Systematic uncertainties in the Monte Carlo calculation of ion chamber replacement correction factors

L. L. W. Wang, D. J. La Russa, and D. W. O. Rogers

Ottawa Carleton Institute of Physics, Carleton University, Campus Ottawa, Ottawa, Ontario K1S 5B6, Canada

(Received 28 November 2008; revised 3 March 2009; accepted for publication 17 March 2009; published 21 April 2009)

In a previous study [Med. Phys. 35, 1747–1755 (2008)], the authors proposed two direct methods of calculating the replacement correction factors \( P_{\text{repl}} \) or \( p_{\text{repl}}p_{\text{dis}} \) for ion chambers by Monte Carlo calculation. By “direct” we meant the stopping-power ratio evaluation is not necessary. The two methods were named as the high-density air (HDA) and low-density water (LDW) methods. Although the accuracy of these methods was briefly discussed, it turns out that the assumption made regarding the dose in an HDA slab as a function of slab thickness is not correct. This issue is reinvestigated in the current study, and the accuracy of the LDW method applied to ion chambers in a \(^{60}\text{Co}\) photon beam is also studied. It is found that the two direct methods are in fact not completely independent of the stopping-power ratio of the two materials involved. There is an implicit dependence of the calculated \( P_{\text{repl}} \) values upon the stopping-power ratio evaluation through the choice of an appropriate energy cutoff \( \Delta \), which characterizes a cavity size in the Spencer-Attix cavity theory. Since the \( \Delta \) value is not accurately defined in the theory, this dependence on the stopping-power ratio results in a systematic uncertainty on the calculated \( P_{\text{repl}} \) values. For phantom materials of similar effective atomic number to air, such as water and graphite, this systematic uncertainty is at most 0.2% for most commonly used chambers for either electron or photon beams. This uncertainty level is good enough for current ion chamber dosimetry, and the merits of the two direct methods of calculating \( P_{\text{repl}} \) values are maintained, i.e., there is no need to do a separate stopping-power ratio calculation. For high-Z materials, the inherent uncertainty would make it practically impossible to calculate reliable \( P_{\text{repl}} \) values using the two direct methods. © 2009 American Association of Physicists in Medicine. [DOI: 10.1118/1.3115982]

Key words: Prepl, replacement correction, ion chamber dosimetry, EGSnrc, Monte Carlo

I. INTRODUCTION

In ionization chamber radiation dosimetry, the replacement correction factor, \( P_{\text{repl}} \) or \( p_{\text{repl}}p_{\text{dis}} \), accounts for the medium of interest being replaced by the air cavity of the chamber. In the AAPM’s TG-21 (Ref. 1) dosimetry protocol, the values of both \( P_{\text{repl}} \) and \( \left[ \frac{\bar{L}_{\Delta}}{\rho_{\text{air}}} \right]_{\text{med}} \), the Spencer-Attix medium/air mean restricted mass collision stopping-power ratio with cutoff energy \( \Delta=10 \) keV, are needed in the equation to determine the absorbed dose to medium [Eq. (9) in TG-21 (Ref. 1)]. In the AAPM’s TG-51 (Ref. 2) dosimetry protocol, the values of both \( P_{\text{repl}} \) and \( \left[ \frac{\bar{L}_{\Delta}}{\rho_{\text{air}}} \right]_{\text{water}} \) are not explicitly required but are implicitly used to determine the beam quality conversion factor \( k_Q \) which is given by

\[
k_Q = \frac{\left[ \frac{\bar{L}_{\Delta}}{\rho_{\text{water}}} \right]_{\text{water}} \left[ \frac{1}{p_{\text{wall}}} \right]_{\text{wall}} P_{\text{cel}} P_{\text{repl}}}{\left[ \frac{\bar{L}_{\Delta}}{\rho_{\text{water}}} \right]_{\text{air}}}, \tag{1}
\]

where the difference (if any) of W/e in different photon beams is neglected, and \( p_{\text{wall}} \) and \( P_{\text{cel}} \) are the wall correction factor and the central electrode correction factor, respectively. Hence in the TG-51 protocol, instead of using individual values of \( P_{\text{repl}} \) or \( \left[ \frac{\bar{L}_{\Delta}}{\rho_{\text{water}}} \right] \) as in TG-21, the ratio of \( P_{\text{repl}} \) or \( \left[ \frac{\bar{L}_{\Delta}}{\rho_{\text{water}}} \right] \) values for a beam of quality \( Q \) to a \(^{60}\text{Co}\) beam is used in the absorbed dose determination. This leads to a more robust system as we demonstrated recently.\(^3\)

