 and 6 MV beams.

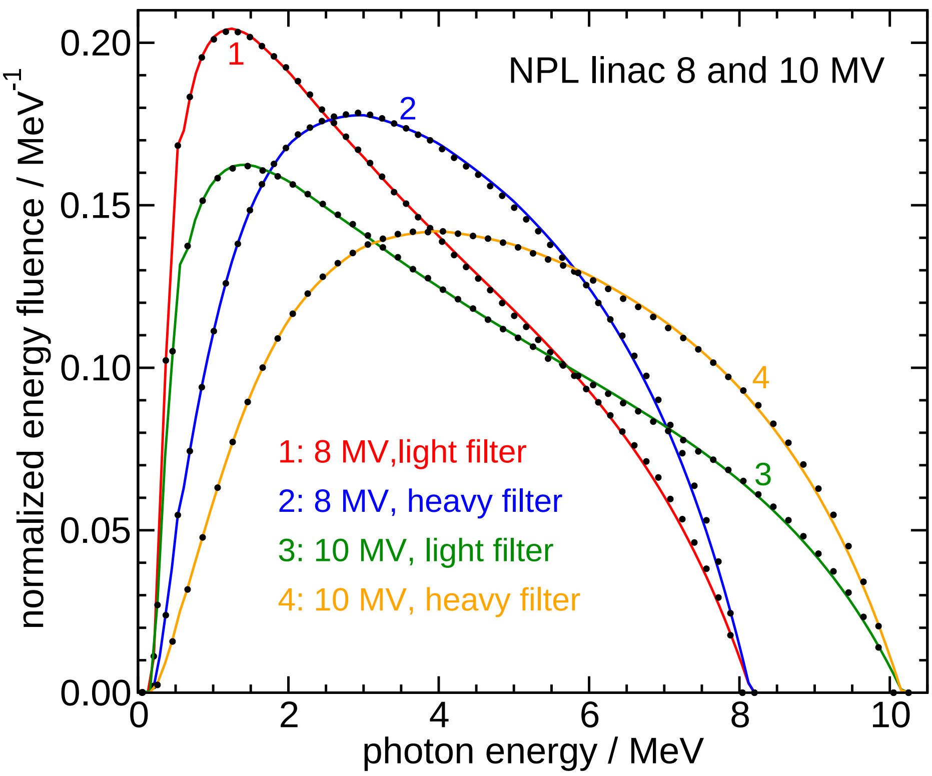

Figure 16: ...same as figure 1 but for the spectra of the NPL 8 and 10 MV beams.

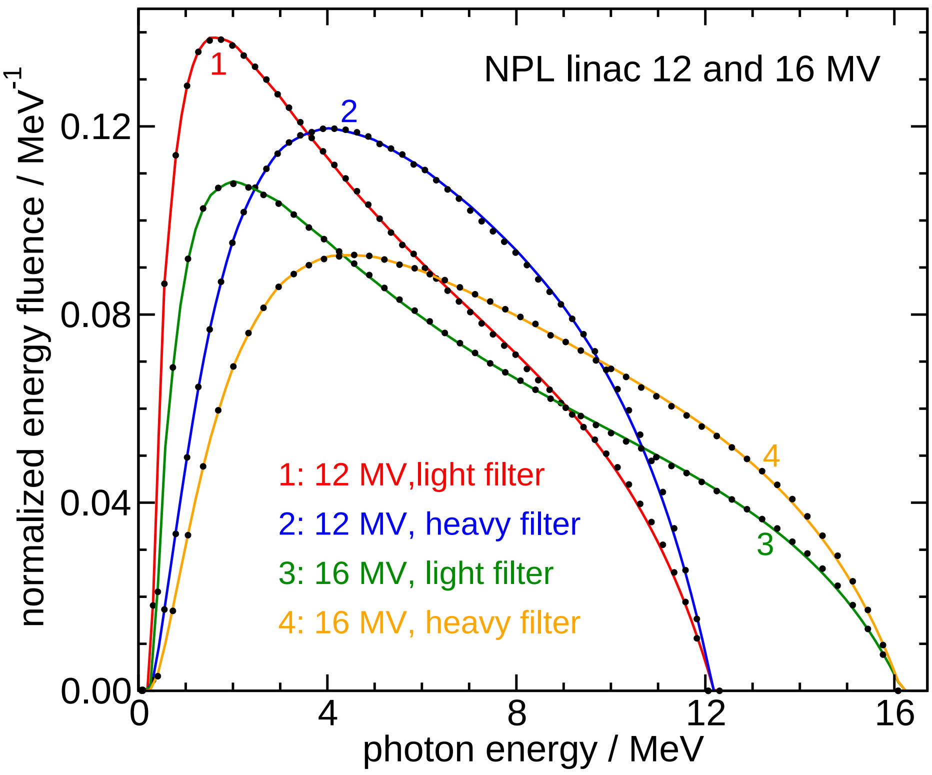

Figure 17: ...same as figure 1 but for the spectra of the NPL 12 and 16 MV beams.

