Re-evaluation of the product of \((W/e)_{\text{air}}\) and the graphite to air stopping-power ratio for \(^{60}\text{Co}\) air kerma standards

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Abstract

Experiments which determine the product of \((W/e)_{\text{air}}\), the average energy deposited per coulomb of charge of one sign released by an electron coming to rest in dry air, and \((\bar{L}/\rho)_{\text{Ca}}\), the Spencer–Attix mean restricted mass collision stopping-power ratio for graphite to air, in a \(^{60}\text{Co}\) or \(^{137}\text{Cs}\) beam are reanalysed. Correction factors, e.g., to account for gaps about a calorimeter core or perturbations due to a cavity’s presence, are calculated using the EGSnrc Monte Carlo code system and these generally decrease the value of \((W/e)_{\text{air}}(\bar{L}/\rho)_{\text{Ca}}\) for each experiment. Stopping-power ratios are calculated for different choices of density correction and average excitation energy \((I\text{-value})\) for graphite. To calculate an average value \((W/e)_{\text{air}}(\bar{L}_{\text{BIPM}}/\rho)_{\text{Ca}}\) for the BIPM air kerma standard, each experimental result is multiplied by the ratio \((\bar{L}_{\text{BIPM}}/\rho)_{\text{Ca}}/(\bar{L}/\rho)_{\text{Ca}}\). While individual values of \((\bar{L}/\rho)_{\text{Ca}}\) are sensitive to the \(I\)-values and density corrections assumed, this ratio varies by less than 0.1% for different choices. Hence, the product \((W/e)_{\text{air}}(\bar{L}_{\text{BIPM}}/\rho)_{\text{Ca}}\) is relatively insensitive to these choices. The weighted mean of the updated data is \((W/e)_{\text{air}}(\bar{L}_{\text{BIPM}}/\rho)_{\text{Ca}} = 33.68 \text{ J C}^{-1} \pm 0.2\%\), suggesting that the accepted value of 33.97 J C\(^{-1}\) ± 0.1% is 0.8% too high. This has implications for primary \(^{60}\text{Co}\) air kerma standards worldwide and potentially for the choice of graphite \(I\)-value and density correction for the calculation of the graphite stopping power, as well as the value of \((W/e)_{\text{air}}\).

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The Spencer–Attix mean restricted mass collision stopping-power ratio for graphite to air in a \(^{60}\text{Co}\) beam, \((\bar{L}/\rho)_{\text{Ca}}\), and the average energy deposited by an electron slowing down to rest in dry air per coulomb of charge of one sign released, \((W/e)_{\text{air}}\), play central roles in radiation dosimetry individually and as a product. The value of \((W/e)_{\text{air}}\) is needed worldwide...
for low-energy x-ray standards for air kerma; the product \((W/e)_{\text{air}}(L_{\Delta}/\rho)_{C}\) enters directly into \(^{60}\text{Co}\) primary standards for air kerma.

The quantity \((W/e)_{\text{air}}\) and its differential counterpart, \(w(E)/e\), are both defined in ICRU Report 31 (1979). \((W/e)_{\text{air}}\) relates the energy deposited and charge released as electrons come to rest in dry air and pertains to situations where there is the full spectrum of slowing electrons present, e.g., electrons set in motion by \(^{60}\text{Co}\) radiation. The quantity \(w(E)/e\) is the differential energy deposited by an electron of energy \(E\) per coulomb of charge released and is important in electron beam dosimetry where the spectrum of electrons in the ion chamber cavity is not the complete spectrum of slowing down electrons. While \((W/e)_{\text{air}}\) and \(w(E)/e\) are conceptually different, they are often taken to be equal due to the assumption that \(w(E)/e\) is constant with energy (see Burns and Buermann (2009) and references therein). There is, however, evidence that this assumption may not be strictly true as small variations \(\sim 0.5\%\) are seen with energy (Svensson and Brahme 1986) or up to 0.8\% for 5 and 6 keV electrons (Büermann et al 2006).

The accepted value of \((W/e)_{\text{air}}, 33.97 \pm 0.05\) J C\(^{-1}\), is an average of the results of 15 experiments (Boutillon and Perroche-Roux 1987). Many of these experiments determine the product \((W/e)_{\text{air}}(L_{\Delta}/\rho)_{C}\) and the value of \((W/e)_{\text{air}}\) is derived assuming a value of \((L_{\Delta}/\rho)_{C}\). Two independent experiments measuring \((W/e)_{\text{air}}(L_{\Delta}/\rho)_{C}\) were performed by Niatel et al (1985) and both results carry significant weight in the determination of the accepted average value of \((W/e)_{\text{air}}\). In the first method, ionometric readings from the BIPM flat cavity ion chamber in a graphite phantom were compared to calorimetric standards from various national laboratories for absorbed dose to graphite irradiated by a \(^{60}\text{Co}\) beam. The product of \((W/e)_{\text{air}}\) and \((L_{\Delta}/\rho)_{C}\) was obtained via

\[
(W/e)_{\text{air}}(L_{\Delta}/\rho)_{C} = \frac{\hat{D}_{\text{cal}}}{JP_{\text{repl}}},
\]

where \(J\) is the quotient of the ionization current collected in the BIPM graphite ion chamber and the mass of air in the cavity, \(P_{\text{repl}}\) is the correction factor which accounts for the perturbation due to the cavity’s presence (Boutillon 1983) and \(\hat{D}_{\text{cal}}\) is the dose rate measured by calorimetry (Niatel et al 1985). Using ICRU Report 37 (ICRU 1984) stopping powers (graphite density correction for 1.70 g cm\(^{-3}\) and \(I\)-value of 78 eV), the value \((W/e)_{\text{air}} = 33.96 \pm 0.08\) J C\(^{-1}\) was calculated by Niatel et al (1985). Accounting for the radial nonuniformity of the \(^{60}\text{Co}\) beam (Boutillon and Perroche-Roux 1989), calculating calorimeter gap corrections (Boutillon 1989) and including measurements from three more absorbed dose calorimeters, the value was revised to \((W/e)_{\text{air}} = 33.99 \pm 0.08\) J C\(^{-1}\) (Boutillon 1990).

The second determination of the product \((W/e)_{\text{air}}(L_{\Delta}/\rho)_{C}\) by Niatel et al was based on measurements of activity and exposure for a \(^{60}\text{Co}\) source. The exposure rate at a distance \(r\) from the source was derived from the measured activity \(A\) and compared to that measured using the BIPM exposure (air kerma) standard, a graphite cavity ionization chamber. The product \((W/e)_{\text{air}}(L_{\Delta}/\rho)_{C}\) was obtained via

\[
(W/e)_{\text{air}}(L_{\Delta}/\rho)_{C} = AE_{\gamma}(\mu_{\text{en}}/\rho)_{C} \frac{1}{4\pi r^{2}J} K_{x}K_{s},
\]

where \(J\) is the quotient of the ionization current and mass of air in the cavity, \((\mu_{\text{en}}/\rho)_{C}\) is the mass energy absorption coefficient for graphite averaged over the incident photon energy fluence spectrum, and \(E_{\gamma}\) is the photon energy emitted at each disintegration. The factor \(K_{x}\) accounts for the attenuation and scatter of radiation in the calculation of the exposure rate from measured activity; \(K_{s}\) is a product of the correction factors for the BIPM cavity ion chamber. Using ICRU Report 37 stopping powers, the value 33.81 \pm 0.14 J C\(^{-1}\) for \((W/e)_{\text{air}}\) was calculated.
Recent work suggests that both results of Niatel et al should be lowered by up to 1%. Wang and Rogers (2008b) recalculated replacement correction factors for the BIPM ion chamber used by Niatel et al. Their results suggest that replacement correction factors used for the BIPM chamber are 1% too small and that the value of \((W/e)_{\text{air}}\) determined using the first method of Niatel et al should be \(33.61 \pm 0.08 \text{ J C}^{-1}\). Burns et al (2007) re-evaluated the BIPM standard for air kerma based on new Monte Carlo calculations of correction factors (Burns 2006), a re-evaluation of the correction factor for saturation and a new evaluation of the chamber’s air volume. This re-evaluation increased the BIPM determination of the air-kerma rate by a factor of 1.0054 (Burns et al 2007), reducing the second result of Niatel et al to \(33.63 \pm 0.14 \text{ J C}^{-1}\).