In a previous study,\(^4\) two direct methods of calculating the value of \( P_{\text{repl}} \) for ion chambers were proposed, the high-density air (HDA) and low-density water (LDW) methods. These methods were applied to either electron or photon beams to calculate \( P_{\text{repl}} \) values and they gave similar results. When using the HDA method, the value of \( P_{\text{repl}} \) is

\[
P_{\text{repl}}^{\text{HDA}} = \frac{D_{\text{HDA}}}{D_{\text{air}}}, \tag{2}
\]

where \( D_{\text{HDA}} \) and \( D_{\text{air}} \) are the doses in a thin slab of HDA at the point of measurement of the ion chamber and in the air cavity of the wall-less chamber, respectively. For the LDW method, the \( P_{\text{repl}} \) value is calculated as

\[
P_{\text{repl}}^{\text{LDW}} = \frac{D_{\text{water}}}{D_{\text{LDW}}}, \tag{3}
\]

where \( D_{\text{water}} \) and \( D_{\text{LDW}} \) are the doses in the water phantom and in the chamber cavity filled with LDW, respectively. Although the accuracy of these methods was discussed briefly, we have found that the conclusion that the dose in an HDA slab becomes constant for sufficiently thin slabs is not correct. In fact the slab thickness will influence the calculation results by the HDA method. For the LDW method, only the accuracy for the NACP02 chamber in an electron beam...
was studied. In this work, the issue of dose in an HDA slab is reinvestigated and the uncertainty of the HDA method is studied; the accuracy of the LDW method applied to photon beams is also examined.

Suppose we have a phantom of material, med1, and a cavity of material, med2, located in the phantom. Assuming no electron fluence perturbation exists, i.e., an ideal Spencer-Attix cavity, the ratio of dose to the phantom without the presence of the cavity, $D_{\text{med1}}$, to dose to the cavity, $D_{\text{med2}}$, can be expressed as

$$D_{\text{med1}} = \frac{\bar{L}_{\Delta}}{\rho} \med1, \quad D_{\text{med2}} = \frac{\bar{L}_{\Delta}}{\rho} \med2,$$

(4)

where $[\bar{L}_{\Delta}/\rho]_{\med1}$ is the Spencer-Attix med1-to-med2 mean restricted mass collision stopping-power ratio. The cutoff energy $\Delta$ is related to cavity size and it is not clearly defined in the Spencer-Attix cavity theory but only vaguely described as the energy of electrons that can just cross the cavity. In practice, for a cavity of volume $V$ and surface area $S$, the cavity size is often specified by the mean chord length $L=4V/S$. For a pancake shaped cavity of radius $r$ and thickness $h$, $L$ is

$$L = \frac{2r}{1 + \frac{h}{r}}$$

(5)

For the cavity of a thimble chamber of radius $r$ and length $l$, $L$ is given by

$$L = \frac{2r}{1 + \frac{r}{l}}$$

(6)

Generally, the value of the stopping-power ratio varies with $\Delta$, thus with the cavity size. Figure 1 shows, as a function of $\Delta$, the graphite/air stopping-power ratio in a $^{60}$Co beam and the water/air stopping-power ratio in either a $^{60}$Co beam or a 6 MeV electron beam. The stopping-power ratios are normalized at $\Delta=10$ keV so that only the variation with $\Delta$ is emphasized. Taking water and HDA as the phantom and cavity materials, respectively, Eq. (4) becomes

$$D_{\text{water}} \quad D_{\text{HDA}} = \frac{\bar{L}_{\Delta}}{\rho \, \sigma_{\text{air}}}.$$  

