# The role of humidity and other correction factors in the AAPM TG-21 dosimetry protocol

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A detailed derivation is presented of the formulas required to determine $N_{\text{gas}}$ and $D_{\text{med}}$ in the AAPM TG-21 dosimetry protocol. This protocol specifies how to determine the absorbed dose in an electron or photon beam when using exposure or absorbed dose calibrated ion chambers. It is shown that the expression given in TG-21's recent letter of clarification is incorrect. Accounting for humidity correctly increases, by 0.4%, all absorbed dose determinations using an exposure calibrated ion chamber. Taking into account other correction factors in the equation for exposure could also have varying, but significant effects (possibly over 1%). These are the stem scatter correction, the axial nonuniformity correction and the electrode correction for electrodes made of different materials from the wall. Attention is drawn to differences in the definitions of the exposure and absorbed dose calibration factors, $N_X$ and $N_D$, respectively, as supplied by the NBS and the NRCC.

## I. INTRODUCTION

According to the definition of exposure, its determination requires the measurement of the charge liberated per unit mass of dry air. For low-energy x rays, an exposure standard is usually established using a free-air chamber, while for high-energy x rays and $\gamma$ rays a cavity chamber is used. In either case, the mass of dry air in the chamber is determined from a knowledge of the sensitive volume, combined with the accepted density of dry air at some reference temperature and pressure. On the other hand, the gas in the chamber is usually humid air which has a significant water vapor content. It is therefore necessary to correct the charge collected from this humid air to what would be obtained if the chamber were filled with dry air. This is a well-known problem which has been extensively studied.[1-6]

The water vapor in the air affects the number of ion pairs produced in two ways. First, the value of $W$, the mean energy required to create an ion pair, decreases by 0.8% in going from dry to saturated conditions. This effect would tend to increase the charge collected. Second, the electron stopping power per molecule (or equivalently the linear stopping power) increases by 0.2% in going from dry to saturated conditions because of the change in the average molecule. This effect would tend to decrease the charge collected. These two effects combine in such a way that the correction for humidity is approximately constant ($0.997 \pm 0.001$) for the range of relative humidities (10% to 70%) typically encountered in a laboratory environment and for the wide variety of electron and photon beam qualities of interest in radiotherapy (see Appendix B for explicit values).

If a calculation of the dose to the gas in the ion chamber is required, it is necessary to know the mass of the gas, or equivalently its density. As water vapor is added to dry air, the density decreases by about 1% in going from dry to saturated conditions because of the decrease in the average molecular weight of the gas.

In its recent "Clarification of the AAPM Task Group 21 Protocol," TG-21[7] has introduced the humidity correction factor which is now required since this has recently been included in the National Bureau of Standards (NBS) primary standard for $^{60}$Co exposure. Unfortunately, NBS has introduced this correction factor differently from the way it has been used for many years in the Canadian standards laboratory. At the National Research Council of Canada (NRCC), the humidity correction is treated in a manner similar to the temperature and pressure corrections of an ion chamber's reading—i.e., the reading is corrected to be as if it were a reading for dry air. Thus, given a chamber with a meter reading of $M^u$ [in meter units (mu)] in the humid air at the reference temperature and pressure for an irradiation in a $^{60}$Co beam and an NRCC exposure calibration factor of $N_X^{\text{CDN}}$($\text{C kg}^{-1}\,\text{mu}^{-1}$), then the exposure is given (assuming similar exposure rates in the users and calibration beams) by the following:

$$X^u = M^u K_h^u N_X^{\text{CDN}}, \tag{1}$$

where superscript $u$ refers to the user's beam and where $K_h^u$ is the humidity correction factor in the user's beam which must be applied by the user. In the general case, using notation developed below, an additional correction factor of $K_{\text{ion}}^u / K_{\text{ion}}^c$ ($= A_{\text{ion}}^c P_{\text{ion}}$ in the AAPM protocol's) is needed to account for changes in ion chamber collection efficiency.

In contrast, since $K_h$ is roughly constant NBS has chosen to define $N_X$ differently:

$$X^u = M^u N_X^{\text{US}}. \tag{2}$$

This means the definitions of the US and Canadian exposure calibration factors are related by

$$N_X^{\text{US}} = K_h N_X^{\text{CDN}}. \tag{3}$$

In view of Eq. (3), for consistency one must *not* include $K_h$ in the new version of Eq. (5) of the AAPM dosimetry protocol when using $N_X^{\text{CDN}}$. Put another way, the new version of Eq. (5) (Ref. 7) is given for $N_X^{\text{US}}$ and Eq. (3) above must be used to get this quantity when using a Canadian calibration factor $N_X^{\text{CDN}}$.

Note that Eq. (3) only holds in the sense that the definitions of $N_X^{\mathrm{CDN}}$ and $N_X^{\mathrm{US}}$ differ by the $K_h$ factor. The numerical values obtained for a particular chamber may differ depending on the experimental uncertainties in measuring $N_X$ and depending on any differences in the practical implementation of the exposure standard at NBS and NRCC (although one BIPM intercomparison[8] has shown that in 1975 the standards agreed within 0.3%).