Boutillon and Perroche-Roux (1987) used the results of a number of other experiments measuring \((W/e)_{\text{air}}(L_\Delta/\rho)^C\) to calculate an average value of \((W/e)_{\text{air}}\). These include experiments by Reid and Johns (1961), Bewley (1963), Petree and Lamperti (1967), Engelke and Hohlfeld (1971), Guiho and Simoen (1975) and Kunze and Hecker (1980). Though details differed, each experiment involved the comparison of ionometric and calorimetric measurements. Generally, dose was measured in a graphite calorimeter (often within a graphite phantom) irradiated by a \(^{60}\)Co or \(^{137}\)Cs source. The calorimeter was replaced by a graphite ion chamber and the ionization current was measured. With the exception of the experiment of Guiho and Simoen (1975), the value of \((W/e)_{\text{air}}(L_\Delta/\rho)^C\) was calculated via equation (1). For the experiment of Guiho and Simoen (1975), ionization currents in 12 different graphite-walled ion chambers in air were measured and used to determine the exposure. The exposure was then related to the dose to graphite measured via calorimetry to calculate \((W/e)_{\text{air}}(L_\Delta/\rho)^C\).

The determination of \((W/e)_{\text{air}}(L_\Delta/\rho)^C\) by investigators required the judicious application of correction factors. These varied with experiment; however, gap correction factors were not applied and, with few exceptions, neither were replacement correction factors. Given the recent work suggesting changes to the two results of Niatel et al (1985), it is possible that the results of all experiments determining \((W/e)_{\text{air}}(L_\Delta/\rho)^C\) require revision. This is important because primary standards of air kerma in a \(^{60}\)Co beam make direct use of this product, with the low uncertainty on the standards achieved by using this measured product.

Knowledge of \((L_\Delta/\rho)^C\) is required to determine \((W/e)_{\text{air}}\) from experiments measuring the product \((W/e)_{\text{air}}(L_\Delta/\rho)^C\). The value of this stopping-power ratio is sensitive to the choice of graphite mean excitation energy (I-value) and density used to calculate the density effect correction. ICRU Report 37 (ICRU 1984) makes use of I = 78 ± 3.5 eV (1σ) and provides tables of stopping powers for graphite of density 1.70 g cm\(^{-3}\), the typical bulk density of graphite used for dosimetry, and 2.265 g cm\(^{-3}\), the crystallite or grain density of graphite. The value I = 78 eV is based on four measurements ranging from 70.8 to 91.7 eV. In 1992, Bichsel and Hiraoka reported I = 86.9 ± 1.2 eV. The weighted average of the four measurements used by the ICRU to calculate I = 78 eV and the more recent result of Bichsel and Hiraoka is I = 84.6 eV. Burns (2009) recently re-evaluated the I-value for graphite based on an analysis of recent work on \((W/e)_{\text{air}}\), \((L_\Delta/\rho)^C\), and cavity perturbation corrections. He argued that the use of I = 82.5 eV along with the grain density of graphite for the density correction can reasonably explain experimental observations. However, part of his analysis required the assumption that \((W/e)_{\text{air}}\) is the same in a \(^{60}\)Co beam as for photons between 4 and 7 keV. Given that the measured values for \((W/e)_{\text{air}}\) of Büermann et al (2006), which are consistent with those of Kato et al (2010), show a trend with energy over this range, this assumed constancy is a conjecture. Variations of up to 1.6% in the value of \((L_\Delta/\rho)^C\) result from different choices of the I-value and density (Rogers and Kawrakow 2003); values of \((W/e)_{\text{air}}\) extracted from experiments measuring the product \((W/e)_{\text{air}}(L_\Delta/\rho)^C\) suffer this uncertainty.
In this paper, we reanalyse the experiments of Bewley (1963), Petree and Lamperti (1967), Engelke and Hohlfeld (1971), Guiho and Simoen (1975) and Niatel et al (1985) determining the product \((W/e)_{\text{air}} (L_{\Delta}/\rho)_{\text{air}}^C\). Correction factors to account for gaps about calorimeter cores and cavity perturbations are calculated, as needed, using Monte Carlo methods. Stopping-power ratios are calculated for each experiment and for different choices of graphite average excitation energy \((I\text{-value})\) and density correction. An average value for the product \((W/e)_{\text{air}} (L_{\Delta}/\rho)_{\text{air}}^C\) for a \(^{60}\text{Co}\) beam in dry air for the BIPM air kerma standard \((\Delta = 14 \text{ keV})\), \((W/e)_{\text{air}} (L_{\text{BIPM}}/\rho)_{\text{air}}^C\), is calculated. This value could be adopted for use with \(^{60}\text{Co}\) air kerma primary standards along with corrections to account for variations due to different cavity sizes. Based on the average value of \((W/e)_{\text{air}} (L_{\text{BIPM}}/\rho)_{\text{air}}^C\) calculated, the implications of different choices of graphite density for the density correction and \(I\text{-value}\) on \((W/e)_{\text{air}}\) are explored.

The experiments of Reid and Johns (1961) and Kunze and Hecker (1980) are omitted from this reanalysis. The results of Reid and Johns (1961) have large uncertainties. There is insufficient information to calculate correction factors for the experiment of Kunze and Hecker (1980). Given these large uncertainties, it is unlikely that including these results in the present analysis would significantly affect the average value of \((W/e)_{\text{air}} (L_{\text{BIPM}}/\rho)_{\text{air}}^C\) calculated.

2. Methods

2.1. Correction factors

Correction factors are calculated for the experiments of Bewley (1963), Petree and Lamperti (1967), Engelke and Hohlfeld (1971) and Guiho and Simoen (1975). Each experiment involved the comparison of calorimetry and ionometry measurements. In this section, correction factor and simulation generalities are discussed before focussing on each experiment.

The replacement correction factor \(P_{\text{repl}}\) (the product \(p_{\text{cav}} p_{\text{dis}}\) in the IAEA’s notation) accounts for the perturbing effect of the ion chamber’s cavity, i.e., the fact that a portion of the medium of interest is replaced by the air cavity of the chamber. The ‘LDW’ method of Wang and Rogers (2008a) is used to calculate \(P_{\text{repl}}\). In this method, the air and central electrode in the chamber are replaced by a low-density graphite material which has all the characteristics of graphite except that its density is that of air. The replacement correction factor is the ratio of \(D_C\), the dose to a slab of graphite (centred on the point of measurement at the mid-line of the cavity), to \(D_{\text{LDC}}\), the dose to the low density graphite:

\[
P_{\text{repl}} = \frac{D_C}{D_{\text{LDC}}}.
\]

Wang et al (2009) reported that systematic uncertainties on \(P_{\text{repl}}\) calculated in this way are at most 0.2%.

Each calorimeter contained a series of gaps (voids) about the calorimeter core to reduce heat leakages. These gaps perturb secondary electrons, thereby changing the fluence reaching the core. The gap correction factor is

\[
k_{\text{gap}} = \frac{D_C}{D_{\text{gap}}},
\]

where \(D_{\text{gap}} (D_C)\) is the dose to the calorimeter core with (without) gaps present (Boutilllon 1989, Owen and DuSautoy 1991). Gap correction factors are termed either ‘compensated’ or ‘uncompensated’. For compensated gap corrections, the on-axis distance through graphite to the centre of the calorimeter core is the same whether the gaps are present or not.
For uncompensated corrections, the on-axis distance through graphite to the centre of the calorimeter core is shorter than when there are gaps present, due to the fact that portions of the graphite are replaced by vacuum gaps.

Both $k_{\text{gap}}$ and $P_{\text{repl}}$ corrections are needed for the experiments of Bewley (1963), Petree and Lamperti (1967) and Engelke and Hohlfeld (1971). Applying both correction factors consistently, only two simulations are required because the ratio of $k_{\text{gap}}$ and $P_{\text{repl}}$ is needed:

$$k_{\text{gap}}/P_{\text{repl}} = (D_C/D_{\text{gap}})(D_{\text{LDC}}/D_C) = D_{\text{LDC}}/D_{\text{gap}}.$$  

One simulation of the calorimeter determines $D_{\text{gap}}$ and another of the ion chamber with low density graphite filling the cavity provides $D_{\text{LDC}}$.

