

Replacement correction factors for cylindrical ion chambers in electron beams

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Purpose: In the TG-21 dosimetry protocol, for cylindrical chambers in electron beams the replacement correction factor P_{repl} (or the product $p_{\text{dis}}p_{\text{cav}}$ in the IAEA's notation), was conceptually separated into two components: the gradient correction (P_{gr}) accounting for the effective point of measurement and the fluence correction (P_{fl}) dealing with the change in the electron fluence spectrum. At the depth of maximum dose (d_{max}), P_{gr} is taken as 1. There are experimental data available at d_{max} for the values of P_{fl} (or P_{repl}). In the TG-51 dosimetry protocol, the calibration is at the reference depth $d_{\text{ref}}=0.6R_{50}-0.1$ (cm) where P_{gr} is required for cylindrical chambers and P_{fl} is unknown and so the measured values at d_{max} are used with the corresponding mean electron energy at d_{ref} . Monte Carlo simulations are employed in this study to investigate the replacement correction factors for cylindrical chambers in electron beams.

Methods: Using previously established Monte Carlo calculation methods, the values of P_{repl} and P_{fl} are calculated with high statistical precision ($<0.1\%$) for cylindrical cavities of a variety of diameters and lengths in a water phantom irradiated by various electron beams. The values of P_{gr} as defined in the TG-51 dosimetry protocol are also calculated.

Results: The calculated values of the fluence correction factors P_{fl} are in good agreement with the measured values when the wall correction factors are taken into account for the plane-parallel chambers used in the measurements. An empirical formula for P_{fl} for cylindrical chambers at d_{ref} in electron beams is derived as a function of the chamber radius and the beam quality specifier R_{50} .

Conclusions: The mean electron energy at depth is a good beam quality specifier for P_{fl} . Thus TG-51's adoption of P_{fl} at d_{max} with the same mean electron energy for use at d_{ref} is proven to be accurate. The values of P_{gr} for a Farmer-type chamber as defined in the TG-51 dosimetry protocol may be wrong by 0.3% for high-energy electron beams and by more than 1% for low-energy electron beams. © 2009 American Association of Physicists in Medicine.

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I. INTRODUCTION

The replacement correction factor (P_{repl}) in ion chamber radiation dosimetry accounts for the effects of the medium being replaced by the air cavity of the chamber. Using the Spencer-Attix formalism, the dose in a water phantom, D_{water} , is related to the dose in the air in a water-walled chamber with no central electrode, D_{air} , by

$$\frac{D_{\text{water}}}{D_{\text{air}}} = \left(\frac{\bar{L}_{\Delta}}{\rho} \right)_{\text{air}}^{\text{water}} P_{\text{repl}}, \quad (1)$$

where $(\bar{L}_{\Delta}/\rho)_{\text{air}}^{\text{water}}$ is the Spencer-Attix water/air mean restricted mass collision stopping-power ratio (SPR) with cut-off energy Δ . Traditionally, P_{repl} can be expressed as¹

$$P_{\text{repl}} = P_{\text{gr}}P_{\text{fl}}, \quad (2)$$

where P_{gr} is the gradient correction and P_{fl} is the fluence correction (corresponding to the displacement correction factor p_{dis} and the fluence perturbation p_{cav} , respectively, in the IAEA's notation). Both P_{gr} and P_{fl} are nonunity for cylindrical chambers in electron beams. In AAPM's TG-51

protocol,² the chamber is positioned with its geometric center at the point of interest in the phantom and a gradient correction P_{gr} is explicitly used. In IAEA's TRS 398 Code of Practice,³ the chamber is positioned with its geometric center at a point downstream from the point of interest in the phantom, i.e., the effective point of measurement (EPOM) approach is used, and thus P_{repl} is simply P_{fl} .

Recently, we have presented systematic and reliable methods of calculating the values of P_{repl} for ion chamber radiation dosimetry by using Monte Carlo methods with systematic uncertainties of 0.2% or less.^{4,5} These methods were applied to calculate P_{repl} values for plane-parallel chambers in both electron and photon beams,⁴ for cylindrical chambers in photon beams,⁶ and for the BIPM chamber in a ⁶⁰Co beam.⁷ However, one issue not addressed previously is the P_{repl} values for cylindrical chambers in electron beams due to the complexity of the problems involved. In a separate work,⁸ we have studied the issue of the EPOM for cylindrical chambers in electron beams. The EPOM concept is actually a different approach for accounting for the gradient correction P_{gr} . In this work, we investigate the overall replacement cor-

rection factors P_{repl} as well as its two components: the fluence correction P_{fl} and the gradient correction P_{gr} . Although cylindrical chambers are not recommended for use in low-energy electron beams,^{2,3} the values of P_{repl} (or P_{fl}) at the depth of maximum dose d_{max} (where $P_{\text{gr}}=1$) are tabulated in TG-21 (Ref. 1) for a variety of chamber radii for electron beams with mean energy at the point of measurement from 2 to 20 MeV. These values are mainly based on the experiments performed by Johansson *et al.*⁹ who measured the electron fluence perturbation factor at d_{max} by comparing ionization readings of various cylindrical chambers to those from a plane-parallel chamber which was assumed to be perturbation-free. In TG-51,² the calibration depth for electron beams is at the reference depth $d_{\text{ref}}=0.6R_{50}-0.1$ (cm), where no P_{fl} values are available for high-energy electron beams and it is assumed that one may use values determined at d_{max} , with the same mean electron energy at depth.

In this study, Monte Carlo simulations using EGSnrc user-codes^{10,11} are employed to study the perturbation effect caused by the introduction of a cylindrical air cavity into a water phantom irradiated by electron beams. With the previously established methods, the values of P_{repl} and/or P_{fl} are calculated with high statistical precision ($<0.1\%$, 1σ) for cylindrical chambers in various electron beams at both d_{max} and d_{ref} and the results are compared to AAPM's TG-21 (Ref. 1) [$\equiv$ TG-51 (Ref. 2)] and/or IAEA's TRS-398 (Ref. 3) values. The systematic uncertainty in these calculated values is 0.2% or less.⁵ The chamber size dependence and the beam quality dependence of P_{fl} values are investigated.