Prompted by the above considerations we began to investigate the role of humidity in other parts of the AAPM protocol. For example, the protocol refers to all stopping-power ratios as being relative to gas whereas all tabulated ratios are for dry air. After examining these questions carefully, we have found that the protocol has conceptual errors concerning humidity which lead to underestimates of the absorbed dose by about 0.4%.

We also wish to point out that to determine $N_{\mathrm{gas}}$ accurately, one should take into account correction factors for stem scatter and point of measurement for the user's ion chamber when used in the calibration beam. Corrections for an aluminum electrode, which are also ignored in the protocol, may also lead to significant ($\sim 0.8\%$) underestimates of $N_{\mathrm{gas}}$. In some cases, changing $N_{\mathrm{gas}}$ may lead to no change in the assigned dose because similar corrections are needed in the final dose equations, but this is not always the case.

In this paper we present a detailed derivation of $N_{\mathrm{gas}}$ and $D_{\mathrm{med}}$ as used in the AAPM protocol so that we can include humidity effects. We emphasize the distinction between measurements made with the user's ion chamber in the standards laboratory's calibration beam (superscript $c$) and those made in the user's beam (superscript $u$). Very few new results are presented since the role of humidity effects has been extensively studied[1-6] and Mijnheer and Williams[6] have already presented a correct analysis of the impact on the AAPM protocol (which was unfortunately overlooked in preparing the TG-21 Letter of Clarification[7]). Our purpose is to develop an adequate notation and to present a unified analysis which is capable of examining the effects of various approximations which can be introduced in order to achieve a simple formulation for use in future protocols.

Appendix A contains a summary of our notation and Appendix B contains a summary of pertinent formulas concerning humidity effects.

## II. DETERMINATION OF ABSORBED DOSE USING AN EXPOSURE CALIBRATED ION CHAMBER

In order to clarify the logic and make explicit the role of humidity we will present a corrected derivation of the procedure used in the AAPM protocol to assign absorbed doses in a phantom using an exposure calibrated ion chamber. Here, the term air always means dry air and gas refers to the actual gas in the users ion chamber (most often humid air). The derivation is based in part on the approach of Attix,[9-11] although our final results differ. For completeness and to define notation we start with some standard definitions and fundamentals.

*Collision kerma* is the kinetic energy per unit mass transferred by a photon beam to electrons and not subsequently lost by radiative processes (i.e., it is the amount of energy dissipated "locally" by ionization). It is a point function and is given by

$$(K_c)_{\mathrm{med}} = E_\gamma \phi_\gamma (\bar{\mu}_{\mathrm{en}}/\rho)_{\mathrm{med}} \quad (\mathrm{J/kg}), \tag{4}$$

where $E_\gamma$ is the average energy of the photons (J), $\phi_\gamma$ is the photon fluence ($\mathrm{m}^{-2}$) at the point and $(\bar{\mu}_{\mathrm{en}}/\rho)_{\mathrm{med}}$ is the mass energy absorption coefficient for the medium averaged over the energy fluence spectrum of the photons ($\mathrm{m}^2/\mathrm{kg}$).

*Exposure* at a point is the amount of charge of one sign released per unit mass of *dry* air by the electrons set in motion by a photon beam. From the definitions of $(K_c)_{\mathrm{air}}$ and $W/e$, and using Eq. (4), exposure is given by

$$\begin{aligned} X &= (K_c)_{\mathrm{air}} (e/W)_{\mathrm{air}} \\ &= E_\gamma \phi_\gamma (\bar{\mu}_{\mathrm{en}}/\rho)_{\mathrm{air}} (e/W)_{\mathrm{air}} \quad (\mathrm{C/kg}), \end{aligned} \tag{5}$$

where $(W/e)_{\mathrm{air}}$ is the mean energy expended by slowing electrons per coulomb of charge liberated in *dry* air (in J/C).

Consider an ion chamber with walls thick enough to produce charged particle equilibrium in the cavity which is filled with some gas (usually humid air). The Spencer–Attix formulation of the Bragg–Gray *cavity theory* tells us the following:

$$D_{\mathrm{wall}} = D_{\mathrm{gas}} (\bar{L}/\rho)^{\mathrm{wall}}_{\mathrm{gas}} \quad (\mathrm{Gy}), \tag{6}$$

where $D_{\mathrm{wall}}$ and $D_{\mathrm{gas}}$ are the doses to the wall (near the cavity) and to the gas in the cavity, and $(\bar{L}/\rho)^{\mathrm{wall}}_{\mathrm{gas}}$ is the value of the restricted mass stopping power for the wall material averaged over the electron spectrum in the cavity divided by that for the gas (the theory assumes the cavity has not disturbed the electron spectrum and for the moment, we ignore photon attenuation and scatter in the wall).