The central electrode correction factor, $P_{\text{cel}}$, accounts for the effect of the central electrode on the ionization in a cavity ion chamber (see, e.g., Nahum (2009)). This factor can be calculated with Monte Carlo methods as the ratio of doses to the air in the chamber without and with the central electrode. The central electrode correction factor is calculated for the experiments of Bewley (1963) and Engelke and Hohlfeld (1971) for which the electrodes are large (descriptions below). The electrode used in the chamber of Petree and Lamperti (1967) is made of graphite and is relatively small; hence, $P_{\text{cel}}$ is expected to be very near unity (Ma and Nahum 1993, Buckley et al 2004). It is not calculated in the present analysis.

Simulations to determine correction factors are performed with EGSnrc user codes. Cylindrical geometries (Bewley 1963, Engelke and Hohlfeld 1971, Guiho and Simoen 1975) are simulated with the user codes CAVRZnrc and DOSRZnrc (Rogers et al 2000) while the user code CAVITY (Kawrakow 2005) is used for the spherical geometry of Petree and Lamperti (1967). Simulation geometries for each experiment are described below. Energy thresholds and cutoffs of 10 keV (kinetic energy) are used for photons and electrons. Sensitivity tests with 1 keV cutoffs are performed for the experiments of Bewley (1963) and Engelke and Hohlfeld (1971); correction factors are consistent within statistical uncertainties of 0.03% with those from simulations with 10 keV cutoffs.

Statistical uncertainties on doses from simulations are less than 0.1%. The overall uncertainty on the correction factors calculated using the EGSnrc user codes is taken as 0.2% in the present work. This uncertainty accounts for statistical and systematic uncertainties (Wang et al 2009) and uncertainties related to the experimental/simulation configuration.

Graphite densities reported by investigators range from 1.7 to 1.8 g cm$^{-3}$. Simulations are performed using data sets with graphite of corresponding density, derived using an $I$-value of 78 eV and the density effect correction for the bulk density of graphite, 1.7 g cm$^{-3}$, from ICRU Report 37. Correction factors for two experiments (Engelke and Hohlfeld 1971; Guiho and Simoen 1975) were recalculated using data sets of the same bulk density as specified below but with the density correction for the grain density (2.265 g cm$^{-3}$) and the $I$-value of 86.8 eV. The $I$-value 86.8 eV is slightly different from the value, 86.9 ± 1.2 eV, measured by Bichsel and Hiraoka (1992); it is used in the present analysis for consistency with Rogers and Kawrakow (2003) and Wang and Rogers (2008b) who mistakenly used it as the value measured by Bichsel and Hiraoka (1992). This difference of 0.1 eV between $I$-values is insignificant as it is much less than the uncertainty of 1.2 eV on Bischel and Hiraoka’s measurement. Further, the correction factors recalculated with the density correction for the grain density (2.265 g cm$^{-3}$) and the $I$-value of 86.8 eV are consistent within statistical uncertainties of 0.07% with those calculated with the usual bulk density data sets with $I = 78$ eV. This is in accord with the results of Wang and Rogers (2008b) and suggests that the correction factors calculated are not sensitive to the graphite density correction and $I$-value used.

Although the heat defect in graphite and its uncertainty are both generally taken to be zero (ICRU 2001), these values are controversial (Rogers 2002 and references therein). Each experiment reanalysed in the present work, as well as the first determination of Niatel et al...
(1985), involves graphite calorimeter measurements and hence would be affected by a non-zero value for the heat defect. The heat defect in water, which is theoretically zero, has been extensively studied and is assigned an uncertainty of 0.15% in water calorimetry (McEwen 2009). Lacking a better justified estimate of the uncertainty, this value is used in the present work as the uncertainty on the graphite heat defect, which is itself taken to be zero.

2.1.1. Bewley (1963). Bewley made measurements inside a graphite phantom irradiated by $^{60}$Co beams using a cylindrical graphite cavity ion chamber and a calorimeter. The product $(W/e)_{air} (\bar{L}/\rho)_{C}^{a}$ was calculated using equation (1) with $P_{repl} = 1$ (no $P_{repl}$ correction factor was applied). Bewley used $^{60}$Co beams with 90, 50 and 60 cm focal lengths and 13 × 13, 10 × 10 and 11 × 11 cm$^2$ field sizes, respectively. Simulations are performed for the 50 and 90 cm cases as extremes; the correction factor $k_{gap}/(P_{repl} P_{cel})$ is calculated for both of these cases. Although measurements were made with these different experimental setups and beams, Bewley averaged ionometric and calorimetric measurements separately before calculating $(W/e)_{air} (\bar{L}/\rho)_{C}^{a}$ using equation (1). The overall correction factor $k_{gap}/(P_{repl} P_{cel})$ used in the present work is the average of the 50 and 90 cm focal length correction factors (see section 3.1).

In the absence of the $^{60}$Co source spectrum, the spectrum of Mora et al (1999) is used. The calculated correction factor is unchanged within statistics of 0.08% if simulations are performed with a 1.25 MeV beam. A point source on axis, incident from the front, is simulated.

The cavity chamber is a flat cylinder, 3 cm in diameter and 5 mm in depth. The chamber is graphite walled. The graphite collecting electrode is 2.6 cm in diameter and 1 mm thick at the chamber’s centre. The calorimeter core is a graphite disc 3 cm in diameter and 8 mm in thickness. There is one gap, 2 mm wide, about the core. Bewley did not report the depth of graphite for measurements; however, he noted that the same quantity of absorbing material was present in the path of radiation for ionometry and calorimetry measurements and that the calorimeter was placed so that the absorber occupied the same position as the chamber. Boutillon and Perroche-Roux (1987) reported a measurement depth of 5 g cm$^{-2}$. For calorimeter simulations, there is 5 g cm$^{-2}$ of graphite on the beam axis to the centre of the calorimeter core. There is some ambiguity in where the ion chamber should be simulated: it could either be positioned such that there is 5 g cm$^{-2}$ of graphite on the beam axis to the centre of the electrode or to the edge of the cavity. These two choices amount to a difference of 0.5 mm in graphite as the electrode is 1 mm wide and this is less than the positional uncertainty of 1 mm reported by Bewley. Simulations are performed for both ion chamber positions and the correction factor used is the average of the two possibilities. The difference resulting from the two ion chamber positions is at most 0.3%. By averaging results, the 0.2% uncertainty assigned to the correction factor accounts for this ambiguity.

2.1.2. Petree and Lamperti (1967). Petree and Lamperti used a configuration consisting of 12 $^{60}$Co source ‘pencils’ equally spaced along the edge of a circle of radius 5.24 cm in a large water tank. Measurements were made with a spherical ion chamber and calorimeter (in a water-tight can) placed at the centre of the circle of sources. The product $(W/e)_{air} (\bar{L}/\rho)_{C}^{a}$ was calculated using equation (1) with a $P_{repl}$ value of unity.

The $^{60}$Co pencils are simulated as cylinders of radius 0.535 cm and length 15.04 cm of $^{60}$Co covered by a 1 mm thick layer of iron. The spectrum of a bare $^{60}$Co source, i.e. two equiprobable lines at 1.175 and 1.334 MeV, is used for the initial particles; the correction factor is consistent within statistics of 0.08% if a 1.25 MeV source is used. For the calculation of
stopping-power ratios, described below, the source spectrum used is that at the location of the ion chamber quoted by Petree and Lamperti (1967). This spectrum is in good agreement with that extracted from simulations using the bare $^{60}$Co source; stopping power ratios calculated with these two spectra differ by at most 0.04%. The ion chamber or calorimeter and $^{60}$Co pencils are simulated at the centre of a cube of water of side length 30 cm.

Two calorimeters, each comprising a spherical core surrounded by a spherical shell (jacket), were used by Petree and Lamperti (1967) and are simulated in the present work. For both calorimeters, the core radius is 0.5 cm, the gap about the core is 0.14 cm wide and the outer radius of the jacket is 1.025 cm. The two calorimeters are identical apart from their cores: one calorimeter has a solid core while the other has a hollow core with a spherical region of radius 0.35 cm filled with air at its centre.

