

Accuracy of the Burns equation for stopping-power ratio as a function of depth and R_{50}

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The accuracy of the Burns *et al.* equation [Med. Phys. **23**, 489–501 (1996)] for the Spencer–Attix water to air stopping-power ratio as a function of depth in a water phantom and electron beam quality in terms of R_{50} is investigated by comparison to the original data on which this fit was based. It is shown that using this equation provides dose estimates on the central axis in a clinical electron beam that are accurate to within 1% of dose maximum for all 24 clinical beams investigated except very close to the surface in swept beams. In contrast, the error in the dose as a percentage of the local dose is much higher for values of the depth/ R_{50} greater than 1.2. © 2004 American Association of Physicists in Medicine. [DOI: 10.1118/1.1803811]

Both the AAPM's TG-51 protocol for reference beam dosimetry¹ and the IAEA's TRS-398 Code of Practice² make use of collision stopping-power ratios calculated for realistic electron beams. This is in contrast to earlier protocols which were based on stopping-power ratios for mono-energetic electron beams. For dosimetry at the reference depth of $d_{\text{ref}} = 0.6R_{50} - 0.1$ cm, both protocols use a function of R_{50} based on a fit to the stopping-power ratios calculated at the reference depth for 24 realistic beams (Burns *et al.*³). This fit is in remarkable agreement with the calculated values for d_{ref} .

However, practical clinical dosimetry also requires knowledge of the stopping-power ratios as a function of depth, z , and beam quality, R_{50} (both in cm). Here again, both the AAPM and IAEA recommend use of another formula developed by Burns *et al.*³ This formula is a fit to values of the stopping-power ratio as a function of depth/ R_{50} for 24 different beams. The stopping-power ratios were calculated^{4,5} for realistic electron beams based on simulations with the BEAM code.⁶ The formula is

$$\left(\frac{\bar{L}}{\rho}\right)_{\text{air}}^{\text{water}}(R_{50}, z) = \frac{a + b(\ln R_{50}) + c(\ln R_{50})^2 + d(z/R_{50})}{1 + e(\ln R_{50}) + f(\ln R_{50})^2 + g(\ln R_{50})^3 + h(z/R_{50})}. \quad (1)$$

The values for the eight coefficients are

$$a = 1.0752, \quad b = -0.50867, \quad c = 0.088670,$$

$$d = -0.08402, \quad e = -0.42806, \quad f = 0.064627,$$

$$g = 0.003085, \quad h = -0.12460.$$

Several observations about this formula are in order. The first is that the fit to the data is not nearly as good as in the case of the fit at the single depth of d_{ref} . This is not surprising given the much more complex nature of the data being fit. The second observation is that this formula is based on a fit to a restricted range of z/R_{50} values. Specifically, the shallowest depth bin extended from 0.0 to 0.05 cm in the phantom (taken as representing a depth of 0.025 cm). This means

that the shallowest values included in the fit were for values of z/R_{50} between 0.0013 and 0.025. The deepest depth used for a given beam corresponded to the last depth before the stopping-power ratios started to decrease (indicating being in the photon tail). This z/R_{50} value ranged between 1.16 and 1.8 but was mostly around 1.2 and 1.3. In the original paper³ the range of acceptable fit was given as values of z/R_{50} between 0.02 and 1.1 and it was pointed out that the quality of the fit deteriorated going to a depth past a value of $z/R_{50} = 1.2$. An earlier internal report⁷ also pointed out that the error in the fit became very substantial for values of $z/R_{50} > 1.2$.

Figures 1 and 2 summarize the earlier data on the difference between the individually calculated stopping-power ra-

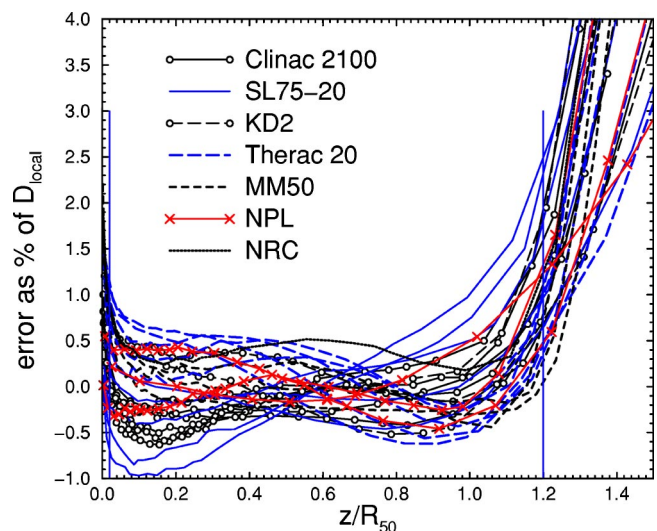


FIG. 1. Error associated with using Eq. (1) instead of the individually calculated stopping-power ratio for a given beam when “measuring” the dose using an ion chamber in a water phantom. The error is given as a percentage of the local dose which is just the % error in the fitted stopping-power ratio compared to the individually calculated value. The vertical lines correspond to the range of applicability of this equation as given in the original paper (Ref. 3). Note that there are multiple beam energies associated with each accelerator model (and the beam qualities may be deduced from the original paper).