Since we have assumed *charged particle equilibrium* in the wall (at least near the cavity), then

$$D_{\mathrm{wall}} = (K_c)_{\mathrm{wall}} = E_\gamma \phi_\gamma (\bar{\mu}_{\mathrm{en}}/\rho)_{\mathrm{wall}} \quad (\mathrm{Gy}). \tag{7}$$

Finally, from definitions we have

$$D_{\mathrm{gas}} = (W/e)_{\mathrm{gas}} (Q_{\mathrm{gas}}/m_{\mathrm{gas}}) \quad (\mathrm{Gy}), \tag{8}$$

where $Q_{\mathrm{gas}}$ refers to the charge liberated in the cavity gas and $m_{\mathrm{gas}}$ is the mass of the gas there. Note that all subscripts refer to the actual gas present. The AAPM protocol introduces $J_{\mathrm{gas}} = Q_{\mathrm{gas}}/m_{\mathrm{gas}}$ but it will be essential to keep the numerator and denominator separated.

Let us now derive a useful expression for the exposure at the center of the chamber. From Eqs. (5) and (7) we find the following:

$$X = D_{\mathrm{wall}} (\bar{\mu}_{\mathrm{en}}/\rho)^{\mathrm{air}}_{\mathrm{wall}} (e/W)_{\mathrm{air}} \quad (\mathrm{C/kg}). \tag{9}$$

Using Eqs. (6) and (8), we get

$$X = \frac{Q_{\mathrm{gas}}}{m_{\mathrm{gas}}} \left(\frac{\bar{L}}{\rho}\right)^{\mathrm{wall}}_{\mathrm{gas}} \left(\frac{\bar{\mu}_{\mathrm{en}}}{\rho}\right)^{\mathrm{air}}_{\mathrm{wall}} \left(\frac{W}{e}\right)^{\mathrm{gas}}_{\mathrm{air}} K \quad (\mathrm{C/kg}), \tag{10}$$

where we have introduced a factor $K$ to represent a variety of correction factors which are needed in this equation because not all the assumptions used in its derivation are completely fulfilled. In this equation, the air terms enter because of the definition of the quantity exposure and the gas terms refer to the actual gas in the ion chamber. It is also common practice[1-4] to introduce a correction factor, $K_h$, into Eq. (10) so that all the factors refer to dry air instead of the gas (usually humid air) in the chamber—except $Q_{\text{gas}}$ which must refer to the charge which is measured in the gas in the chamber. One then gets

$$X = \frac{Q_{\text{gas}}}{m_{\text{air}}} \left(\frac{\bar{L}}{\rho}\right)_{\text{air}}^{\text{wall}} \left(\frac{\bar{\mu}_{\text{en}}}{\rho}\right)_{\text{wall}}^{\text{air}} K_h K \quad (\text{C/kg}). \tag{11}$$

The quantity $K_h$ is called the humidity correction factor because it is usually used to correct for the humidity in the air, but it can also be used in the general case of any gas. Comparing Eqs. (10) and (11) gives the commonly used result that

$$K_h = \frac{m_{\text{air}}}{m_{\text{gas}}} \left(\frac{W}{e}\right)_{\text{air}}^{\text{gas}} \left(\frac{\bar{L}}{\rho}\right)_{\text{gas}}^{\text{air}} = \left(\frac{W}{e}\right)_{\text{air}}^{\text{gas}} (\bar{L})_{\text{gas}}^{\text{air}}, \tag{12}$$

where $m_{\text{air}}$ is the mass of dry air that would fill the ion chamber, $m_{\text{gas}}$ is the mass of the actual gas in the ion chamber, and Eq. (B4) in Appendix B is used to get the right-hand expression.

Three points need to be made. First, consider the other correction factors included in Eqs. (10) and (11), viz., $K = K_{\text{wall}} K_{\text{an}} K_{\text{st}} K_{\text{el}} K_{\text{comp}}$ with (see, e.g., Ref. 12):

| | |
|---|---|
| $K_{\text{wall}}$ | A correction which is greater than 1.0 (usually) to take into account photon attenuation and scatter in the walls of the ion chamber[13,14]; and includes the correction $\beta$, for the center of electron production.[7] |
| $K_{\text{an}}$ | A correction (less than 1.0) to account for the axial nonuniformity of the photon beam (i.e., it decreases as $1/r^2$)—this can be thought of as a correction for the point of measurement. |
| $K_{\text{st}}$ | A correction factor (less than 1.0) to account for the photons scattered into the chamber by the stem. |
| $K_{\text{el}}$ | A correction factor which is needed to account for an electrode which is made of a material different from the wall. For a graphite Farmer chamber with a 1-mm-diam aluminum electrode, $K_{\text{el}} \approx 0.992$.[14,15] Its form would be similar to the correction factor required for a composite wall. |
| $K_{\text{comp}}$ | A correction factor to take into account composite wall materials [i.e., a buildup cap of different material, see Eq. (6) of the AAPM protocol[16]]. |

Note that we have already implicitly utilized a correction factor $K_{\text{ion}}$ (greater than 1.0) to account for ion recombination and to convert the charge collected, $M_a$ (in C) into $Q_{\text{gas}}$, the charge of ions released, i.e.,

$$Q_{\text{gas}} = M_a K_{\text{ion}} \quad (\text{C}), \tag{13}$$

where

$$M_a = Mf \quad (\text{C}), \tag{14}$$

and $f$ is the electrical calibration factor in C/mu which converts the instrument's reading $M$ in meter units (mu) into coulombs.