II. MATERIALS AND METHODS

II.A. Calculation of P_{repl} values

Previously⁴ we have described four methods [SPR, high-density air (HDA), low-density water (LDW), and fluence (FLU)] of calculating P_{repl} values. The SPR method is the typical way of calculating P_{repl} by using the Spencer-Attix relationship. Making the depth dependences explicit, one may rewrite Eq. (1) to give

$$P_{\text{repl}}^{\text{SPR}}(z) = \frac{D_{\text{water}}(z)}{D_{\text{air}}(z)} \left/ \left(\frac{\bar{L}_{\Delta}(z)}{\rho} \right)_{\text{air}}^{\text{water}} \right. \quad (3)$$

Using Eq. (3), the value of P_{repl} , which is denoted here as $P_{\text{repl}}^{\text{SPR}}$ to indicate it is obtained by the SPR method, is calculated as the quotient of the phantom-to-cavity (here it is water-to-air) dose ratio and the mean restricted stopping-power ratio of the two media (phantom and cavity). The phantom-to-cavity dose ratio is the ratio of dose in phantom at the reference point, in the absence of the cavity, to the dose in the collecting volume of the cavity, when the cavity is centered at the reference point. Here all values are evaluated at depth z . The HDA and LDW methods are two direct methods of calculating P_{repl} without the need of the stopping-power ratio calculation. For the HDA method, the dose in phantom is calculated as the dose in a thin slab centered at the reference point, with the slab being replaced by high-density air which has all the characteristics of air except its

density is equal to that of the phantom. The value of P_{repl} is

$$P_{\text{repl}}^{\text{HDA}}(z) = \frac{D_{\text{HDA}}(z)}{D_{\text{air}}(z)}, \quad (4)$$

where D_{HDA} is the dose in the HDA slab in a water phantom. For the LDW method, the air in the cavity is replaced by a low-density water material which has all the characteristics of water except its density is the same as that of air. The value of P_{repl} is

$$P_{\text{repl}}^{\text{LDW}}(z) = \frac{D_{\text{water}}(z)}{D_{\text{LDW}}(z)}, \quad (5)$$

where D_{LDW} is the dose in the chamber cavity filled with LDW. For both the direct methods, the phantom-to-cavity dose ratio will give P_{repl} directly since the stopping-power ratio vanishes as the materials are the same. The FLU method calculates P_{repl} as the ratio of the total electron fluence in the phantom to that in the cavity.

Unless otherwise specified, the cylindrical air cavity is of length 2 cm and radius 3 mm (i.e., a wall-less Farmer-type chamber without central electrode). The air cavity is in a water phantom (a cube of 30 cm sides) with the cavity's geometric center at the point of measurement. The radiation sources (electron beams from 6 to 22 MeV) are at 100 cm source-surface distance (SSD) and have a 10×10 cm² field size. The spectra of the incident electron beams are from Monte Carlo simulations of a Varian Clinac 2100C linac.¹² A spectrum source from a 22 MeV Elekta SL25 electron beam is also used in the calculations. The EGSnrc user-code CAVITY (Refs. 13 and 14) is used in all calculations. When using the LDW method the ratios required are independent of the exact cutoffs used⁴ and the electron and photon energy thresholds for production and tracking (AE and ECUT and AP and PCUT) are 521 and 10 keV, respectively. However, for the HDA method it is necessary to use 512 keV electron energy cutoff because the slab thickness is very thin.⁵ For the HDA slab dose calculation, the HDA slab thickness depends on the size of the chamber air cavity.⁵ For cavity radii from 1 to 5 mm, corresponding to HDA thicknesses of about 1 to 5 μm , the water/air stopping-power ratio only varies by about 0.1% due to the difference in energy cutoffs Δ (from 10 to 20 keV) for different cavity sizes. Thus the HDA slab thickness is chosen as 3 μm for all cavity sizes and this causes less than 0.1% uncertainty. The transverse size of the phantom voxel or the HDA slab is 1 cm in radius.

Table I lists P_{repl} values calculated by the four methods at d_{ref} in both 6 and 18 MeV electron beams. The SPR method and the HDA method give the same P_{repl} values for both beams. The FLU method gives similar results but it could be coincidental as the electron fluence spectra are very different in the cavity from those in the phantom.⁸ The most notable thing is that the LDW method, a direct method of calculating P_{repl} , gives a value which is 0.6% higher in the 6 MeV beam and 0.3% higher in the 18 MeV beam, than the other direct method (HDA). This seems to be inconsistent with the previous study⁴ in which all methods give the same values for cylindrical chambers in photon beams or for plane-parallel

TABLE I. Comparison of calculated P_{repl} values using four methods for a Farmer chamber cavity (length of 2 cm and radius of 3 mm) centered at d_{ref} in a 6 MeV and in an 18 MeV electron beam. The HDA slab thickness is 3 μm for the HDA method. The uncertainties are statistical only.

Beam energy (MeV)	SPR	FLU	P_{repl}	HDA	LDW
6	$0.9604 \pm 0.04\%$	$0.9627 \pm 0.04\%$		$0.9618 \pm 0.07\%$	$0.9674 \pm 0.04\%$
18	$0.9832 \pm 0.07\%$	$0.9841 \pm 0.12\%$		$0.9829 \pm 0.07\%$	$0.9858 \pm 0.07\%$

chambers in either type of beam. In Sec. II B, this discrepancy is investigated and the relationships among P_{repl} values calculated by different methods will be derived.

II.B. Relationships between P_{repl} values calculated by different methods

In our separate study on the effective point of measurement for ion chambers in electron beams,⁸ we have demonstrated that for a cylindrical chamber of radius r to be considered as a Spencer-Attix cavity, the following equation should hold approximately (based on Fig. 10 in Ref. 8):

$$\frac{D_{\text{LDW}}(z)}{D_{\text{air}}(z)} = \left(\frac{\bar{L}_{\Delta}(z-s)}{\rho} \right)_{\text{air}}^{\text{water}}, \quad (6)$$

where z is the depth at which the center of the chamber is located and $s=0.8r$ is the shift for matching the primary electron fluence spectrum.⁸ This equation recognizes that the spectrum in the cavity corresponds to that at the effective point of measurement and hence the water/air stopping-power ratio $\bar{L}_{\Delta}/\rho$ is evaluated at depth $z-s$. From our previous studies,^{4,5} the electron fluence spectrum in a thin enough ($<20 \mu\text{m}$) HDA slab is very similar to that at the same point of measurement in a water phantom. This means the ratio of the dose to water (D_{water}) to the dose to the HDA slab (D_{HDA}) is very close to the water/air stopping-power ratio⁵ at the point of measurement at z , i.e.,

$$\frac{D_{\text{water}}(z)}{D_{\text{HDA}}(z)} = \left(\frac{\bar{L}_{\Delta}(z)}{\rho} \right)_{\text{air}}^{\text{water}}. \quad (7)$$

Based on these equations, relations among P_{repl} values calculated by different methods can be derived. Dividing Eq. (5) by Eq. (4) and using Eqs. (6) and (7), one arrives at

$$P_{\text{repl}}^{\text{LDW}}(z) = P_{\text{repl}}^{\text{HDA}}(z) \frac{\left(\frac{\bar{L}_{\Delta}(z)}{\rho} \right)_{\text{air}}^{\text{water}}}{\left(\frac{\bar{L}_{\Delta}(z-s)}{\rho} \right)_{\text{air}}^{\text{water}}}. \quad (8)$$

Equation (8) suggests that the two direct methods of calculating P_{repl} for cylindrical chambers in electron beams no longer give the same values because s is not zero and $(\bar{L}_{\Delta}/\rho)_{\text{air}}^{\text{water}}$ varies with depth. Equation (8) is generally applicable to both cylindrical chambers and plane-parallel chambers in either electron or photon beams. In photon beams, $(\bar{L}_{\Delta}/\rho)_{\text{air}}^{\text{water}}$ does not vary with depth z , and for plane-parallel chambers in electron beams $s=0$, so that in those two

cases the two direct methods give the same P_{repl} values as shown in our earlier work.⁴ For cylindrical chambers in electron beams, the LDW method generally gives a higher value of P_{repl} than the HDA method as the water/air stopping-power ratio is smaller at shallower depth as shown in Fig. 1, and hence the ratio of the stopping-power ratios in Eq. (8) is greater than 1. This explains the discrepancy in Table I between the P_{repl} values calculated by the two direct methods.