The ion chamber used and modelled is spherical. The cavity is at the centre of the sphere and has a radius of 0.635 cm, filled with air. The wall surrounding the cavity has nearly the same thickness as the calorimeter jacket with an outer radius of 1.035 cm. Though the ion chamber used by Petree and Lamperti (1967) contained a graphite collecting electrode (consisting of a cylinder of diameter 0.1 cm with a hemisphere of diameter 0.2 cm at its end; overall length 1.1 cm), it is omitted from simulations as its effect is expected to be very small (Ma and Nahum 1993, Buckley et al 2004).

The calorimeter cores and jacket and ion chamber walls are all simulated as graphite of density $1.80 \text{ g cm}^{-3}$; results are consistent within statistics of 0.1% if $1.70 \text{ g cm}^{-3}$ is used. Petree and Lamperti quoted a density of $1.76 \text{ g cm}^{-3}$. Petree and Lamperti applied corrections to account for gamma ray attenuation in the calorimeter core, calculated based on the linear attenuation and average length of gamma-ray paths in the graphite cores. These corrections are removed in the present analysis as this attenuation is accounted for in the simulations.

2.1.3. Engelke and Hohlfeld (1971). Engelke and Hohlfeld (1971) performed measurements using a cylindrical graphite ion chamber and calorimeter in graphite irradiated by a $^{137}$Cs beam. The product $(W/e)_{air}(L_{BIPM}/\rho)_C$ was calculated using equation (1). They applied a $P_{\text{repl}}$-type correction to account for the fact that the dose found by calorimetry was 0.15% too small when set in relation to the actual measuring point for the ionometric measurement; effectively, their correction amounted to taking $P_{\text{repl}} = 0.9985$. This correction is replaced by the correction factor $k_{\text{gap}}/(P_{\text{repl}}P_{\text{cel}})$ calculated in the present work.

The calorimeter core (absorber) is 3.25 mm in thickness and 2.5 cm in diameter. Gaps 2 mm wide separate the jacket from the core and the jacket from the surrounding mantle. Engelke and Hohlfeld (1971) did not specify the jacket thickness; it is taken to be 4 mm. Simulations with a 6 mm wide jacket yield a correction factor consistent within statistical uncertainties of 0.06%. The ion chamber is of identical dimensions and material to the absorber-jacket system used for the calorimetric measurement: the ion chamber’s electrode is of identical dimensions to the calorimeter’s absorber, the cavity dimensions are identical to those of the gap about the calorimeter core and the walls correspond (or are identical) to the calorimeter jacket. The gap separating the jacket from the mantle is present for ion chamber measurements (i.e., there is a 2 mm gap between the cavity walls and the phantom).

In simulations, the spectrum for the $^{137}$Cs source is from the EGSnrc distribution (average energy 613 keV). The distance from the source to the centre of the calorimeter core, stated as about 50 cm, is taken as 50 cm. This is also taken as the distance to the centre of the ion chamber cavity/electrode. Engelke and Hohlfeld did not report the measurement depth; however, Boutillon and Perroche-Roux (1987) reported it as 5 g cm$^{-2}$. In simulations, the on-axis distance through graphite to the centre of the calorimeter core and ion chamber electrode is $5 \text{ g cm}^{-2}$. The calculated correction factor $P_{\text{repl}}/k_{\text{gap}}$ is unchanged within statistics of 0.06%.
if the on-axis distance through graphite is changed by 2 mm. The density of graphite (not reported) is taken to be 1.7 g cm$^{-3}$.

2.1.4. Guiho and Simoen (1975). Guiho and Simoen determined the exposure (air kerma) for a $^{60}$Co beam using measurements of ionization current for 12 different graphite-walled ion chambers. The dose rate to the core of a graphite calorimeter was measured for the same source. The product $(W/e)_{air} (L_{Δ}/ρ)$ was calculated from these measurements, as described in the appendix, and depends on a derived factor, denoted $t_c$ by Guiho and Simoen, which relates the dose to a point in graphite to the air kerma at a nearby point in the absence of the calorimeter.

In the present analysis, the factor $t_c$ is recalculated and the gap correction factor is also calculated, as described in the following paragraphs. In addition to these two factors, Guiho and Simoen’s value for $(W/e)_{air} (L_{Δ}/ρ)$ is further modified. In 1992, Bielajew and Rogers reported that properly accounting for point of measurement effects and wall attenuation and scatter effects in a variety of differently shaped ion chambers increased exposure standards of national laboratories by 0.6%, on average. In the present work, it is assumed that the exposure should increase by this amount since a wide variety of chambers were used by Guiho and Simoen. This implies that Guiho and Simoen’s value of $(W/e)_{air} (L_{Δ}/ρ)$ should decrease by 0.6%. A 0.3% uncertainty is assigned to this correction.

The calorimeter used by Guiho and Simoen (1975) and simulated is cylindrical with an outer diameter of 2.8 cm and width of 1.5 cm and consists of a core, jacket and mantle separated by gaps. Measurements were made with the calorimeter in air, not within a graphite phantom. Gap widths are not indicated by Guiho and Simoen in their 1975 article; however, a later article describes a calorimeter of the same external dimensions (Guiho et al 1978). Based on the data in the 1978 article, the calorimeter is simulated with a core of 3 mm thickness and of diameter 1.6 cm. The gaps are both 1 mm wide. The inner (outer) diameter of the jacket is 1.8 cm (2.2 cm) and that of the mantle is 2.4 cm (2.8 cm). The graphite density is 1.70 g cm$^{-3}$. The gap correction factor is calculated as the ratio of two doses, $D_C$ and $D_{gap}$: $D_C$ is the dose to a disc-shaped region the size of the calorimeter core at the centre of a small homogeneous phantom (1.1 cm thick and of diameter 2.8 cm); $D_{gap}$ is the dose to the calorimeter core with the gaps present. This is a ‘compensated’ gap correction factor. The distance from the source to the centre of the calorimeter core (not stated) and graphite phantom is taken to be 100 cm. The $^{60}$Co source was the one used to establish the French primary standard of exposure (air kerma). The spectrum used in simulations is taken to be that quoted by Guiho et al (1978); results are unchanged within statistics of 0.1% if 1.25 MeV is used.

The factor $t_c$ is recalculated as $\tilde{t}_c$, defined in equation (A.5) in the appendix, in the present analysis. The dose to a graphite disc, $D_C$, was calculated as described in the previous paragraph. The air kerma at a point $O$, 0.83 g cm$^{-2}$ (0.49 cm) upstream from the point $G$ (defined in the appendix, figure A1), and all other quantities in equation (A.5) were calculated using the EGSnrc user code g.

2.2. Stopping-power ratios

Spencer–Attix stopping-power ratios are calculated with the EGSnrc user code SPRRZnrc (Rogers et al 2000) using (i) the bulk density of graphite (1.70 g cm$^{-3}$) and $I = 78$ eV and (ii) the grain density of graphite (2.265 g cm$^{-3}$) and $I = 86.8$ eV. Source details and the depth of the point of measurement are described above for each experiment. The cutoff energies, presented in table 2, are those listed by Boutillon and Perroche-Roux (1987). For two determinations of $(W/e)_{air} (L_{Δ}/ρ)_{air}$, no depth in graphite is specified; these
determinations involved ion chambers free in air and ion chamber response theory requires stopping power ratios for the unattenuated primary photon beam (see Borg et al (2000) and references therein). For these calculations of the stopping power ratio, the ‘photon regeneration’ option in SPRZnrc is used: after each interaction, the original properties of photons are restored and secondary particles are discarded.

Statistical uncertainties on the stopping-power ratios are less than 0.01%. A 0.1% uncertainty is assigned to the ratio \( \bar{L}_{\Delta}/\rho \) \( (\text{W/e})_{\text{air}}(\text{BIPM}/\rho)_{\text{C}} \), as discussed below in section 3.2. This is a correlated uncertainty for all six experiments considered.