The second point is that Eq. (11) is considerably different from the corresponding equation used implicitly in the revised AAPM protocol,[7] namely

$$X^{\text{AAPM}} = \frac{Q_{\text{gas}}}{m_{\text{gas}}} \left(\frac{\bar{L}}{\rho}\right)_{\text{gas}}^{\text{wall}} \left(\frac{\bar{\mu}_{\text{en}}}{\rho}\right)_{\text{wall}}^{\text{air}} K_h K. \tag{15}$$

In particular the value of $m_{\text{gas}}$ varies by up to 1% compared to $m_{\text{air}}$ which appears in the correct Eq. (11).

The third point is to realize that Eq. (11) is used in two separate instances in the dose measurement procedure. In the first instance, the standards laboratory uses this equation as applied to their standard ion chamber(s) to establish the primary standard for exposure and from this a value of $N_X$ is determined for each user's chamber. In order to apply the AAPM protocol, the user indirectly makes use of Eq. (11) again, because it is needed to derive an expression for $N_{\text{gas}}$. To do this, it is applied to the user's chamber when placed in the calibration beam at the standards laboratory. As we shall see below, this is to find the ratio of the mass of gas in the ion chamber and the instrument's electrical calibration factor $f$.

The primary purpose of the AAPM protocol is to determine the dose to a medium in an arbitrary user's beam by using an ion chamber calibrated in a $^{60}$Co beam. For clarity we will derive how this is done on the assumption that the chamber is "thin walled" (i.e., $P_{\text{wall}} = 1.00$) although this assumption is not essential and we assume a buildup cap of the same material as the chamber wall is used for the exposure calibration.

In the user's beam, denoted by a superscript $u$, the Spencer–Attix formulation of Bragg–Gray cavity theory gives

$$D_{\text{med}}^{u} = D_{\text{gas}}^{u} \left(\frac{\bar{L}^{u}}{\rho}\right)_{\text{gas}}^{\text{med}} P_{\text{repl}} \quad (\text{Gy}), \tag{16}$$

where $P_{\text{repl}}$ is a correction factor introduced to take into account the replacement of phantom material by the ion chamber. Note that $P_{\text{repl}}$ ($\leqslant 1.0$) takes into account the lack of attenuation in the cavity gas where as Eq. (6) is eventually corrected, using $K_{\text{wall}}$, by removing the effects of attenuation in the wall material.The essential element of the protocol is to recognize that $D_{\text{gas}}$ can be written as the product of a corrected meter reading from the ion chamber and a constant called the cavity gas calibration factor, $N_{\text{gas}}$:

$$D_{\text{gas}}^{u} = M^{u} K_{\text{ion}}^{u} N_{\text{gas}} \quad (\text{Gy}), \tag{17}$$

where $K_{\text{ion}}^{u}$, defined in Eq. (13), corrects the measured charge to the actual charge released in the cavity (in the protocol it is written as $P_{\text{ion}}$). From the definition of $D_{\text{gas}}$, [Eq. (8)] and from the relationship between the instrument reading and the charge liberated [Eqs. (13) and (14)] we find:

$$D_{\text{gas}}^{u} = M^{u} K_{\text{ion}}^{u} (W/e)_{\text{gas},u} f / m_{\text{gas}}^{u}, \tag{18}$$

and hence

$$N_{\text{gas}} = (W/e)_{\text{gas},u} f / m_{\text{gas}}^{u} \quad (\text{Gy mu}^{-1}). \tag{19}$$

To determine the ratio $f/m_{\text{gas}}^{u}$ we use the ion chamber's exposure calibration factor as follows. For the ion chamber exposed in the calibration field we have [from Eq. (2)]:

$$N_X^{\text{US}} = X/M^{c} \quad (\text{C kg}^{-1}\,\text{mu}^{-1}). \tag{20}$$

Using Eqs. (11), (13), and (14) for $X$ and $M^c$, we get

$$\frac{f}{m_{\text{air}}} = \frac{N_X^{\text{US}}}{(\bar{L}^{c}/\rho)_{\text{air}}^{\text{wall}} (\bar{\mu}_{\text{en}}/\rho)_{\text{wall}}^{\text{air}} K_h^{c} K K_{\text{ion}}^{c}}. \tag{21}$$