(7)

As $\Delta$ is a monotonic function of cavity size, if we treat the HDA slab as the cavity, $\Delta$ will be a monotonic function of the slab thickness. As $D_{\text{water}}$ is a constant independent of slab thickness at a specific position in a phantom, Eq. (7), together with Fig. 1, tells us that the dose in an HDA slab cannot be a constant while one is reducing the HDA slab thickness which corresponds with decreasing $\Delta$. This is on the assumption that the slab is thin enough that it does not perturb the electron fluence. This lack of a constant value of $D_{\text{HDA}}$ contradicts our previous study where calculations showed that, as one goes to a thinner slab, the dose in the HDA slab becomes constant. This paper investigates this issue further.

II. UNCERTAINTY OF $P_{\text{repl}}$ IN THE HDA METHOD

II.A. Dose and dose ratio in HDA slab

Figure 4 of the previous study showed the variation of the dose in the HDA slab in either a $^{60}$Co beam or a 6 MeV electron beam as a function of the slab thickness. In those calculations, the electron energy threshold and cutoff were set at $AE=ECUT=10$ keV (kinetic energy). We did a comparison of 10 to 1 keV cutoffs for the water/HDA dose ratio in 1 $\mu$m slabs in the 6 MeV electron beam and found that they agreed within the 0.1% statistical uncertainties and the dose ratio was very close to the water/air stopping-power ratio. This led to the conclusion that there was no difference using either a 10 or 1 keV electron energy threshold and cutoff. However, this is not correct. We have recalculated the dose in the HDA slab (radius of 1 cm) with $AE=ECUT=1$ keV down to a thickness of 0.1 $\mu$m. The results are shown in Fig. 2 with the previous data calculated with $AE=ECUT=10$ keV. Again, all values are normalized at a thickness of 0.2 mm. The dose in the water slab remains constant down to the 0.1 $\mu$m thickness (not shown in the figure). For the calculations with a 1 keV cutoff, there is no tendency to a constant dose in the HDA slab. This is expected from Eq. (7) and Fig. 1. Note also that the difference between a 10 and 1 keV calculation at 1 $\mu$m slab thickness is not more than 0.2% for the electron beam and it is the same for the $^{60}$Co beam. For 10 keV electrons, the CSDA range in HDA or water is about 3 $\mu$m. This means that when

Medical Physics, Vol. 36, No. 5, May 2009
AE=10 keV the spatial resolution is about 3 μm and setting an HDA slab thinner than that is inappropriate. For 1 keV electrons, the range is well below 0.1 μm.

The dose in the HDA slab is in fact affected by both the water/air stopping-power ratio and a possible electron fluence perturbation. In the previous paper, the constant dose for thin slabs was taken to imply that there was no further fluence perturbation. Now we need to find when the electron fluence perturbation becomes negligible in the case that the dose does not become constant. To do so, we have to eliminate the influence of the water/air stopping-power ratio on the dose in the HDA slab. From Eq. (7), for an unperturbed cavity, we expect the quotient $(D_{\text{water}}/D_{\text{HDA}})/(L/\rho)$ to be unity; otherwise there would be an electron fluence perturbation. Before proceeding, one still needs to find how the cutoff energy $\Delta$ is related to the HDA slab thickness. We used a series of broad, parallel, monoenergetic electron beams incident on a semi-infinite HDA slab and calculated the depth-dose curves, from which the practical ranges of the electrons are found by extending the maximum tangent line on the dose fall-off area to the abscissa. Figure 3 shows the calculation results of the practical ranges which are slightly less than the corresponding CSDA ranges. Traditionally, $\Delta$ is taken as the energy of electrons that can just cross the cavity. The HDA slab thickness corresponding to a given cutoff $\Delta$ can be taken as the slab thickness equal to the practical range from this figure. Based on this assumption, we calculated the quantity $(D_{\text{water}}/D_{\text{HDA}})/(L/\rho)$ as a function of HDA slab thickness as shown by dashed lines in Fig. 4 for the 6 MeV electron beam and the $^{60}$Co beam. It is seen that in both cases the quantity $(D_{\text{water}}/D_{\text{HDA}})/(L/\rho)$ stabilizes and is within 0.2% of 1 as the thickness becomes smaller than 20 μm (for electron beam, it is within 0.2% of 1 just below 0.1 mm thickness). This is taken to mean that the electron fluence perturbation is negligible for HDA thicknesses less