The selection of which P_{repl} value to use depends upon the depth at which the water/air stopping-power ratio is evaluated and upon the relative position of the point of measurement and the chamber center. Assuming the chamber center is taken as the point of measurement, if one chooses the water/air stopping-power ratio at the effective point of measurement, i.e., a point upstream from the chamber center, then based on Eqs. (5) and (6) the P_{repl} values calculated by the LDW method should be used, but if one chooses the water/air stopping-power ratio at the same depth as the chamber center (this is the method used in the AAPM dosimetry protocols), based on Eqs. (4) and (7) the value calculated by the HDA method should be used. In other words, whenever the water/air stopping-power ratio is evaluated at the depth of the center of the chamber in the phantom, the HDA method should be used.

Rearranging Eq. (3) and using Eqs. (7) and (4), one has

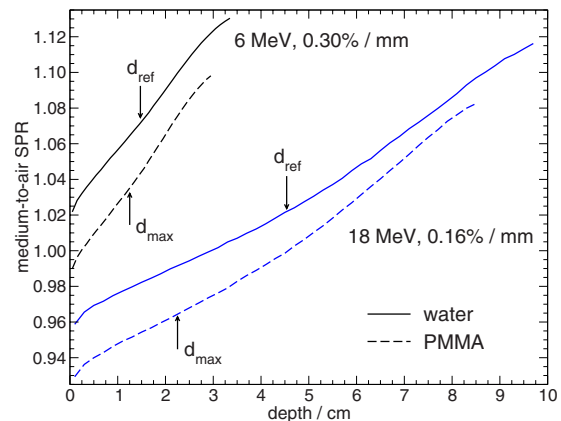


FIG. 1. The medium-to-air SPR vs depth for the 6 MeV and the 18 MeV electron beams. The medium is either water (solid line) or PMMA (dashed line).

TABLE II. Comparison of the calculated fluence correction factors $P_{f,0}$ at d_{ref} using the four methods for the same scenarios as in Table I. The depth for phantom calculation is shifted upstream by $0.8r$ to match the spectra of the primary electrons (see Sec. II C for details on the calculation methods).

Beam energy (MeV)	SPR	FLU	$P_{f,0}$ ($s=0.8r$)	HDA	LDW
6	$0.9645 \pm 0.04\%$	$0.9666 \pm 0.07\%$		$0.9652 \pm 0.06\%$	$0.9647 \pm 0.04\%$
18	$0.9947 \pm 0.07\%$	$0.9944 \pm 0.12\%$		$0.9941 \pm 0.07\%$	$0.9939 \pm 0.07\%$

$$P_{\text{repl}}^{\text{SPR}}(z) = \frac{D_{\text{HDA}}(z)}{D_{\text{air}}(z)} \frac{D_{\text{water}}(z)}{D_{\text{HDA}}(z)} \left/ \left(\frac{\bar{L}_{\Delta}(z)}{\rho} \right)_{\text{air}}^{\text{water}} \right. = P_{\text{repl}}^{\text{HDA}}(z), \quad (9)$$

that is, in principle, the SPR method gives the same P_{repl} values as the HDA method. The results in Table I support this derivation. The FLU method relies on the similarity of the electron fluence spectrum in the cavity to that in the phantom at the point of measurement, which is obviously not correct;⁸ thus this method should not be used for cylindrical chambers in electron beams even though it may give approximately the same value as other methods.

II.C. Calculation of the fluence correction factor

As the electron fluence spectrum in the cavity is similar to that at a depth shifted $0.8r$ upstream from the center of the cavity,⁸ strictly speaking, Eq. (3) should not be used. Rather, one may write the Spencer-Attix equation using an effective point of measurement approach, to eliminate the gradient effect P_{gr} , as

$$\frac{D_{\text{water}}(z-0.8r)}{D_{\text{air}}(z)} = \left(\frac{\bar{L}_{\Delta}(z-0.8r)}{\rho} \right)_{\text{air}}^{\text{water}} P_{f,0}(z), \quad (10)$$

where the water/air stopping-power ratio is now evaluated at a depth $z-0.8r$, since this is where the electron spectrum in the phantom resembles that in the cavity centered at z . $P_{f,0}$ (rather than P_{repl}) is the “true” fluence perturbation factor which only accounts for the minor difference in the electron fluence spectrum in the cavity at depth z compared to that at depth $z-0.8r$ in the phantom without the cavity. One may write Eq. (10) in a more general form for any shift s as

$$\frac{D_{\text{water}}(z-s)}{D_{\text{air}}(z)} = \left(\frac{\bar{L}_{\Delta}(z-s)}{\rho} \right)_{\text{air}}^{\text{water}} P_{\text{fl}}(z). \quad (11)$$

The symbol P_{fl} is used instead of $P_{f,0}$ to indicate that there is still at least partial gradient corrections involved in the value of P_{fl} if $s \neq 0.8r$. Equation (11) reduces to Eq. (3) when the effective point of measurement is not used, i.e., $s=0$ and then $P_{\text{fl}}=P_{\text{repl}}$. As was done before in calculating P_{repl} , all four methods can be applied to calculate the fluence correction factor P_{fl} ; the only difference is that the phantom dose is calculated at a depth $z-s$. Specifically, for LDW and HDA methods, the following equations are used to calculate the values of P_{fl} :

$$P_{\text{fl}}^{\text{LDW}}(z) = \frac{D_{\text{water}}(z-s)}{D_{\text{LDW}}(z)}, \quad (12)$$

$$P_{\text{fl}}^{\text{HDA}}(z) = \frac{D_{\text{HDA}}(z-s)}{D_{\text{air}}(z)}. \quad (13)$$

Table II lists the $P_{f,0}$ values (i.e., P_{fl} values with $s=0.8r$) calculated by the four methods for the same scenarios as described in Table I. All methods give excellent agreement for the values of $P_{f,0}$ at d_{ref} for the two electron beams.