Using Eq. (12) for $K_h$ we write

$$\frac{f}{m^u_{\rm gas}} = \frac{f}{m_{\rm air}}\frac{m_{\rm air}}{m^u_{\rm gas}}$$
$$= \frac{f}{m_{\rm air}} K^u_h \left(\frac{W}{e}\right)^{\rm air}_{{\rm gas},u} \left(\frac{\bar{L}^u}{\rho}\right)^{{\rm gas},u}_{\rm air}. \qquad (22)$$

Combining (19), (21), (22) gives

$$N_{{\rm gas},u} = \frac{N^{\rm US}_X \left(\frac{W}{e}\right)_{\rm air} \left(\frac{\bar{L}^u}{\rho}\right)^{{\rm gas},u}_{\rm air}}{\left(\frac{\bar{L}^c}{\rho}\right)^{\rm wall}_{\rm air} \left(\frac{\bar{\mu}_{\rm en}}{\rho}\right)^{\rm air}_{\rm wall} KK^c_{\rm ion}} \frac{K^u_h}{K^c_h} \quad ({\rm Gy\ mu}^{-1}). \qquad (23)$$

and

$$D^u_{\rm med} = M^u N_{{\rm gas},u} (\bar{L}^u/\rho)^{\rm med}_{{\rm gas},u} P_{\rm repl} K^u_{\rm ion} \quad ({\rm Gy}), \qquad (24)$$

where we have introduced the additional subscript $u$ with $N_{{\rm gas},u}$ to indicate its (usually small) dependence on the gas or spectrum in the users beam. Recall that $M^u$ is the instrument reading (in mu) corrected for leakage and polarity effects, and normalized to standard temperature and pressure conditions.

The AAPM protocol assumes that *gas, u* and *gas, c* are the same (denoted by gas), and correspond to air with a relative humidity of 50%. With this simplification, the dose equation given by the protocol is the same as Eq. (24) but $N_{\rm gas}$ is given in the letter of clarification[7] by

$$N^{\rm AAPM86}_{\rm gas} = \frac{N^{\rm US}_X (W/e)_{50\%}}{(\bar{L}^c/\rho)^{\rm wall}_{50\%} (\bar{\mu}_{\rm en}/\rho)^{\rm air}_{\rm wall} KK_h K^c_{\rm ion}} \quad ({\rm Gy\ mu}^{-1}). \qquad (25)$$

The superscript, $c$, on the stopping-power ratio indicates that it is to be evaluated for the electron spectrum appropriate to the calibration beam. The recommended values for $(W/e)_{50\%}$ and $K_h$ are 33.77 J/C and 0.997, respectively. Note that Eq. (25) and the other equations for $N_{\rm gas}$ assume that the chamber wall and buildup cap are of the same material (i.e., $K_{\rm comp} = 1.0$). When they are of different materials, Eq. (25) would be modified as shown in Eq. (6) of the protocol.[16]

Using Eqs. (23) and (25) we find that the ratio of the AAPM dose estimate to the correct dose estimate is given by

$$\frac{D^{\rm AAPM86}}{D^{\rm correct}} = \frac{K^c_h}{K^u_h} \frac{(W/e)^{50\%}_{\rm air}}{K_h} \left(\frac{\bar{L}^c}{\rho}\right)^{50\%}_{\rm air} \left(\frac{\bar{L}^u}{\rho}\right)^{\rm air}_{50\%}. \qquad (26)$$

From Table B1 we find that the product of stopping-power ratios appearing in Eq. (26) is unity to within 0.02% and that the ratio $K^c_h/K^u_h$ lies between 0.9985 and 1.0015 for combinations of relative humidities between 30% and 80%. Thus Eq. (26) indicates that the AAPM protocol underestimates the absorbed dose by anywhere from 0.23% to 0.53%, and even if the humidity is constant at 50%, it is low by 0.4%.

*For normal ranges of humidity variation, the AAPM protocol systematically underestimates the dose by about 0.4%.*

An approach which is correct to within ± 0.15% for the extremes of humidity variation defined above and which is simple to use is to define

$$N^A_{\rm gas} = \frac{N^{\rm US}_X (W/e)_{\rm air}}{(\bar{L}^c/\rho)^{\rm wall}_{\rm air} (\bar{\mu}_{\rm en}/\rho)^{\rm air}_{\rm wall} KK^c_{\rm ion}} \quad ({\rm Gy\ mu}^{-1}), \qquad (27)$$

and write

$$D^{A,u}_{\rm med} = M^u N^A_{\rm gas} (\bar{L}^u/\rho)^{\rm med}_{\rm air} P_{\rm repl} K^u_{\rm ion} \quad ({\rm Gy}). \qquad (28)$$