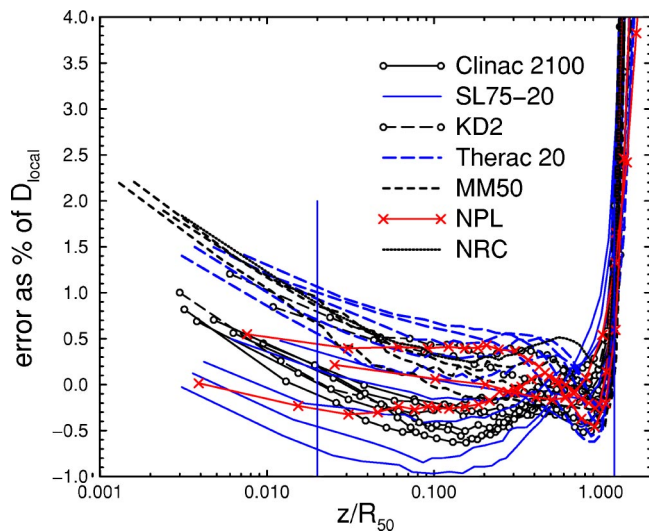


FIG. 2. Same as Fig. 1 except for a logarithmic scale to emphasize values near the surface.

tios and the values determined from Eq. (1). The plots actually show the percentage error in the local dose determined using the two different stopping powers. This dose error is given by the percentage difference in the two stopping powers.

However, a 10% error in a dose which is only 2% of the dose maximum, $D_{\max}$, is not of practical importance in most situations and thus the error as a percentage of $D_{\max}$ is more useful. This quantity is given by

$$\frac{D_{fn}(z) - D_{real}(z)}{D_{\max,real}} 100\% = \frac{(spr_{fn} - spr_{real})}{spr_{real}} \%DD, \quad (2)$$

where $\%DD$ is the percentage dose at depth z and $D_{real}(z)$ and $D_{fn}(z)$ are the doses determined at depth z using either the realistic calculated stopping-power ratio at that depth for that specific beam or using the stopping-power ratio from Eq. (1). This quantity is shown as a function of z/R_{50} for all

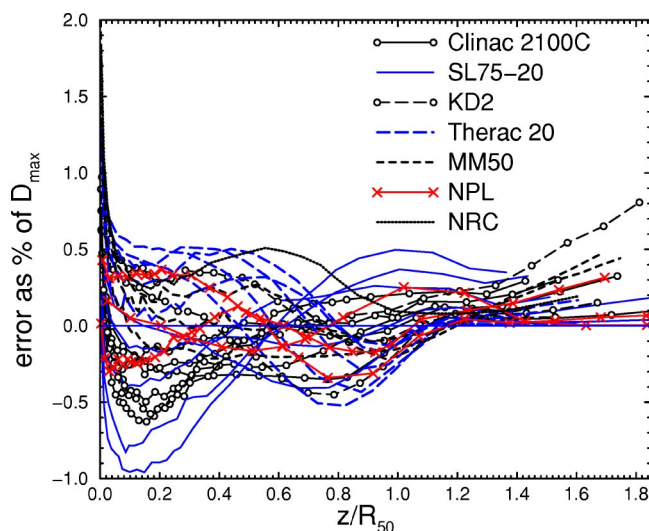


FIG. 3. Same as Fig. 1 but as a percentage of dose maximum, $D_{\max}$.

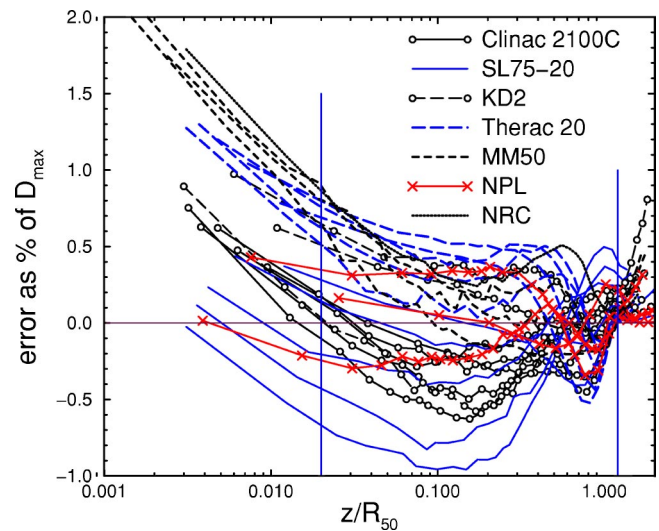


FIG. 4. Same as Fig. 3 except for a logarithmic scale to emphasize values near the surface.

24 beams in Figs. 3 and 4. The values of $\%DD$ as a function of depth are obtained from Monte Carlo calculations using the same beam simulations.⁸

In all cases, for large values of z/R_{50} , the error caused by using Eq. (1) is less than 0.8% of $D_{\max}$, and in many cases less than 0.2%. However, the error for very shallow depths remains somewhat larger, the worst offenders being the relatively rare machines with swept electron beams (which have nearly mono-energetic beams). Aside from these beams, the error is 1% of $D_{\max}$ or less, even coming very close to the surface.

It is clear that Eq. (1) is certainly accurate enough for clinical work under all circumstances with the possible exception of magnetically swept beams very close to the surface. However, for careful research work, it is also clear that the best approach is to simulate the electron beam in question and directly calculate the stopping-power ratio needed (e.g., using the code SPRZRnc which is freely available⁹) since Eq. (1) has uncertainties of up to 1%. In particular, any work needing values very close to the surface should be recalculated since the stopping-power ratios determined in the original calculations are for a bin of 0.05 cm thickness.

A final comment is to note that all of this discussion is about stopping-power ratios needed when measuring depth-dose curves. For reference dosimetry, the equations used by the major protocols are very accurate representations of the individually calculated data at d_{ref} (as discussed by Burns *et al.*³).

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