In the AAPM dosimetry protocols,^{1,2} cylindrical chambers are positioned such that the point of interest, i.e. the reference depth z_{ref} , is at the center of the chamber cavity and the water/air stopping-power ratio is also evaluated at this point; thus the P_{repl} value calculated by the HDA method should be used according to the results in Sec. II B. In TG-51, a gradient correction factor P_{gr} [the same as defined by Eq. (14) below] is explicitly used in the formula, so actually only the values of P_{fl} at the depth z_{ref} are needed in the protocol. In IAEA’s TRS-398,³ the dose at the point of interest z_{ref} is measured by shifting the chamber center downstream by an amount $s=0.5r$, i.e., at a depth $z_{\text{ref}}+0.5r$; thus the gradient correction is accounted for by the chamber shift.

II.D. The gradient correction factors P_{gr}

In Eq. (2), P_{repl} was written as the product of a gradient correction and a fluence correction. In TG-51,² the gradient correction factor at d_{ref} is determined by taking the ratio of the chamber reading at depth $d_{\text{ref}}+0.5r$ to that at d_{ref} , i.e., $P_{\text{gr}}(d_{\text{ref}}) = M(d_{\text{ref}}+0.5r)/M(d_{\text{ref}})$, where $M(z)$ is the chamber reading by a cylindrical chamber with its center at depth z . This is based on the assumption that the chamber reading represents approximately the dose from a point in the phantom upstream by a distance $0.5r$, that is, $M(z) \propto D_{\text{water}}(z-0.5r)$, where $D_{\text{water}}(z-0.5r)$ is the actual phantom dose at depth $z-0.5r$. Then one may write approximately

$$P_{\text{gr}}(z) = \frac{M(z+0.5r)}{M(z)} \approx \frac{D_{\text{water}}(z)}{D_{\text{water}}(z-0.5r)}. \quad (14)$$

Now the question is, how accurately can the dose ratio in Eq. (14) be represented by the chamber-reading ratio? Using an NE2571 chamber model (3.14 mm radius) as an example, the ratios of the chamber reading (i.e., the cavity dose) at $d_{\text{ref}}+0.5r$ to that at d_{ref} are calculated for various electron beams from 6 to 22 MeV. If the cavity radius is very small

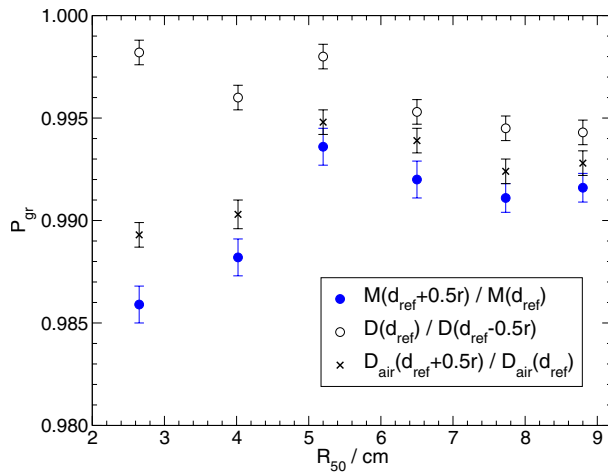


FIG. 2. The calculated gradient correction P_{gr} at d_{ref} as defined in the TG-51 dosimetry protocol (Ref. 2) for the NE2571 chamber (3.14 mm cavity radius) in various electron beams (filled circles). The ratio of phantom doses (open circles) is presumed to be represented by the ratio of the chamber readings according to Eq. (14). Cross symbols are the results for the air cavity only, i.e., the NE2571 chamber without wall and central electrode.

and the first derivative of dose vs depth is a continuous function, then, mathematically, $D_{water}(z+0.5r)/D_{water}(z)$ should be very close to $D_{water}(z)/D_{water}(z-0.5r)$ as defined in Eq. (14). Using Eq. (2), the value of P_{gr} may also be expressed as P_{repl}/P_{fl} , where P_{repl} and P_{fl} can be calculated separately. The values of P_{gr} for all of these definitions are calculated and compared to the ratios of the chamber reading for the NE2571 chamber at depths close to d_{ref} for both a 6 MeV beam and an 18 MeV beam.

III. RESULTS AND DISCUSSION

III.A. P_{gr} at the reference depth

The calculation results for the verification of Eq. (14) for various electron beams are shown in Fig. 2. The ratio of the readings for the fully modeled NE2571 chamber, $M(d_{ref}+0.5r)/M(d_{ref})$, is systematically lower than the corresponding dose ratio, $D(d_{ref})/D(d_{ref}-0.5r)$. For high-energy beams, the discrepancy is 0.3%; and for low-energy beams, the discrepancy is more than 1%. The calculation results for the NE2571 air cavity (no wall and central electrode), $D_{air}(d_{ref}+0.5r)/D_{air}(d_{ref})$, are close to the full chamber simulation results which suggests that the discrepancy is mainly from the replacement effect but partly from the wall or electrode effects. These results demonstrate that the gradient correction factor P_{gr} as defined in Eq. (14) (also in TG-51) for the NE2571 chamber is wrong by 0.3% for high-energy beams and more than 1% for low-energy beams.

Figure 3 compares the P_{gr} values as a function of depth near d_{ref} for different definitions of P_{gr} in (a) a 6 MeV electron beam and (b) an 18 MeV electron beam. The figure shows that the P_{gr} values vary among different calculation methods by more than 1% for the 6 MeV beam and by 0.4% for the 18 MeV beam. It is expected that the P_{gr} value should be 1 at d_{max} and Fig. 3(a) shows that the P_{gr} values pass through 1 at a depth of d_{max} based on the calculated P_{repl}/P_{fl}

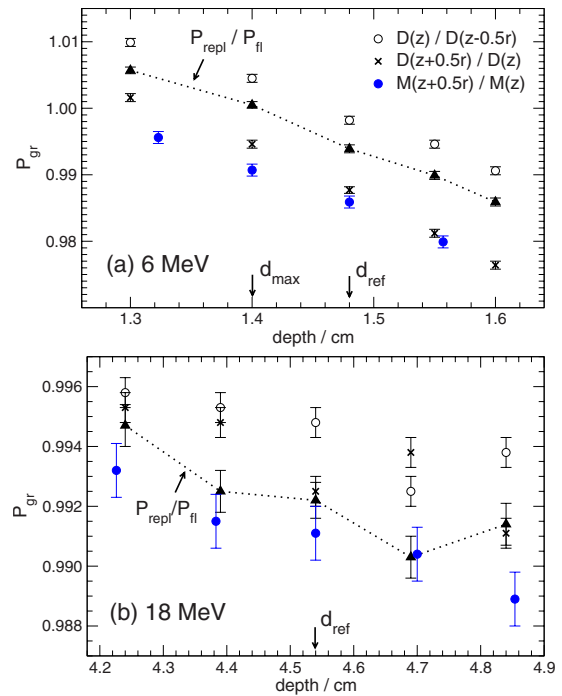


FIG. 3. Comparison of the values of P_{gr} calculated by different definitions for the NE2571 chamber at depths near d_{ref} for (a) a 6 MeV electron beam and (b) an 18 MeV electron beam.