These equations follow from Eqs. (23) and (24) by setting $K^u_h = K^c_h$ and recalling that $(\bar{L}^u/\rho)^{{\rm gas},u}_{\rm air} (\bar{L}^u/\rho)^{\rm med}_{{\rm gas},u} = (\bar{L}^u/\rho)^{\rm med}_{\rm air}$. These equations are much more accurate than the equations in the AAPM protocol—in fact the range of error, ± 0.15%, is entirely due to the assumption that the humidity correction is a constant—an assumption which is also built into the NBS calibration factor. These equations also have the distinct advantage of using stopping-power ratios for dry air, which is what all tabulations provide. This equation for $N^A_{\rm gas}$ also has the advantage over the exact one of making $N^A_{\rm gas}$ a constant. One minor disadvantage of these equations is that the simple relationships for $N_{\rm gas}$ and $D_{\rm gas}$ [Eqs. (17) and (19)] no longer hold (see Table I). A more serious disadvantage is that Eqs. (27) and (28) do not apply in the case of a general gas in the ion chamber because in general $K^c_h \neq K^u_h$, even if the gas remains the same because the stopping-power ratios in $K_h$ depend on the electron spectrum being averaged over. It is only in the special case of humid air that these stopping ratios happen to be independent of the electron spectrum. For example, the stopping-power ratio for tissue-equivalent gas to air varies by 2% for electrons between 300 keV and 20 MeV.

In Sec. II we will derive two further formulations. The first has been recommended by Attix,[9,10] as applicable in the general case, but we will show that if it is applied to the case of humid air it can lead to errors of up to ± 0.15% since it becomes equivalent to formulation $A$ above. The second is a formulation which is both exact in the general case, and retains a simple interpretation of $N_{\rm gas}$ as the dose to dry air in the ion chamber per meter unit [cf. Eq. (19)].

## III. OTHER FORMULATIONS AND A SUMMARY

The general equations for $N_{{\rm gas},u}$ and $D^u_{\rm med}$ [Eqs. (23) and (24)] are formally correct for any gas filling the ion chamber although there is a more natural formulation for the case of an ion chamber in which the mass of the gas (after correction to standard temperature and pressure) is independent of external conditions (e.g., a closed chamber or one with flowing tissue-equivalent gas). In general, after substituting Eq. (12) for $K_h$ into Eq. (23), one gets

$$N_{{\rm gas},u} = \frac{N^{\rm US}_X \left(\frac{W}{e}\right)_{\rm air} \frac{m^c_{\rm gas}}{m^u_{\rm gas}} \left(\frac{W}{e}\right)^{{\rm gas},u}_{{\rm gas},c}}{\left(\frac{\bar{L}^c}{\rho}\right)^{\rm wall}_{{\rm gas},c} \left(\frac{\bar{\mu}_{\rm en}}{\rho}\right)^{\rm air}_{\rm wall} KK^c_{\rm ion}} \quad ({\rm Gy\ mu}^{-1}), \qquad (29)$$

where $(W/e)^{{\rm gas},u}_{{\rm gas},c}$ is the ratio of the values of $W/e$ for the gas in the ion chamber at the time of the measurement in the user's beam to that for the gas in the ion chamber during the calibration in the standards laboratory. Equation (29) still holds in the general case, but if the gas in the ion chamber does not change, then one can write the following:

TABLE I. Summary of equations for determining dose based on exposure-calibrated ion chambers. Recall that $N_X^{\mathrm{US}} = K_h^c N_X^{\mathrm{CDN}}$.