![Figure 2](image.png)  
**Fig. 2.** Dose in an HDA slab as a function of the slab thickness. The calculations are at $d_{\text{ref}}$ and $R_{50}$ in the 6 MeV electron beam and at depth of 5 cm in the $^{60}$Co beam. The doses are normalized at a slab thickness of 0.2 mm. The dashed lines are the data calculated with energy threshold $AE=10$ keV [from Fig. 4 in our previous study (Ref. 4)]. The solid lines are calculated with $AE=1$ keV in this study.

![Figure 3](image.png)  
**Fig. 3.** Relationship between the practical range of electrons in HDA and the energy of the monoenergetic, parallel-incident broad electron beams. The solid line is calculated from an empirical fitted formula between the electron energy and the practical range.

![Figure 4](image.png)  
**Fig. 4.** Ratio of the dose ratio (water to HDA) and the restricted water/air stopping-power ratio as a function of the HDA slab thickness at (a) $R_{50}$ in a 6 MeV electron beam and (b) 5 cm depth in a $^{60}$Co beam. The cutoff energy $\Delta$ for the stopping-power ratio evaluation is equal to the energy of electrons that have a practical range either equal to the HDA thickness (dashed lines) or two times the thickness (solid lines).
than 20 μm for either electron or photon beams. The Δ value selected here is based on the practical range being equal to the HDA slab thickness. If the Δ value is taken as the mean chord length as defined in Eq. (5), which is two times the HDA thickness (since \( h \ll r \)), then the results are represented by the solid lines in Fig. 4. The difference between these two is at most 0.1%–0.2%. The solid lines are even closer to 1, suggesting that the mean chord length \( L \) is a reasonably good cavity size specifier. This is consistent with the results of La Russa and Rogers\(^6\) who found that this prescription for Δ was valid for low-\( Z \) cavities (but not high \( Z \)).

### II.B. Calculation of \( P_{\text{repl}} \) by the HDA method

As \( D_{\text{HDA}} \) is shown not to be a constant while the HDA thickness is decreasing, it appears that \( P_{\text{HDA}}^{\text{repl}} \) will not have a unique value based on Eq. (2), even in the situations when there is no electron fluence perturbation. We have to use a particular thickness of the HDA slab in order to have a unique value. Since all dosimetry protocols\(^2,7,8\) adopted a single value of Δ = 10 keV for the evaluation of the water/air stopping-power ratio for all kinds of commonly used chambers, the reasonable choice of the HDA slab thickness is such that it equals the practical range or CSDA range of electrons having an energy of 10 keV. This is strictly not correct, but practically it is acceptable since the water/air stopping-power ratio only varies by 0.1% when Δ varies from 10 to 20 keV (see Fig. 1), corresponding to an air cavity size from 2 to 7 mm. With Δ = 10 keV, the appropriate HDA slab thickness is 2.5 μm based on the practical range (Fig. 3). On the other hand, if the mean chord length is used to specify the cavity size, the appropriate HDA slab thickness would have to be 1.25 μm. Either way, the thickness is well below the value of 20 μm where the electron fluence perturbation diminishes. The difference between the dose in the HDA slab of 2.5 μm and of 1.25 μm is only 0.1%–0.2%. This means that the systematic uncertainty of \( P_{\text{repl}} \) values resulting from the uncertainty of selecting an appropriate HDA slab thickness is 0.1%–0.2%. Our previous study\(^4\) compared \( P_{\text{repl}} \) values calculated by different methods and obtained good agreement. For the HDA method, a slab thickness of 1 μm was used, which is just below the thickness suggested in this study; so one would expect a slightly underestimated \( P_{\text{repl}} \) value. But since AE = 10 keV was used in the calculation of dose in the HDA slab, this led to an overestimation by 0.1%–0.2% of \( P_{\text{repl}} \) values. Hence the net effect was in agreement within 0.1% between the HDA method and the other methods.