The stopping power ratio, \( \bar{L}_{\text{BIPM}}/\rho \) \( (\text{W/e})_{\text{air}}(\text{BIPM}/\rho)_{\text{C}} \), is computed for the BIPM air kerma chamber \( (\Delta = 14 \text{ keV}) \) for the graphite bulk and grain densities and for \( I \)-values of 78 eV, 82.5 eV, 84.5 eV and 86.8 eV. Following Wang and Rogers (2008b), the \( ^{60}\text{Co} \) spectrum of Mora et al (1999) is used. The distance from the source to the point of measurement is 1.12 m, as noted by Niatel et al (1985). In principle, the stopping power ratio \( \bar{L}_{\text{BIPM}}/\rho \) \( (\text{W/e})_{\text{air}}(\text{BIPM}/\rho)_{\text{C}} \) is sensitive to the \( ^{60}\text{Co} \) spectrum which depends on the \( ^{60}\text{Co} \) source, the field size, etc. Rogers and Kawrakow (2003) studied the variation of the \( ^{60}\text{Co} \) graphite to air stopping-power ratio with the spectrum incident on the phantom. Although there is a 0.13% change in the stopping-power ratio if a realistic spectrum rather than a mono-energetic or near mono-energetic spectrum is used, they found that the variation is less than 0.03% for a wide variety of realistic spectra with the scattered energy fluence representing 17 to 24% of the total fluence. Hence, though there are many different \( ^{60}\text{Co} \) spectra, variations in \( \bar{L}_{\text{BIPM}}/\rho \) \( (\text{W/e})_{\text{air}}(\text{BIPM}/\rho)_{\text{C}} \) due to these different spectra should be small.

2.3. Average value of \( (W/e)_{\text{air}}(\bar{L}_{\text{BIPM}}/\rho)_{\text{C}} \)

Following Jones (2006) and the methods of (Müller 2000b), the arithmetic mean, weighted mean and weighted median of \( (W/e)_{\text{air}}(\bar{L}_{\text{BIPM}}/\rho)_{\text{C}} \) are calculated for the six updated experimental results. The uncertainty quoted with each sample mean or median is that calculated as described below and added in quadrature with the 0.1% correlated uncertainty on the ratio \( \bar{L}_{\Delta}/\rho \) \( (\text{W/e})_{\text{air}}(\text{BIPM}/\rho)_{\text{C}} \).

As usual, the arithmetic mean ignores the statistical weights (uncertainties). The uncertainty quoted with the arithmetic mean is the standard deviation of the mean.

The weighted mean, \( \bar{z} \), accounts for uncertainties, \( s_i \), on experimental results, \( z_i \). For each experimental result \( i = 1, 2, \ldots, 6 \), the weight is \( w_i = s_i^{-2} \) and the normalized weight is \( p_i = w_i/\sum w_i \). The weighted mean is then \( \bar{z} = \sum p_i z_i \) and the internal uncertainty is \( s_{int} = \sqrt{\sum w_i} \). The statistical confidence in the results requires the \( t \)-distribution because of the limited number of independent data points (Rogers 1975, Drosg 2007); a more realistic estimate of the 68% confidence interval is the external uncertainty (Rogers 1975), \( s_{ext} = s_{int} \sqrt{n-1}/\sqrt{n} \) \( (\chi^2/(n-1)) \), where \( t(n-1) \) is Student’s \( t \) distribution with \( n-1 \) degrees of freedom and \( \chi^2 = \sum ((z_i - \bar{z})/s_i)^2 \). For the six data points considered, Student’s \( t \) distribution for the 68% (1σ) confidence interval is \( t(5) = 1.101 \) (Rogers 1975).

The mean value is traditionally used as a location parameter for comparisons; however, it suffers from a lack of stability due to the effects of outliers (Müller 2000a). For these reasons, Müller (2000a, 2000b) proposed that the median should replace the mean value. Traditionally, the evaluation of the median of a data set ignores the statistical weights of results; however, Müller (2000b) described a technique to extend the evaluation of the median to the case where statistical weights are available. The weighted sample median \( \tilde{z} \) will minimize the sum \( \sum p_i |z_i - \tilde{z}| \), with an uncertainty given by \( s(\tilde{z}) \simeq 1.9 \text{MAD}/\sqrt{n-\bar{T}} \), where MAD (‘median of the absolute deviations’) minimizes \( \sum |z_i - \tilde{z}| = \text{MAD} \).
Table 1. Correction factors calculated using the EGSnrc code system. Statistical uncertainties are listed. The overall uncertainty on these correction factors is taken as 0.2%.

<table>
<thead>
<tr>
<th>Authors</th>
<th>Type</th>
<th>Configuration</th>
<th>Correction</th>
<th>Authors</th>
<th>Type</th>
<th>Configuration</th>
<th>Correction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bewley (1963)</td>
<td>$k_{\text{gap}} / (P_{\text{repl}} P_{\text{cel}})$</td>
<td>focal distance: 50 cm</td>
<td>0.9912 ± 0.07%</td>
<td>Petree and Lamperti (1967)</td>
<td>$k_{\text{gap}} / P_{\text{repl}}$</td>
<td>calorimeter core: hollow</td>
<td>1.0041 ± 0.08%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>90 cm</td>
<td>0.9921 ± 0.08%</td>
<td></td>
<td></td>
<td>solid</td>
<td>1.0102 ± 0.08%</td>
</tr>
<tr>
<td>Petree and Lamperti</td>
<td></td>
<td></td>
<td></td>
<td>Engelse and Hohlfeld (1971)</td>
<td>$k_{\text{gap}} / (P_{\text{repl}} P_{\text{cel}})$</td>
<td>–</td>
<td>0.9963 ± 0.06%</td>
</tr>
<tr>
<td>Guiho and Simoen (1975)</td>
<td>$k_{\text{gap}} / P_{\text{repl}}$</td>
<td>–</td>
<td>1.0039 ± 0.01%</td>
<td></td>
<td>$k_{\text{gap}}$</td>
<td>–</td>
<td>0.9943 ± 0.07%</td>
</tr>
<tr>
<td></td>
<td>$\tilde{t}_c$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Results and discussion

3.1. Correction factors

A summary of the calculated correction factors is given in table 1.

For Bewley’s experiment, the difference between correction factors for the 50 and 90 cm focal lengths is less than 0.1%, demonstrating that these factors are not very sensitive to focal length. The central electrode correction factors are nearly unity: 0.9993 (±0.05%) and 0.9999 (±0.06%) for the 50 and 90 cm focal lengths, respectively. The average value of $k_{\text{gap}} / (P_{\text{repl}} P_{\text{cel}})$ for the two focal lengths is 0.9917 (±0.1%).

For Petree and Lamperti’s experiment, the $k_{\text{gap}} / P_{\text{repl}}$ corrections are 1.0041 (1.0102) for the hollow (solid) core. Using these corrections obviates the need for those used by Petree and Lamperti for gamma ray attenuation in the calorimeter core which were $1 / 0.9888 = 1.0113$ (1/0.9830 = 1.0173) for the hollow (solid) core.

For the experiment of Engelse and Hohlfeld, the $k_{\text{gap}} / (P_{\text{repl}} P_{\text{cel}})$ correction is 0.9963. This supersedes Engelse and Hohlfeld’s 0.15% point of measurement correction; as noted in section 2.1.3, they took $P_{\text{repl}} = 0.9985$ with $k_{\text{gap}}$ and $P_{\text{cel}}$ values of unity. The central electrode correction (calculated in the present study) is a little greater than unity at 1.0025 (±0.04%).

The gap correction factor for Guiho and Simoen’s experiment is 1.0039. The correction $\tilde{t}_c$ is 0.9943, which differs by 1.5% from Guiho and Simoen’s value of 0.980 which was quoted with an uncertainty of 0.3% with 99.7% confidence.

3.2. Stopping-power ratios

Stopping-power ratios are presented in table 2. For case i ($I = 78$ eV and 1.70 g cm$^{-3}$), the stopping-power ratios are generally within 0.1% of those reported by Boutillon and Perroche-Roux (1987). Guiho and Simoen’s experiment is the exception with a difference of 0.2% and this is due to the use of the photon regeneration option in SPRRRZnc for the calculation of $(\bar{L}/\rho)_{\text{C}}$ for this in-air measurement (section 2.2); if this option is not used, the value for $(\bar{L}/\rho)_{\text{C}}$ is within 0.1% of that reported by Boutillon and Perroche-Roux (1987). Stopping-power ratios for case ii ($I = 86.8$ eV and 2.265 g cm$^{-3}$) are on average 1.6% lower than those for case i, in agreement with the results of Rogers and Kawrakow (2003) for ion chambers in air.