| Label | $N_{\mathrm{gas}}$ | $D_{\mathrm{med}}$ | $\dfrac{D_{\mathrm{med}}}{D_{\mathrm{med}}^{\mathrm{exact}}}$ [a] | Comments |
|---|---|---|---|---|
| – | $\dfrac{N_X^{\mathrm{US}}(W/e)_{\mathrm{air}}(\bar{L}^u/\rho)_{\mathrm{air}}^{\mathrm{gas},u}}{(\bar{L}^c/\rho)_{\mathrm{air}}^{\mathrm{wall}}(\bar{\mu}_{\mathrm{en}}/\rho)_{\mathrm{wall}}^{\mathrm{air}}KK_{\mathrm{ion}}^c}\dfrac{K_h^u}{K_h^c}=\left(\dfrac{W}{e}\right)_{\mathrm{gas},u}\dfrac{f}{m_{\mathrm{gas}}^u}$ | $M^u N_{\mathrm{gas},u}\left(\dfrac{\bar{L}^u}{\rho}\right)_{\mathrm{gas},u}^{\mathrm{med}}P_{\mathrm{repl}}K_{\mathrm{ion}}^u$ | 1.0000 | Exact.<br>Simple interpretation of $N_{\mathrm{gas}}$. |
| *A* | $\dfrac{N_X^{\mathrm{US}}(W/e)_{\mathrm{air}}}{(\bar{L}^c/\rho)_{\mathrm{air}}^{\mathrm{wall}}(\bar{\mu}_{\mathrm{en}}/\rho)_{\mathrm{wall}}^{\mathrm{air}}KK_{\mathrm{ion}}^c}=\left(\dfrac{W}{e}\right)_{\mathrm{gas},c}\left(\dfrac{L^c}{\rho}\right)_{\mathrm{gas},c}^{\mathrm{air}}\dfrac{f}{m_{\mathrm{gas}}^c}$ | $M^u N_{\mathrm{gas}}^A\left(\dfrac{\bar{L}^u}{\rho}\right)_{\mathrm{air}}^{\mathrm{med}}P_{\mathrm{repl}}K_{\mathrm{ion}}^u$ | 0.9985→1.0015[b] | Assumes $K_h^c = K_h^u$.<br>All quantities are with respect to air.<br>Only applies to humid air. |
| *B* | $\dfrac{N_X^{\mathrm{US}}(W/e)_{\mathrm{air}}}{(\bar{L}^c/\rho)_{\mathrm{gas}}^{\mathrm{wall}}(\bar{\mu}_{\mathrm{en}}/\rho)_{\mathrm{wall}}^{\mathrm{air}}KK_{\mathrm{ion}}^c}=\left(\dfrac{W}{e}\right)_{\mathrm{gas}}\dfrac{f}{m_{\mathrm{gas}}}$ | $M^u N_{\mathrm{gas}}^B\left(\dfrac{\bar{L}^u}{\rho}\right)_{\mathrm{gas}}^{\mathrm{med}}P_{\mathrm{repl}}K_{\mathrm{ion}}^u$ | 0.9985→1.0015[b] | Only holds exactly for gases which do not change; i.e., when $m_{\mathrm{gas}}$ is constant.<br>Note that in general, even for an unchanging gas, $K_h^c \neq K_h^u$. |
| *C* | $\dfrac{N_X^{\mathrm{CDN}}(W/e)_{\mathrm{air}}}{(\bar{L}^c/\rho)_{\mathrm{air}}^{\mathrm{wall}}(\bar{\mu}_{\mathrm{en}}/\rho)_{\mathrm{wall}}^{\mathrm{air}}KK_{\mathrm{ion}}^c}=\left(\dfrac{W}{e}\right)_{\mathrm{air}}\dfrac{f}{m_{\mathrm{air}}}$ | $M^u K_h^u N_{\mathrm{gas}}^C\left(\dfrac{\bar{L}^u}{\rho}\right)_{\mathrm{air}}^{\mathrm{med}}P_{\mathrm{repl}}K_{\mathrm{ion}}^u$ | 1.0000 | Exact.<br>All quantities are with respect to air.<br>Holds exactly for any gas. |
| AAPM86 | $\dfrac{N_X^{\mathrm{US}}(W/e)_{50\%}}{(\bar{L}^c/\rho)_{50\%}^{\mathrm{wall}}(\bar{\mu}_{\mathrm{en}}/\rho)_{\mathrm{wall}}^{\mathrm{air}}KK_hK_{\mathrm{ion}}^c}$ | $M^u N_{\mathrm{gas}}^{\mathrm{AAPM}}\left(\dfrac{\bar{L}^u}{\rho}\right)_{50\%}^{\mathrm{med}}P_{\mathrm{repl}}K_{\mathrm{ion}}^c$ | 0.9947→0.9977[b] | Low by 0.4%. |

[a] $D_{\mathrm{med}}^{\mathrm{exact}}$ is given by the first row or row *C*. $D_{\mathrm{med}}$ is the dose assigned using the respective formulations when applied to ion chamber filled with humid air.

[b] Range of values assuming 30% and 80% humidity in the calibration and user's beams, respectively, and vice versa.

$$N_{\mathrm{gas}}^B = \frac{N_X^{\mathrm{US}}(W/e)_{\mathrm{air}}}{(\bar{L}^c/\rho)_{\mathrm{gas}}^{\mathrm{wall}}(\bar{\mu}_{\mathrm{en}}/\rho)_{\mathrm{wall}}^{\mathrm{air}}KK_{\mathrm{ion}}^c} \quad (\mathrm{Gy\ mu}^{-1}), \tag{30}$$

where we utilize the assumptions that $m_{\mathrm{gas}}^c = m_{\mathrm{gas}}^u$ and that $(W/e)_{\mathrm{gas}}$ is independent of the spectrum for a given gas [remember that $m_{\mathrm{gas}}$ and $(W/e)$ *do* change for humid air if the humidity changes; and in particular, this is not the same as assuming that $K_h^c = K_h^u$]. Equation (30) is substantially the same as that given by Attix for $N_{\mathrm{gas}}$,[9,10] however, it is not exact in the case of humid air if the humidity varies. Note that even in this case with no air in the ion chamber, the $(W/e)$ and $(\bar{\mu}_{\mathrm{en}}/\rho)$ quantities are for *air* because of the definition of exposure. Equation (30) for $N_{\mathrm{gas}}^B$, is used with Eq. (24) for the dose to the medium, with *gas*, *u* replaced by *gas* in the stopping-power ratio.

Suppose that for the case of an ion chamber filled with humid air this formulation were used and the gas were treated as dry air. This case then reduces to being the same as using $N_{\mathrm{gas}}^A$ [Eqs. (27) and (28)] with the attendant range of error of $\pm 0.15\%$ associated with ignoring variations in humidity.