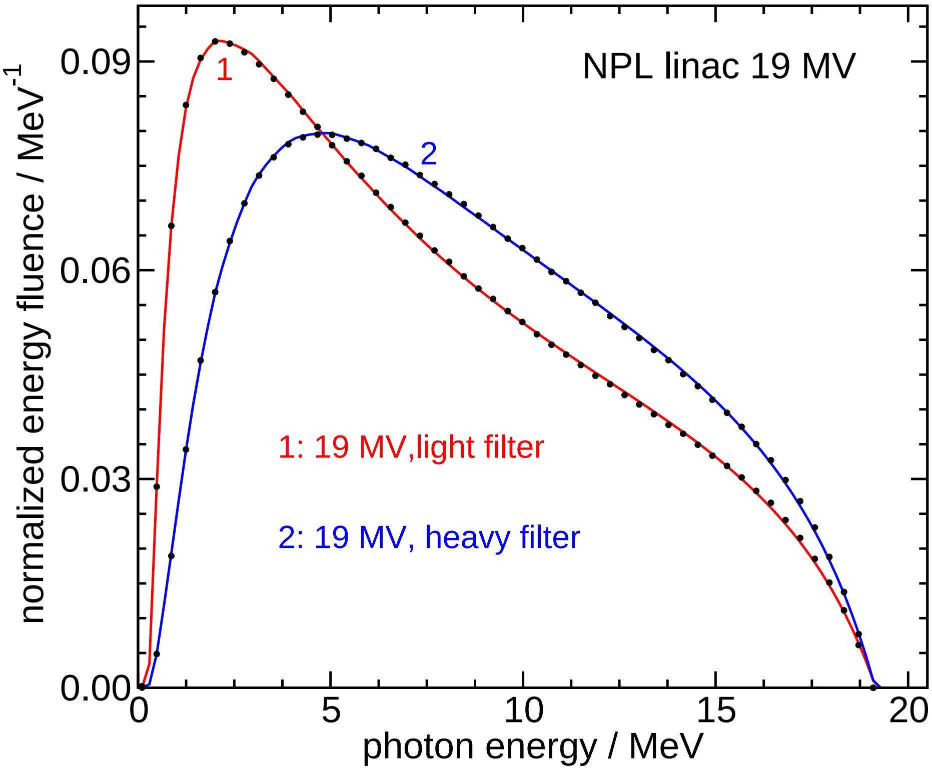

Figure 18: ...same as figure 1 but for the spectra of the NPL 19 MV beams.

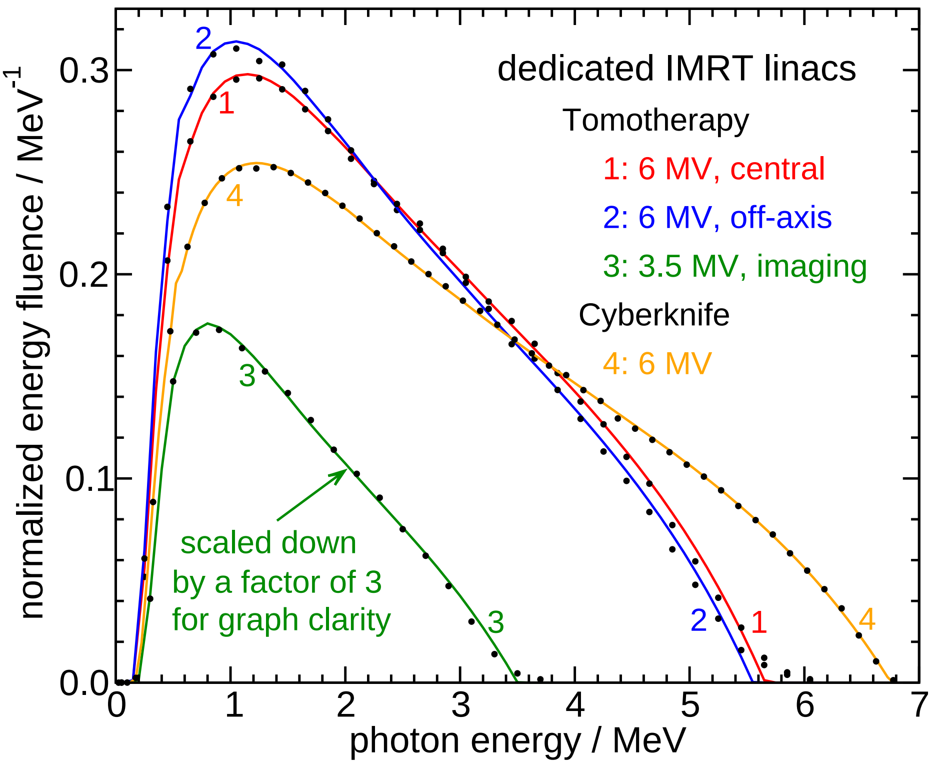

Figure 19: ...same as figure 1 but for the spectra of Tomotherapy and Cyberknife.