The uncertainty related to the selection of an appropriate HDA slab thickness is inherent in Spencer-Attix cavity theory. It introduces a systematic uncertainty in the calculation of \( P_{\text{repl}} \), which is in addition to the statistical uncertainty of Monte Carlo calculation method. For high-\( Z \) materials such as lead, the stopping-power ratio changes very rapidly with cutoff energy Δ (about 1% from 10 to 20 keV for lead) and the electron fluence perturbation is also expected to be very large for an HDA slab in the high-\( Z \) material which is consistent with the large fluence corrections found necessary for high-\( Z \) walled chambers in a \( ^{60}\text{Co} \) beam if one uses the traditional \( L = 4V/S \) prescription.\(^6\) These issues make it impossible to calculate the \( P_{\text{repl}} \) values by the HDA method for high-\( Z \) materials.

### III. Uncertainty of \( P_{\text{repl}} \) in the LDW method

For the low-density water (LDW) method, the perturbation effect caused by replacing air with LDW in an NACP02 cavity in a 6 MeV electron beam was studied previously.\(^4\) The conclusion was that the fluence perturbation or the uncertainty related to the selection of Δ for the stopping-power ratio evaluation was at most 0.2% at all depths. In our study of \( P_{\text{repl}} \) values for the BIPM chamber in a graphite phantom irradiated by a \( ^{60}\text{Co} \) beam,\(^9\) we used the LDW method for the \( P_{\text{repl}} \) calculations. To establish if there is a possible electron fluence perturbation in the LDW filled cavity compared to the air-filled cavity, there is also an issue associated with the selection of the appropriate Δ for the graphite/air stopping-power ratio evaluation. To investigate this, we calculate the ratio of doses in the BIPM chamber cavity filled with low-density graphite (LDG) to that filled with air at two depths in a graphite phantom in a \( ^{60}\text{Co} \) beam. The source-chamber distance is kept at 100 cm, and the field size at the chamber is 10 × 10 cm\(^2\). The graphite/air stopping-power ratios are evaluated for Δ = 14 keV.

<table>
<thead>
<tr>
<th>Depth (g/cm(^2))</th>
<th>( D_{\text{LDW}} )</th>
<th>( \left( \frac{L_D}{\rho} \right)^{\text{repl}} )</th>
<th>( \frac{D_{\text{LDW}}}{D_{\text{air}}} )</th>
<th>( \frac{\left( L_D \right)^{\text{repl}}}{\left( L_D \right)^{\text{air}}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0</td>
<td>1.0014 ± 0.0008</td>
<td>1.002 75 ± 0.0007</td>
<td>0.9986 ± 0.0008</td>
<td></td>
</tr>
<tr>
<td>15.0</td>
<td>1.0016 ± 0.0008</td>
<td>1.003 65 ± 0.0008</td>
<td>0.9980 ± 0.0008</td>
<td></td>
</tr>
</tbody>
</table>

TABLE I. The ratio of dose in the BIPM chamber cavity filled with low-density graphite (LDG) to that filled with air at two depths in a graphite phantom in a \( ^{60}\text{Co} \) beam. The source-chamber distance is kept at 100 cm, and the field size at the chamber is 10 × 10 cm\(^2\). The graphite/air stopping-power ratios are evaluated for Δ = 14 keV.
dose ratio and the water/air stopping-power ratio. When the cutoff energy $\Delta = 10$ keV is used for the stopping-power ratio evaluation for all the chambers, the quotient varies by 0.16% from 1 to 10 mm chamber. However, if the value of $\Delta$ characterizing each cavity size is used in the stopping-power ratio evaluation, the quotients are essentially unity for all chamber sizes. This demonstrates that the 0.16% variation when using a single $\Delta$ value for all chambers comes from the stopping-power ratio variation with $\Delta$, and there is a negligible change in the electron fluence perturbation for these chambers in photon beams when air is replaced by LDW. Thus the calculated values of $P_{\text{repl}}^{\text{LDW}}$ are correct but must be used with stopping-power ratios using the value of $\Delta$ corresponding to the mean chord length $\bar{L}_{s} = 4V/S$. As current dosimetry protocols use a single value of $\Delta = 10$ keV for all chambers used for calibration, one way to account for this is to assume that the calculation of $P_{\text{repl}}$ values by the LDW method for commonly used thimble chambers has a systematic uncertainty of something less than 0.2%.