value. However, the uncertainty of P_{gr} values from different definitions is 1%. For the 18 MeV beam, all P_{gr} values are 1.00 within 0.2% at d_{max} (not shown in the figure at a depth of 2.4 cm). The large variation in low-energy beams is due to the steep dose gradient of the depth-dose curve and finite size of the Farmer-type chamber. A single shift of $0.5r$, adopted in current dosimetry protocols for all electron beams, also contributes to the uncertainty in the P_{gr} values since it is demonstrated that for electron beams of different energies the amount of shift should also be different.^{9,8}

III.B. P_{repl} and P_{fl} vs depth

Figure 4 shows P_{repl} values calculated by the two direct methods, HDA [Eq. (4)] and LDW [Eq. (5)], for the cavity of a Farmer chamber as a function of depth in both a 6 MeV and an 18 MeV electron beam. The LDW method gives a higher value at all depths as expected from Eq. (8). The very large P_{repl} values at deeper depths are due to the steep dose gradient near R_{50} for electron beams. Since in both the AAPM and IAEA dosimetry protocols the gradient effects have been separated out, only the values of P_{fl} are needed and they are calculated by Eqs. (12) and (13). The depth dependence of P_{fl} values is illustrated in Fig. 5 for two different shifts of the point of measurement, $s=0.8r$ and $s=0.5r$, in electron beams of energies of (a) 6 MeV and (b) 18 MeV. In this figure, the chamber cavity is located at the specified depth but the phantom dose is calculated at a depth shifted upstream by s . For an $s=0.8r$ shift, P_{fl} is the true fluence correction factor, $P_{fl,0}$, since the electron fluence spectra are similar as discussed in a separate work.⁸ The variation in P_{fl} vs depth (from d_{ref} to R_{50}) can be as large as

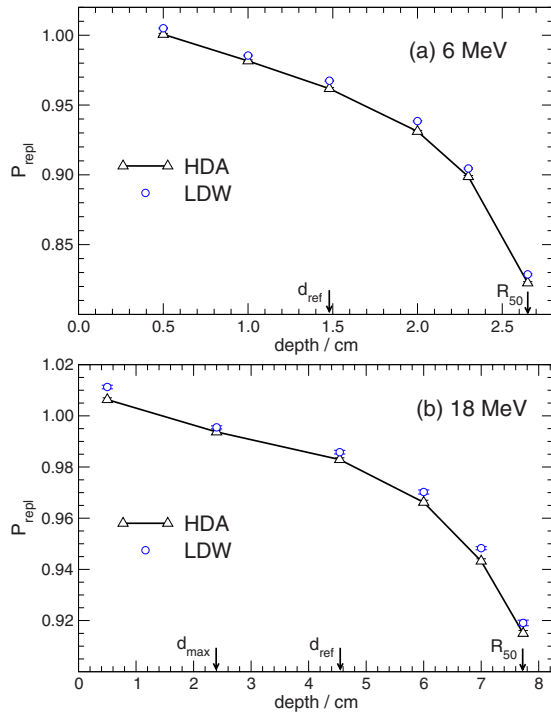


FIG. 4. P_{repl} values calculated by the two direct methods [Eqs. (5) and (4)] for the cavity of a Farmer chamber (radius of 3 mm) as a function of depth in (a) a 6 MeV beam and (b) an 18 MeV electron beam. The depth is specified by the location of the center of the chamber cavity. For the HDA method, the HDA slab thickness is 3 μm . The differences between the two methods are predicted by Eq. (8).

15% for the 6 MeV beam and 5% for the 18 MeV beam, even though it is relatively constant from the surface to d_{ref} . The very large values of P_{fl} at depths close to R_{50} mean that the scattered electrons are becoming dominant over the primary electrons, and the actual shift needed should be shorter than $0.8r$ or $0.5r$. For an $s=0.5r$ shift, which is currently used by dosimetry protocols, the variation in P_{fl} values vs depth is greatly reduced. This is reasonable since the shift value of $0.5r$ was obtained by matching the depth-dose curves,⁸ which effectively makes the averaged variation in the P_{fl} value a minimum. For $s=0.8r$, P_{fl} remains almost constant from the surface to d_{ref} for the 18 MeV beam. For $s=0.5r$, P_{fl} is not a true fluence correction factor, it partly contains a contribution from the gradient effect, which accounts for the decrease in P_{fl} by 1% from the surface to a depth beyond d_{ref} for the 18 MeV beam. Since a good match of the electron fluence spectrum is reached at $s=0.8r$, a $0.5r$ shift will result in a small mismatch of the electron fluence spectrum as discussed in Sec. III A. This will inevitably lead to a discrepancy in values of P_{fl} calculated by different methods. It can be seen from Fig. 5 that the difference in the calculated P_{fl} values from the HDA vs LDW methods is more observable when the shift is $s=0.5r$.

III.C. Beam quality dependence of P_{repl}

Figure 6 shows the beam quality dependence of the calculated P_{repl} values for the cavity of a Farmer chamber at both d_{max} and d_{ref} in a water phantom for either real linac

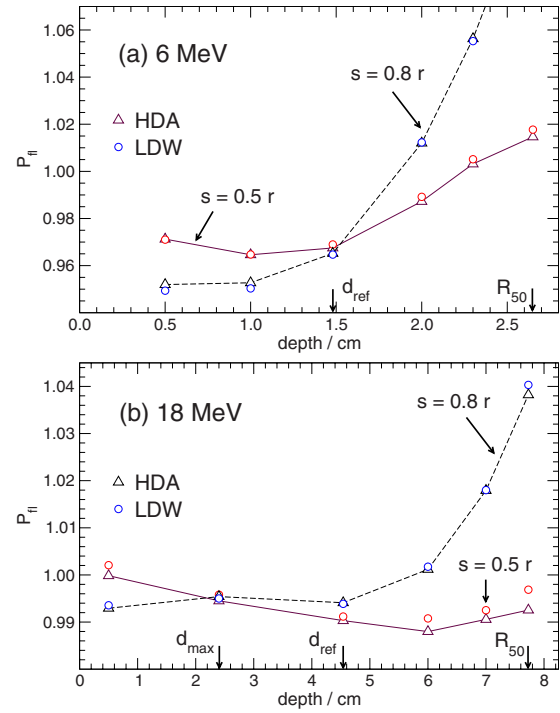


FIG. 5. The values of fluence correction (P_{fl}) calculated by the two direct methods [Eqs. (12) and (13)] for the cavity of a Farmer chamber (radius of 3 mm) as a function of depth in (a) 6 MeV and (b) 18 MeV electron beams. The depth is specified by the location of the center of the chamber cavity. For the phantom dose calculation, the depth is shifted upstream by either $s=0.5r$ or $s=0.8r$ (see Sec. II C for details on P_{fl} calculations). For the HDA method, the HDA slab thickness is 3 μm . The calculation statistical uncertainty is less than 0.1% (1σ) at all depths for both beams.