The ratios $(\bar{L}/\rho)_{\text{C}} / (\bar{L}_{\text{BIPM}}/\rho)_{\text{C}}$ are nearly the same for cases i and ii for each experiment, differing by 0.09% or less (table 2). In section 2.1 it was noted that the correction factors are insensitive to the choice of graphite density correction and $I$-value for the evaluation of stopping powers. Hence, the product $(W/e)_{\text{air}} (\bar{L}_{\text{BIPM}}/\rho)_{\text{C}}$ is relatively insensitive to these choices given the relatively limited range of cavity sizes used in these experiments. In the
### Table 2

Stopping-power ratios from Boutillon and Perroche-Roux (1987) and the current analysis with (i) graphite of density 1.70 g cm$^{-3}$ and $I = 78$ eV and (ii) 2.265 g cm$^{-3}$ and $I = 86.8$ eV. The values of $(\bar{L}_\Delta/\rho)_C$ for the first experiment of Niatel et al (1985) were calculated by Wang and Rogers (2008b). The final column gives the per cent difference between the ratio $(\bar{L}_\Delta/\rho)_C/(\bar{L}_{BIPM}/\rho)_C$ for cases i and ii. Statistical uncertainties on stopping-power ratios calculated in the current analysis are less than 0.01%.

<table>
<thead>
<tr>
<th>Authors</th>
<th>$\Delta$ (keV)</th>
<th>Depth (g cm$^{-2}$)</th>
<th>$\bar{L}_\Delta/\rho$</th>
<th>$(\bar{L}_\Delta/\rho)_C$</th>
<th>$(\bar{L}_\Delta/\rho)<em>C/(\bar{L}</em>{BIPM}/\rho)_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>1987 (i)</td>
<td>(ii)</td>
<td>(i)</td>
</tr>
<tr>
<td>Bewley (1963)</td>
<td>16</td>
<td>5</td>
<td>1.003</td>
<td>1.0030</td>
<td>1.0016</td>
</tr>
<tr>
<td>Petree and Lamperti (1967)</td>
<td>21</td>
<td>0.7</td>
<td>1.001</td>
<td>0.9849</td>
<td>0.9999</td>
</tr>
<tr>
<td>Engelke and Holhfeld (1971)</td>
<td>18</td>
<td>5</td>
<td>1.012</td>
<td>0.9939</td>
<td>1.0091</td>
</tr>
<tr>
<td>Guiho and Simoen (1975)</td>
<td>25</td>
<td>–</td>
<td>1.002</td>
<td>0.9842</td>
<td>0.9984</td>
</tr>
<tr>
<td>Niatel et al (1985) (1)</td>
<td>14</td>
<td>1–17</td>
<td>1.002 to 1.004</td>
<td>0.9861 to 1.0038 to 0.9877</td>
<td>1.0009 to 1.0025 to 1.0027</td>
</tr>
<tr>
<td>Niatel et al (1985) (2)</td>
<td>14</td>
<td>–</td>
<td>1.000</td>
<td>0.9849</td>
<td>1.0000</td>
</tr>
</tbody>
</table>
The values of \(\langle W/e \rangle_{\text{air}} \langle \bar{L}_{\text{BIPM}} / \rho \rangle_C^C\) from Boutillon and Perroche-Roux (1987) and the current analysis are presented in Table 3 and Figure 1. Values are only presented for case i (\(I = 78\) eV and 1.70 g cm\(^{-3}\)) as those for case ii (\(I = 86.8\) eV and 2.265 g cm\(^{-3}\)) are nearly identical, as discussed above. The uncertainties on the results for the current study, listed for each author, do not include the 0.1% correlated uncertainty due to the ratio \(\langle L_{\Delta} / \rho \rangle_C^C / \langle \bar{L}_{\text{BIPM}} / \rho \rangle_C^C\) calculated with case i. A 0.1% correlated uncertainty due to \(\langle L_{\Delta} / \rho \rangle_C^C / \langle \bar{L}_{\text{BIPM}} / \rho \rangle_C^C\) is assigned to the final, calculated value of \(\langle W/e \rangle_{\text{air}} \langle \bar{L}_{\text{BIPM}} / \rho \rangle_C^C\), as mentioned in Section 2.2.

### Table 3. Values of \(\langle W/e \rangle_{\text{air}} \langle \bar{L}_{\text{BIPM}} / \rho \rangle_C^C\) in \(\text{J C}^{-1}\) from the present study and from Boutillon and Perroche-Roux (1987). The means and weighted median are listed for each data set. Per cent uncertainties for each value of the product \(\langle W/e \rangle_{\text{air}} \langle \bar{L}_{\text{BIPM}} / \rho \rangle_C^C\) are listed in brackets after the absolute (1 \(\sigma\)) uncertainties. Per cent differences between values in each row are indicated, as are the values of \(\chi^2\) per degree of freedom (dof) for the means and median.

<table>
<thead>
<tr>
<th>Authors</th>
<th>Boutillon and Perroche-Roux</th>
<th>Present study</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bewley (1963)</td>
<td>34.21 ± 0.21 (0.60%)</td>
<td>33.97 ± 0.23 (0.66%)</td>
<td>0.70%</td>
</tr>
<tr>
<td>Petree and Lamperti (1967)</td>
<td>33.80 ± 0.05 (0.15%)</td>
<td>33.62 ± 0.10 (0.29%)</td>
<td>0.52%</td>
</tr>
<tr>
<td>Engelke and Hohlfeld (1971)</td>
<td>33.87 ± 0.06 (0.18%)</td>
<td>33.79 ± 0.10 (0.31%)</td>
<td>0.23%</td>
</tr>
<tr>
<td>Guiho and Simoen (1975)</td>
<td>34.02 ± 0.10 (0.29%)</td>
<td>33.58 ± 0.16 (0.49%)</td>
<td>1.30%</td>
</tr>
<tr>
<td>Niatel et al (1985) (1)</td>
<td>33.96 ± 0.04 (0.12%)</td>
<td>33.66 ± 0.09 (0.27%)</td>
<td>0.90%</td>
</tr>
<tr>
<td>Niatel et al (1985) (2)</td>
<td>33.81 ± 0.12 (0.35%)</td>
<td>33.63 ± 0.18 (0.52%)</td>
<td>0.54%</td>
</tr>
</tbody>
</table>

The values of \(\langle W/e \rangle_{\text{air}} \langle \bar{L}_{\text{BIPM}} / \rho \rangle_C^C\) from Boutillon and Perroche-Roux (1987) and the current analysis are presented in Table 3 and Figure 1. Values are only presented for case i (\(I = 78\) eV and 1.70 g cm\(^{-3}\)) as those for case ii (\(I = 86.8\) eV and 2.265 g cm\(^{-3}\)) are nearly identical, as discussed above. The uncertainties on the results for the current study, listed for each author, do not include the 0.1% correlated uncertainty due to the ratio \(\langle L_{\Delta} / \rho \rangle_C^C / \langle \bar{L}_{\text{BIPM}} / \rho \rangle_C^C\) calculated with case i. A 0.1% correlated uncertainty due to \(\langle L_{\Delta} / \rho \rangle_C^C / \langle \bar{L}_{\text{BIPM}} / \rho \rangle_C^C\) is assigned to the final, calculated value of \(\langle W/e \rangle_{\text{air}} \langle \bar{L}_{\text{BIPM}} / \rho \rangle_C^C\), as mentioned in Section 2.2.