For high-Z materials, it is expected that the electron fluence perturbation is not negligible when replacing air by the low-density high-Z material. In addition, the rapid change of the stopping-power ratios with the cutoff energy $\Delta$ makes the uncertainty on $P_{\text{repl}}$ values calculated by the LDW method very large.

### IV. CONCLUSIONS

The accuracy associated with the HDA and LDW direct methods of calculating the replacement correction factors ($P_{\text{repl}}$) for ion chambers is not completely independent of the stopping-power ratio of the two materials involved. There is an implicit dependence of the calculated $P_{\text{repl}}$ values upon the stopping-power ratio through the determination of an appropriate energy cutoff $\Delta$. As the selection of $\Delta$ is not accurately defined in the Spencer-Attix cavity theory, this dependence on the stopping-power ratio results in a systematic uncertainty on the calculated $P_{\text{repl}}$ values. This uncertainty is inherent in the Spencer-Attix cavity theory. For phantom materials of similar effective atomic number to air (e.g., water, graphite), this systematic uncertainty is at most 0.2% for most commonly used chambers in either electron or photon beams. This level of accuracy is good enough for current ion chamber dosimetry, and the merits of the two direct methods of calculating $P_{\text{repl}}$ values are maintained, i.e., there is no need to do a separate stopping-power ratio calculation to calculate the $P_{\text{repl}}$ values. For high-Z materials, the inherent uncertainty would be so large that in practice it is impossible to calculate $P_{\text{repl}}$ values by the two direct methods.

### ACKNOWLEDGMENTS

This work is supported by an OGS scholarship, NSERC, the Canada Research Chairs program, CFI, and OIT.

---

**Table II.** The quotient of the dose ratio and the water/air stopping-power ratio for cylindrical chambers of different radii at 5 cm depth in a water phantom irradiated by a $^{60}$Co beam. The dose ratio is the ratio of dose in the chamber cavity filled with LDW to that filled with air. The source-surface distance is at 80 cm and the field size is $10 \times 10$ cm$^2$. The length for all the chambers is 2 cm. The mean chord length is calculated by Eq. (6). $\Delta$ is found from Fig. 3, after scaling density from air to HDA. In the first row, $\Delta = 10$ keV is used for the stopping-power ratio evaluation for all the chambers. In the last row, the value of $\Delta$ characterizing each cavity size is used in the stopping-power ratio evaluation.

<table>
<thead>
<tr>
<th>Radius (mm)</th>
<th>1 mm</th>
<th>3 mm</th>
<th>10 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{D_{\text{LDW}}}{D_{\text{air}}} / \left[ \frac{\bar{L}<em>{s}}{\rho} \right]</em>{a}^{\Delta}$</td>
<td>0.9998 ± 0.0008</td>
<td>0.9988 ± 0.0009</td>
<td>0.9982 ± 0.0008</td>
</tr>
<tr>
<td>Mean chord length (mm)</td>
<td>2</td>
<td>5</td>
<td>13</td>
</tr>
<tr>
<td>$\Delta$ (keV)</td>
<td>9.6</td>
<td>16.2</td>
<td>28.1</td>
</tr>
<tr>
<td>$\frac{D_{\text{LDW}}}{D_{\text{air}}} / \left[ \frac{L_{s}}{\rho} \right]_{a}^{\Delta}$</td>
<td>0.9997 ± 0.0008</td>
<td>0.9977 ± 0.0009</td>
<td>1.0000 ± 0.0008</td>
</tr>
</tbody>
</table>