electron beams¹² or monoenergetic electron beams (with energies of 6, 12, 18, and 24 MeV). The HDA method is used in the calculations. In Fig. 6(a), the abscissa is the mean electron energy at depth z , $\bar{E}_z$, determined from Harder's relation:¹

$$\bar{E}_z = \bar{E}_0 \left(1 - \frac{z}{R_p} \right), \quad (15)$$

where $\bar{E}_0 = 2.33R_{50}$ is the mean electron energy at the surface which was used as the electron beam quality specifier in the TG-21 dosimetry protocol,¹ and R_p is the practical range. The large discrepancies in Fig. 6(b) between values for realistic vs monoenergetic electron beams suggest that R_{50} is not a good beam quality specifier for the values of P_{repl} . Although the mean electron energy at depth makes agreement look better as shown in Fig. 6(a) for different beams, the discrepancy is still large especially for low-energy beams. This is most likely because the gradient effect at the reference depth varies for different beams having the same R_{50} . This is why the gradient correction is generally explicitly accounted for in dosimetry protocols.

III.D. Beam quality dependence of P_{fl}

Figure 7 shows the calculated P_{fl} values for the Farmer chamber at both d_{max} and d_{ref} in a water phantom irradiated

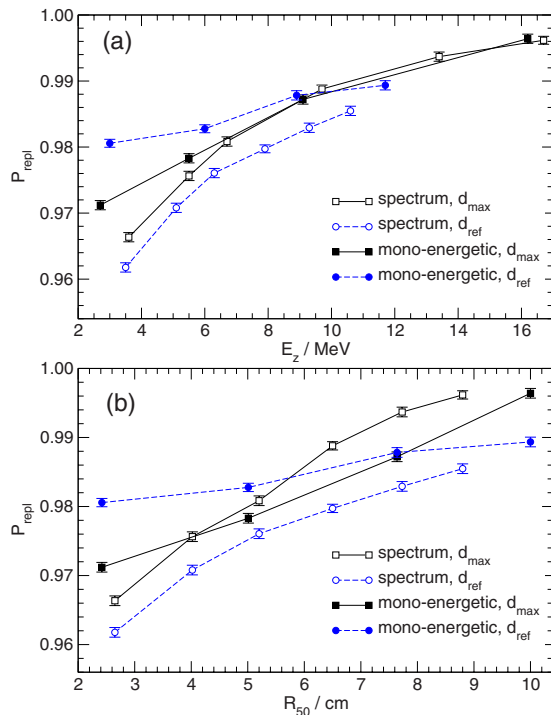


FIG. 6. Comparison of P_{repl} values for a Farmer chamber (radius of 3 mm) at either d_{max} or d_{ref} in a water phantom for both realistic electron beam spectra (open symbols, with nominal energy ranging from 6 to 22 MeV) and monoenergetic incident electron beams (corresponding solid symbols, with energies of 6, 12, 18, and 24 MeV) as a function of (a) the mean electron energy at depth and (b) R_{50} . The HDA method is used in the calculation [Eq. (4)], and the HDA slab thickness is 3 μm .

by a realistic linac spectrum source¹² (nominal energy from 6 to 22 MeV) and/or monoenergetic electron beams (6, 12, 18, and 24 MeV). In Fig. 7(a), the results are plotted as a function of the mean electron energy at depth and are compared to AAPM's TG-21/51 values¹ which were based on the experimental work of Johansson *et al.*⁹ The same sets of data are plotted in Fig. 7(b) where R_{50} is used as the beam quality specifier and the results are also compared to the P_{fl} (i.e., p_{cav}) values used in IAEA's TRS-398 (Ref. 3) and AAPM's TG-21/51 values, which were recast from the experimental measurements of P_{fl} at d_{max} . The HDA method is used in the calculation [Eq. (13) in Sec. II C] since in the experiments the water/air stopping-power ratio was evaluated at the center of the cavity which is the point of measurement. The values of P_{fl} are calculated by Eq. (13) where a shift of $0.5r$ is used to correspond to the value adopted in dosimetry protocols. In Fig. 7(a), the calculated values of P_{fl} for the spectrum source at d_{ref} lie on the same curve as those at d_{max} , as well as the P_{fl} values at d_{ref} for the monoenergetic beams. This means that the mean electron energy at depth, $\bar{E}_z$, is a good beam quality specifier for P_{fl} . These results demonstrate that, consistent with experimental findings,¹⁵ TG-51's adoption of P_{fl} values at d_{max} with the same mean electron energy for use at d_{ref} is reasonable, although there is a clear discrepancy compared to the values actually used. In Fig. 7(b), R_{50} is a good beam quality specifier if we are only interested in the P_{fl} values at d_{ref} . Figure 7 also shows that

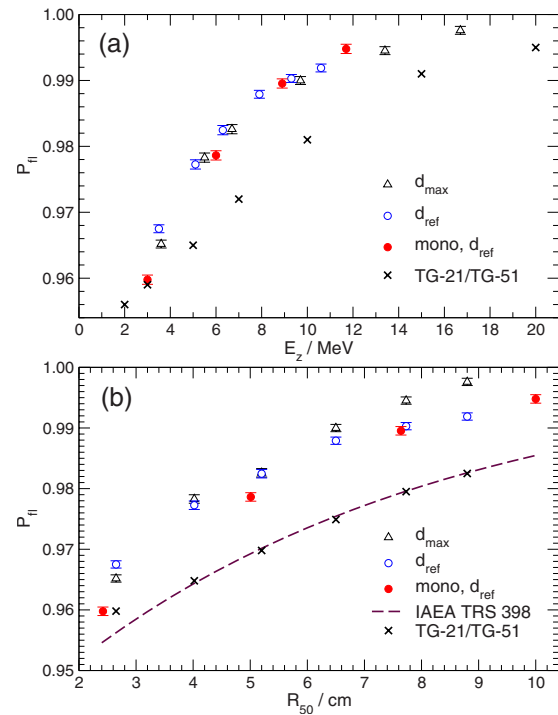


FIG. 7. (a) The calculated P_{fl} values for a Farmer chamber's cavity in electron beams as a function of the mean electron energy at depth. P_{fl} values at d_{max} and/or at d_{ref} for both realistic electron beam spectra (open symbols) and monoenergetic beams (solid symbols) are shown. The HDA method is used in the calculation [Eq. (13)] with $s=0.5r$. The HDA slab thickness is 3 μm . The cross symbols are the TG-21 values (also used in TG-51) based on the measurement of Johansson *et al.* (Ref. 9) at d_{max} . (b) Same data sets as in (a) but with R_{50} as the beam quality specifier. The dashed line is the P_{fl} (p_{cav}) values used in IAEA's TRS-398 (Ref. 3).