3.3. Value of \(\langle W/e \rangle_{\text{air}} \langle \bar{L}_{\text{BIPM}} / \rho \rangle_C^C\)

The values of \(\langle W/e \rangle_{\text{air}} \langle \bar{L}_{\text{BIPM}} / \rho \rangle_C^C\) from Boutillon and Perroche-Roux (1987) and the current analysis are presented in Table 3 and Figure 1. Values are only presented for case i (\(I = 78\) eV and 1.70 g cm\(^{-3}\)) as those for case ii (\(I = 86.8\) eV and 2.265 g cm\(^{-3}\)) are nearly identical, as discussed above. The uncertainties on the results for the current study, listed for each author, do not include the 0.1% correlated uncertainty due to the ratio \(\langle L_{\Delta} / \rho \rangle_C^C / \langle \bar{L}_{\text{BIPM}} / \rho \rangle_C^C\) calculated with case i. A 0.1% correlated uncertainty due to \(\langle L_{\Delta} / \rho \rangle_C^C / \langle \bar{L}_{\text{BIPM}} / \rho \rangle_C^C\) is assigned to the final, calculated value of \(\langle W/e \rangle_{\text{air}} \langle \bar{L}_{\text{BIPM}} / \rho \rangle_C^C\), as mentioned in Section 2.2.

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</tr>
<tr>
<td>Guiho and Simoen (1975)</td>
<td>34.02 ± 0.10 (0.29%)</td>
<td>1.30%</td>
</tr>
<tr>
<td>Niatel et al (1985) (1)</td>
<td>33.96 ± 0.04 (0.12%)</td>
<td>0.90%</td>
</tr>
<tr>
<td>Niatel et al (1985) (2)</td>
<td>33.81 ± 0.12 (0.35%)</td>
<td>0.54%</td>
</tr>
</tbody>
</table>

The results for the two determinations of \(\langle W/e \rangle_{\text{air}} \langle \bar{L}_{\text{BIPM}} / \rho \rangle_C^C\) by Niatel et al, as modified by the recent analysis of Wang and Rogers (2008b), based in part on the work of Burns et al (2007), are listed in Table 3. The uncertainties on these values have been modified from those listed by Boutillon and Perroche-Roux (1987). For the first determination of Niatel et al, the 1\(\sigma\) uncertainty on \(P_{\text{rep}}\) was originally taken to be 0.06% and this is replaced by a 0.2% uncertainty (Wang et al 2009). Additionally, a 0.15% uncertainty is assigned to the graphite heat defect. Overall, the uncertainty on this result increases from 0.12% to 0.27%. The uncertainty on the second determination of \(\langle W/e \rangle_{\text{air}} \langle \bar{L}_{\text{BIPM}} / \rho \rangle_C^C\) by Niatel et al is dominated by the uncertainty on the mass-energy absorption coefficient for graphite, \((\mu_{en} / \rho)_C\), which was originally taken to be 0.33% (1\(\sigma\)). Hubbell (1982) suggests that the uncertainty on \((\mu_{en} / \rho)_C\) should be 1% with no confidence limit specified. The interpretation of this uncertainty in the present work is guided by the discussion of uncertainties on \(I\)-values appearing in ICRU Report 37 (footnote 10). This 1% uncertainty is interpreted as a 90% (\(\sim 2\sigma\)) confidence limit and hence the 1\(\sigma\) uncertainty is 0.5%. The total uncertainty on correction factors for the BIPM air kerma standard is reduced from 0.11% to 0.06% (Burns et al 2007, Burns 2006). Overall,
Re-evaluation of \((W/e)_{\text{air}}(\bar{L}_{\text{BIPM}}/\rho)^C_a\)

Figure 1. Values of \((W/e)_{\text{air}}(\bar{L}_{\text{BIPM}}/\rho)^C_a\) reported by Boutillon and Perroche-Roux (1987) and from the current analysis. The accepted value, based on the results of 15 experiments (Boutillon and Perroche-Roux 1987) and the uncertainty of 0.11% recommended by the CCRI (1999), and the weighted mean for the updated results are indicated.

The uncertainty on \((W/e)_{\text{air}}(\bar{L}_{\text{BIPM}}/\rho)^C_a\) increases from 0.35% (Boutillon and Perroche-Roux 1987) to 0.52% for the second determination of Niatel et al.

All revised values of \((W/e)_{\text{air}}(\bar{L}_{\text{BIPM}}/\rho)^C_a\) are lower than those quoted by Boutillon and Perroche-Roux (1987). Five of the six updated results are more than 0.5% lower; three updated results are outside the range of the 1σ error bars of both evaluations and all are outside the original 68% confidence interval.

The lower portion of table 3 presents arithmetic and weighted means and the weighted median and associated uncertainties for \((W/e)_{\text{air}}(\bar{L}_{\text{BIPM}}/\rho)^C_a\) for the updated data and the data quoted by Boutillon and Perroche-Roux (1987). These uncertainties include the 0.1% correlated uncertainty on the ratios \((\bar{L}_{\text{BIPM}}/\rho)^C_a/(\bar{L}_{\Delta}/\rho)^C_a\). The means and median are separately internally consistent both for the Boutillon and Perroche-Roux (1987) and for the updated data sets. However, the means and median are consistently lower (by 0.6% or more) for the updated data set and are outside the range of uncertainties on the means and median of the 1987 data set. Using the average value \((W/e)_{\text{air}}(\bar{L}_{\text{BIPM}}/\rho)^C_a = 33.97 \pm 0.05 \text{ J C}^{-1}\) and \((\bar{L}_{\text{BIPM}}/\rho)^C_a = 1.000 \pm 0.3\%\), both calculated by Boutillon and Perroche-Roux (1987), the accepted value of \((W/e)_{\text{air}}(\bar{L}_{\text{BIPM}}/\rho)^C_a\) is 33.97 ± 0.04 J C\(^{-1}\) where the uncertainty (0.11%) is that recommended by the CCRI(1) at its 1999 meeting (CCRI 1999). This value is consistent with the means and median calculated here with the six 1987 data points. The means and median for the revised data are 0.8 to 0.9% lower than this accepted value. The accepted value and weighted mean of the updated data are indicated in figure 1.

Table 3 also presents values of \(\chi^2\) per degree of freedom. While these values are 2 or greater for the 1987 data, suggesting further systematic uncertainties, they are nearer unity for the updated data set, primarily due to the increased uncertainties.

3.4. \(I\)-value and density for graphite and the value of \((W/e)_{\text{air}}\)

A decrease in the value of \((W/e)_{\text{air}}(\bar{L}_{\text{BIPM}}/\rho)^C_a\) may have implications for the choice of graphite \(I\)-value and density correction and the value of \((W/e)_{\text{air}}\). Table 4 lists values of \((W/e)_{\text{air}}\)
corresponding to stopping powers calculated for different choices of graphite I-value and density used for the density correction, based on the weighted mean \((W/e)_{\text{air}}(L_{\text{BIPM}}/\rho)_{C}^{C} = 33.68 \pm 0.07 \text{ J C}^{-1}\) computed in the current study.

<table>
<thead>
<tr>
<th>I-value (eV)</th>
<th>Density (g cm(^{-3}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>78.0</td>
<td>33.63 33.71</td>
</tr>
<tr>
<td>82.5</td>
<td>33.89 33.96</td>
</tr>
<tr>
<td>84.5</td>
<td>34.00 34.07</td>
</tr>
<tr>
<td>86.8</td>
<td>34.13 34.20</td>
</tr>
</tbody>
</table>

Table 4. Values of \((W/e)_{\text{air}}\) for different choices of graphite I-value and density for the density correction based on the weighted mean of \((W/e)_{\text{air}}(L_{\text{BIPM}}/\rho)_{C}^{C} = 33.68 \pm 0.07 \text{ J C}^{-1}\) computed in the current study.
unless one assumes a value of \((W/e)_{air}\) with very small uncertainties, as done by Burns (2009). Such an assumption is not supported by experimental measurements. A reanalysis of these experimental results might yield insights into these issues.