the calculated P_{fl} values are systematically higher by 0.5%–1% than TG-21/51 and/or IAEA's TRS-398 values. The values in the dosimetry protocols are based on measurements in a PMMA phantom, while P_{fl} values in this study are calculated in a water phantom. To see if this difference accounts at least partly for the discrepancy, we calculated P_{repl} values by the SPR method at d_{max} in a PMMA phantom in a few electron beams and found that the P_{repl} values for PMMA are only 0.1% lower than those for water. This demonstrates that TG-21's recommendation of P_{repl} values at d_{max} for PMMA being used for water is reasonable.

The 0.5%–1% systematic difference can be explained as follows. The experimental values were based on the assumption that the plane-parallel chamber used in the comparison with the cylindrical chamber was perturbation-free; specifically, the wall correction factor P_{wall} is 1. However, recent studies^{16–19} have shown that P_{wall} values for most plane-parallel chambers in a water phantom in electron beams are in the range of 1.005–1.015 for electron energies of 20 to 6 MeV. Although there is no detailed information about the plane-parallel chamber used in the experiments of Johansson *et al.*, it is highly probable that it had a similar wall effect. For example, we calculated the wall corrections for the NACP02 chamber in a PMMA phantom irradiated by a 6 or a 12 MeV electron beam. The values of P_{wall} are found to be 1.005 and 1.004 with 0.1% statistics. On the other hand, the

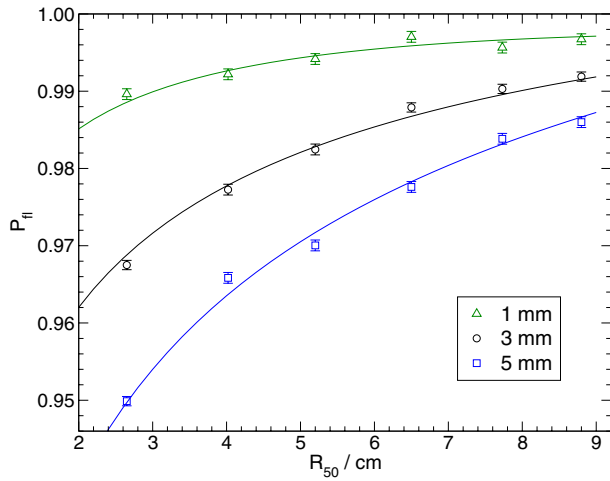


FIG. 8. The beam quality dependence of the calculated P_{ri} values at d_{ref} for cylindrical chambers of length of 2 cm for three cavity radii (1, 3, and 5 mm) in realistic electron beams (Ref. 12) ranging from 6 to 22 MeV. R_{50} is used as the beam quality specifier. The chamber cavity center is defined as the point of measurement or the reference point. The HDA method [Eq. (13)] is used in the calculations with an HDA slab thickness of 3 μm and a shift of $0.5r$. The solid lines are the values calculated for the three cavity radii from Eq. (16). The calculation statistical uncertainty is less than 0.07% (1σ).

values of P_{repl} for well-guarded plane-parallel chambers have shown to be very close to unity (within 0.4%) at d_{max} in electron beams,⁴ and no wall correction was needed for the cylindrical chambers as the wall material (PMMA) was the same as the phantom. Therefore, the actual measured values by Johansson *et al.* (i.e., TG-21/51 or TRS-398 values) should be roughly 0.5%–1.5% larger than those shown in Fig. 7, in which case the agreement with the calculated values would be better.

III.E. Cavity size dependences and an empirical formula for P_{ri}

Figure 8 shows the beam quality dependence of the P_{ri} values calculated at d_{ref} for realistic electron beam spectra¹² ranging from 6 to 22 MeV, for cylindrical chambers of three different cavity radii, when R_{50} is used as the beam quality specifier. The HDA method was used in the calculations. The solid lines are the values calculated by the following empirical formula (from TABLE CURVE 3D v4.0) expressing P_{ri} values for a cavity of length of 2 cm as a function of R_{50} (in cm) and the radius of the chamber cavity r (in mm):

$$P_{ri}(d_{ref}) = 0.9902 - 0.016r + 0.01218 \ln R_{50} + 0.000083r^2 - 0.0035(\ln R_{50})^2 + 0.00593r(\ln R_{50}). \quad (16)$$

With this formula, the maximum deviation from the Monte Carlo calculated values is less than 0.2% for all the data points. Equation (16) is valid for $r=1$ mm to $r=5$ mm and for R_{50} between 2.5 and 10 cm. It is applicable to situations where the center of the chamber's cavity is defined as the point of measurement and is placed at d_{ref} , i.e., the P_{ri} values calculated by the formula are only applicable to the AAPM

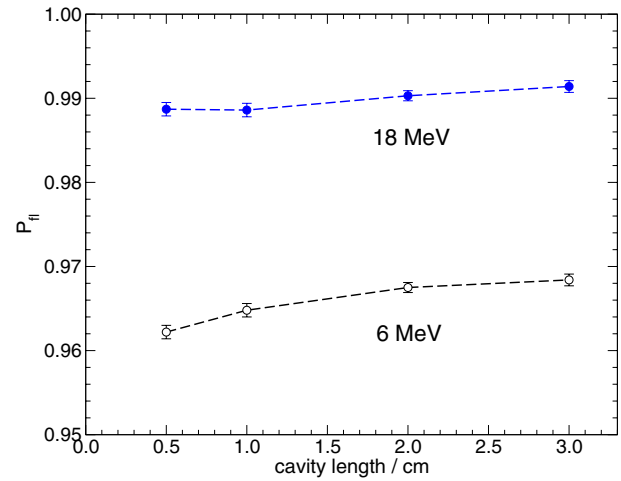


FIG. 9. Calculated P_{ri} values vs the length of the cavity for cylindrical chambers of radius of 3 mm at d_{ref} in a 6 MeV (open symbols) and in an 18 MeV (filled symbols) electron beam. The center of the chamber cavity is the point of measurement. The HDA method [Eq. (13)] is used in the calculations with an HDA slab thickness of 3 μm and a shift of $0.5r$.

TG-51 dosimetry protocol but not IAEA's TRS-398 in which case the P_{ri} values at depth $d_{ref}+0.5r$ are needed. However, the P_{ri} values from Eq. (16) may still be used in IAEA's Code of Practice if we ignore the difference between the P_{ri} values at the two depths. According to Fig. 5, the difference is negligible near d_{ref} for high-energy electron beams.

Figure 9 shows the calculated $P_{ri}(d_{ref})$ values in electron beams for different cavity lengths for cylindrical chambers of radius 3 mm. For the low-energy beam (6 MeV), the variation in P_{ri} values for cavity lengths from 0.5 to 3 cm can be as large as 0.6%. For the high-energy electron beam (18 MeV), the variation is at most 0.3%. In contrast, for the same cylindrical chambers in photon beams, the variation is only 0.2%.⁴

IV. CONCLUSIONS

Monte Carlo simulations using EGSnrc user-codes have been employed to study the perturbation effect caused by the introduction of a cylindrical air cavity into a water phantom irradiated by electron beams. The four previously established techniques^{4,5} of calculating the replacement correction factors by Monte Carlo methods have been applied to the cylindrical cavities of different diameters and lengths in various electron beams. It is found that the value of calculated replacement correction factor depends on the calculation method when the cavity center is the point of measurement. This is related to the choice of the depth used to evaluate the water/air stopping-power ratio. However, if only the fluence perturbation is considered, i.e., the gradient effect is separated out, the different methods for calculating P_{ri} give the same value within 0.1% statistics. The calculated values of the fluence correction factors are in good agreement with the revised measured values when the wall correction factors for plane-parallel chambers are considered. The results also show that the mean electron energy at depth is a good beam quality specifier for the fluence correction factors, and it is

demonstrated that TG-51's adoption of the fluence correction factors for cylindrical chambers as a function of mean electron energy at depths is reasonably accurate. The dependencies of the calculated fluence correction factors on the chamber radius and on the electron beam quality are investigated. An empirical formula [Eq. (16)] is given for the fluence correction factors for cylindrical chambers when the cavity center is at the reference point in the phantom and the water/air stopping-power ratio is evaluated at the same point. The variation in the calculated fluence correction factors with the cavity length is studied for cavities of radius of 3 mm; the results show that there is a 0.6% change from 0.5 to 3.0 cm cavity length in a 6 MeV electron beam. Finally, as shown in Fig. 2, the gradient correction factors P_{gr} for a Farmer-type chamber as defined in the TG-51 dosimetry protocol are 0.3% low for high-energy electron beams and are more than 1% low for low-energy electron beams.

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¹AAPM TG-21, "A protocol for the determination of absorbed dose from high-energy photon and electron beams," *Med. Phys.* **10**, 741–771 (1983).

²P. R. Almond, P. J. Biggs, B. M. Coursey, W. F. Hanson, M. S. Huq, R. Nath, and D. W. O. Rogers, "AAPM's TG-51 protocol for clinical reference dosimetry of high-energy photon and electron beams," *Med. Phys.* **26**, 1847–1870 (1999).

³IAEA, *Absorbed Dose Determination in External Beam Radiotherapy: An International Code of Practice for Dosimetry Based on Standards of Absorbed Dose to Water*, Technical Report Series Vol. 398 (IAEA, Vienna, 2001).

⁴L. L. W. Wang and D. W. O. Rogers, "Calculation of the replacement correction factors for ion chambers in megavoltage beams by Monte Carlo simulation," *Med. Phys.* **35**, 1747–1755 (2008).

⁵L. L. W. Wang, D. J. La Russa, and D. W. O. Rogers, "Systematic uncertainties in the Monte Carlo calculation of ion chamber replacement correction factors," *Med. Phys.* **36**, 1785–1789 (2009).

⁶L. L. W. Wang and D. W. O. Rogers, "The replacement correction factors for cylindrical chambers in high-energy photon beams," *Phys. Med. Biol.* **54**, 1609–1620 (2009).

⁷L. L. W. Wang and D. W. O. Rogers, "The replacement correction factor for the BIPM flat cavity ion chamber and the value of W/e ," *Med. Phys.* **35**, 4410–4416 (2008).

⁸L. L. W. Wang and D. W. O. Rogers, "Study of the effective point of measurement for ion chambers in electron beams by Monte Carlo simulation," *Med. Phys.* **36**, 2034–2042 (2009).

⁹K. A. Johansson, L. O. Mattsson, L. Lindborg, and H. Svensson, "Absorbed-dose determination with ionization chambers in electron and photon beams having energies between 1 and 50 MeV," *IAEA Symposium Proceedings* (IAEA, Vienna, 1977), Paper No. IAEA-SM-222/35, pp. 243–270.

¹⁰I. Kawrakow and D. W. O. Rogers, "The EGSnrc code system: Monte Carlo simulation of electron and photon transport," National Research Council of Canada Technical Report No. PIRS-701, 2000 (see <http://www.irs.inms.nrc.ca/inms/irs/EGSnrc/EGSnrc.html>).

¹¹I. Kawrakow, "Accurate condensed history Monte Carlo simulation of electron transport. I. EGSnrc, the new EGS4 version," *Med. Phys.* **27**, 485–498 (2000).

¹²G. X. Ding and D. W. O. Rogers, "Energy spectra, angular spread, and dose distributions of electron beams from various accelerators used in radiotherapy," NRC Report No. PIRS-0439, 1995.

¹³I. Kawrakow, "EGSnrc C++ class library," National Research Council of Canada Technical Report No. PIRS-898, 2005 (unpublished) (<http://www.irs.inms.nrc.ca/EGSnrc/pirs898/>).

¹⁴I. Kawrakow, "egspp: The EGSnrc C++ class library," National Research Council of Canada Technical Report No. PIRS-899, 2005.

¹⁵M. S. Huq, N. Yue, and N. Suntharalingam, "Experimental determination of fluence correction factors at depths beyond d_{max} for a Farmer type cylindrical ionization chamber in clinical electron beams," *Med. Phys.* **24**, 1609–1613 (1997).

¹⁶E. Chin, H. Palmans, D. Shipley, M. Bailey, and F. Verhaegen, "Analysis of dose perturbation factors of a NACP-02 ionization chamber in clinical electron beams," *Phys. Med. Biol.* **54**, 307–326 (2009).

¹⁷K. Zink and J. Wulff, "Monte Carlo calculations of beam quality correction factors k_Q for electron dosimetry with a parallel-plate Roos chamber," *Phys. Med. Biol.* **53**, 1595–1607 (2008).

¹⁸F. Verhaegen, R. Zakikhani, A. DuSautoy, H. Palmans, G. Bostock, D. Shipley, and J. Seuntjens, "Perturbation correction factors for the NACP-02 plane-parallel ionization chamber in water in high-energy electron beams," *Phys. Med. Biol.* **51**, 1221–1235 (2006).

¹⁹L. A. Buckley and D. W. O. Rogers, "Wall correction factors, P_{wall} , for parallel-plate ionization chambers," *Med. Phys.* **33**, 1788–1796 (2006).