4. Conclusions

Correction factors calculated using the EGSnrc code system consistently decrease values of \((W/e)_{air}(L_{BIPM}/\rho)_{C}^{I}\) for the experiments of Bewley (1963), Petree and Lamperti (1967), Engelke and Hohlfeld (1971) and Guiho and Simoen (1975), compared to those quoted by Boutillon and Perroche-Roux (1987). The ratio \((L_{\Delta}/\rho)_{C}^{I}/(L_{BIPM}/\rho)_{C}^{I}\) calculated with the extreme \(I\)-value \((I = 86.8 \text{ eV})\) and the grain density for graphite \((2.265 \text{ g cm}^{-3})\) is consistent within 0.1% of that calculated using ICRU Report 37 stopping powers \((I = 78 \text{ eV} \text{ and the bulk density } 1.70 \text{ g cm}^{-3})\) for each experiment considered (Bewley 1963, Petree and Lamperti 1967, Engelke and Hohlfeld 1971, Guiho and Simoen 1975, Niatel et al 1985). Thus, the value of the product \((W/e)_{air}(L_{BIPM}/\rho)_{C}^{I}\) is relatively insensitive to choices of graphite \(I\)-value and density for the range of cavities employed in these experiments. The updated values of \((W/e)_{air}(L_{BIPM}/\rho)_{C}^{I}\) for these experiments are consistently lower than those of Boutillon and Perroche-Roux (1987). The present analysis suggests that the accepted value of the product \((W/e)_{air}(L_{BIPM}/\rho)_{C}^{I}, 33.97 \text{ J C}^{-1} \pm 0.11\%\) is 0.8–0.9% too high, whether the arithmetic mean \((33.71 \text{ J C}^{-1} \pm 0.2\%)\), weighted mean \((33.68 \text{ J C}^{-1} \pm 0.2\%)\), or weighted median \((33.66 \text{ J C}^{-1} \pm 0.2\%)\) of the updated data is adopted.

In principle, a reduction in the value of the product \((W/e)_{air}(L_{BIPM}/\rho)_{C}^{I}\) has implications for the graphite mean excitation energy and density (used in the calculation of the graphite stopping power) and for the value of \((W/e)_{air}\). However, the large uncertainties on the value of direct determinations of \((W/e)_{air}\) preclude the possibility of determining the \(I\)-value and density correction for graphite based on the revised value of \((W/e)_{air}(L_{BIPM}/\rho)_{C}^{I}\). None of the graphite \(I\)-values or the graphite densities used to evaluate the density effect listed in table 4 may be correct; the granular nature of graphite may mean that neither the grain nor the bulk density of graphite should be used to calculate the density effect correction. New insights into the values of the graphite \(I\)-value and density correction and the value of \((W/e)_{air}\) might be achieved through a reanalysis of existing experimental data for direct determinations of \((W/e)_{air}\), but more particularly through new measurements in the energy range of interest.

The normalization of the product \((W/e)_{air}(L_{BIPM}/\rho)_{C}^{I}\) to the BIPM \({}^{60}\text{Co}\) air kerma standard \((\Delta = 14 \text{ keV})\), \((W/e)_{air}(L_{BIPM}/\rho)_{C}^{I}\), was introduced in this paper. The value of \((W/e)_{air}(L_{BIPM}/\rho)_{C}^{I}\) could be adopted for use with \({}^{60}\text{Co}\) air kerma standards worldwide which depend directly on this product. Individual standards laboratories would need to adopt a correction factor \((L_{\Delta}/\rho)_{C}^{I}/(L_{BIPM}/\rho)_{C}^{I}\), as plotted in figure 2, to convert the accepted value of \((W/e)_{air}(L_{BIPM}/\rho)_{C}^{I}\) to that required for their standard. This method provides standards laboratories with a consistent way to use the value of \((W/e)_{air}(L_{\Delta}/\rho)_{C}^{I}\) for their \({}^{60}\text{Co}\) air kerma standards, ensuring a rigorous, standardized and consistent assignment of uncertainties.

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Figure 2. Values of $(\bar{L}/\rho)_{a}^C / (\bar{L}_{\text{BIPM}}/\rho)_{a}^C$ for different values of $\Delta$ for the two extreme $I$-values and density corrections. These values are based on those calculated by Rogers and Kawrakow (2003) using the EGSnrc user-code SPRZnrc with photon regeneration on in a mini-phantom with 5 mm of buildup and the $10 \times 10 \text{ cm}^2$ spectrum of Mora et al. (1999); as usual, $\Delta$ is the low-energy transport cutoff used in calculations, corresponding to the mean chord length of particle tracks in the chamber.

Wang for information regarding his work on the BIPM chamber (Wang and Rogers 2008b) and Ernesto Mainegra-Hing for his assistance with the German-language article of Kunze and Hecker (1980).

Appendix. Calculation of $(W/e)_{\text{air}} (\bar{L}/\rho)_{a}^C$ by Guiho and Simoen (1975)

At a point $O$, the exposure $X$ and air kerma $K_{\text{air}}$ are related via

$$X(O) = \left(1 - \bar{g}_{\text{air}}\right) \frac{(W/e)_{\text{air}}}{K_{\text{air}}(O)}, \quad (A.1)$$

where $\bar{g}_{\text{air}}$ is the average fraction of a secondary electron’s energy lost in air via radiative processes. For a graphite-walled ion chamber, the exposure rate can be written as

$$\dot{X}(O) = \frac{IP}{V\rho_a} \left(\bar{L}_{\Delta}/\rho\right)_{a}^C \left(\frac{\bar{\mu}_{en}}{\rho}\right)_{a}^C, \quad (A.2)$$

where $I$ is the ionization current, $V$ is the cavity volume, $\rho_a$ is the air density and $P$ represents the product of correction factors for the chamber. Guiho and Simoen measured the current, $I$, for 12 different graphite-walled cavities, and calculated the product

$$j = \frac{IP}{V} \left(\frac{\bar{L}_{\Delta}}{\rho}\right)_{a}^C \quad (A.3)$$

for each cavity, using Spencer–Attix cavity theory and cavity size to calculate the stopping power ratio. They argued that the consistency of the value of $j$ for all 12 cavities suggested that correction factors were applied appropriately. The average $\bar{j}$ of the $j$-values was calculated to determine the exposure rate:

$$\dot{X}(O) = \frac{\bar{j}}{\rho_a} \left(\frac{\bar{\mu}_{en}}{\rho}\right)_{a}^C. \quad (A.4)$$
Guiho and Simoen related the air kerma at the point \( O \) to the absorbed dose in graphite at a point \( G \) (figure A1) using a factor they derived denoted by \( t_c \). Guiho and Simoen calculated \( t_c \) as the product of factors accounting for the beam’s attenuation and scatter in graphite. In the present work, this factor is replaced by \( \tilde{t}_c \),

\[
\tilde{t}_c = \frac{1}{K_{air}(O)} \frac{D_c(G)}{1 - \bar{g}_{air}} \left( \frac{\mu_{en}}{\rho} \right)_C^{a}.
\]

which is calculated using Monte Carlo simulations. With this factor, equation (A.1) can be re-expressed as

\[
\left( \frac{W}{e} \right)_{air} = \frac{1}{t_c} \ \hat{D}_c(G) \ \left( \frac{\mu_{en}}{\rho} \right)_C^{a}.
\]

Along with \((W/e)_{air}\), Guiho and Simoen (1975) calculated \((W/e)_{air}(L/\rho)^C_{a}\). In principle, they might have reported values of \((W/e)_{air}(L/\rho)^C_{a}\) for each chamber; however, they reported only one result for \((W/e)_{air}(L/\rho)^C_{a}\) representing the average value. Guiho and Simoen did not report the value of \( \Delta \) for their chambers, only that \((L/\rho)^C_{a}\) values range from 1.003 to 1.005, with an average of 1.0035. Boutillon and Perroche-Roux (1987) reported that \( \Delta = 25 \) keV for the experiment of Guiho and Simoen and this is used in the present analysis. Writing

\[
\hat{j} = \left( \frac{L/\rho}{\rho_a} \right)_a^{C}.
\]

where \( \hat{j} \) can be regarded as the average current per unit mass of air in the cavity, the exposure rate can be written as

\[
\hat{X}(O) = \hat{j} \left( \frac{L/\rho}{\rho_a} \right)_a^{C} \left( \frac{\mu_{en}}{\rho} \right)_C^{a}.
\]
Substituting this expression into equation (A.6), putting $\dot{D}_c(G) = \frac{k_{\text{gap}}}{t_c} \dot{D}_{\text{cal}}$ and rearranging yields

$$\left( \frac{W}{e} \right)_{\text{air}} \left( \frac{L_\Delta}{\rho} \right)_{\text{a}}^{C} = \frac{k_{\text{gap}}}{t_c} \frac{\dot{D}_{\text{cal}}}{J}. \quad (A.9)$$

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