Let us now consider a third formulation, which retains a simpler form than Eq. (23) and yet is exact in the general case. It follows naturally if the humidity correction is thought of as a correction to the instrument's reading:

$$N_{\mathrm{gas}}^C = \frac{N_X^{\mathrm{CDN}}(W/e)_{\mathrm{air}}}{(\bar{L}^c/\rho)_{\mathrm{air}}^{\mathrm{wall}}(\bar{\mu}_{\mathrm{en}}/\rho)_{\mathrm{wall}}^{\mathrm{air}}KK_{\mathrm{ion}}^c} \quad (\mathrm{Gy\ mu}^{-1}), \tag{31}$$

and

$$D_{\mathrm{med}}^{C,u} = M^u K_h^u N_{\mathrm{gas}}^C (\bar{L}^u/\rho)_{\mathrm{air}}^{\mathrm{med}} P_{\mathrm{repl}} K_{\mathrm{ion}}^u \quad (\mathrm{Gy}). \tag{32}$$

The need to correct the instrument's reading by the humidity correction factor in the user's beam is consistent with the approach needed to measure exposure using an NRCC-calibrated ion chamber [see Eq. (1)]. Equation (31) can equally well be written in terms of $N_X^{\mathrm{US}}$ by using Eq. (3).

The advantages of this formulation are that $N_{\mathrm{gas}}^C$ is a constant; the expression for $D_{\mathrm{med}}^{C,u}$ is exact both for humid air or a general gas in the ion chamber (although this is only a $\pm 0.15\%$ improvement over formulation *A*); the quantities are all with respect to air; and $N_{\mathrm{gas}}^C$ has a conceptually simple interpretation, it is the dose that would be delivered to dry air in the ion chamber per unit instrument reading, i.e.,

$$N_{\mathrm{gas}}^C = (W/e)_{\mathrm{air}}\ f/m_{\mathrm{air}} \quad (\mathrm{Gy\ mu}^{-1}). \tag{33}$$

Table I presents a summary of all the formulations of $N_{\mathrm{gas}}$.

## IV. $N_{\mathrm{gas}}$ FROM ABSORBED DOSE CALIBRATION FACTORS

NRCC and NBS both offer calibration services based on $^{60}$Co absorbed dose to water standards. Once again, the definitions of the absorbed dose calibration factors differ in the way the humidity correction factor is included. In a water phantom at the appropriate depth in a $^{60}$Co beam one has, in either the calibration laboratory or the user's beam:

$$D_{\rm water} = MK_h N_D^{\rm CDN}(K^u_{\rm ion}/K^c_{\rm ion}), \qquad (34)$$

or

$$D_{\rm water} = MN_D^{\rm US}(K^u_{\rm ion}/K^c_{\rm ion}), \qquad (35)$$

where here, unlike in Eqs. (1) and (3), we have made explicit the corrections required to account for changes in ion chamber collection efficiency (in the calibration beam $u \equiv c$ and hence there is no correction required). In order to get an exact answer in what follows, it is easiest to consider the Canadian formulation.

Consider an absorbed dose measurement in the user's $^{60}$Co beam under conditions similar to the calibration beam. We consider the user's beam because we are interested in determining $N_{{\rm gas},u}$ under the user's humidity conditions. From Eq. (24), and now including the $P_{\rm wall}$ correction factor (which was ignored earlier for simplicity, but which plays a more central role here), one has

$$D^u_{\rm water} = M^u N_{{\rm gas},u}\left(\frac{\bar{L}^u}{\rho}\right)^{\rm water}_{{\rm gas},u} P^u_{\rm repl} K^u_{\rm ion} P^u_{\rm wall}. \qquad (36)$$

Comparing this with Eq. (34) we find

$$N_{{\rm gas},u} = \frac{K^u_h N_D^{\rm CDN}}{K^c_{\rm ion} P^c_{\rm repl} P^c_{\rm wall} (\bar{L}^c/\rho)^{\rm water}_{{\rm gas},u}} \quad ({\rm Gy\ mu}^{-1}), \qquad (37)$$

where we have changed $u$ to $c$ in three places since $P^c_{\rm repl}$ and $P^c_{\rm wall}$ are those values which apply in a $^{60}$Co beam (which is both the calibration conditions and the user's conditions) and the stopping-power ratio is calculated for a $^{60}$Co spectrum although for the gas in the user's beam. Equivalently, in terms of $N_D^{\rm US}$ we have

$$N_{{\rm gas},u} = \frac{K^u_h}{K^c_h}\,\frac{N_D^{\rm US}}{K^c_{\rm ion} P^c_{\rm repl} P^c_{\rm wall} (\bar{L}^c/\rho)^{\rm water}_{{\rm gas},u}} \quad ({\rm Gy\ mu}^{-1}). \qquad (38)$$

Equation (8) of the AAPM protocol is equivalent to Eq. (38) except it does not include the ratio of humidity correction factors which introduce errors of up to ± 0.15% (as we saw before).

In the above derivation we had to introduce the construct of a $^{60}$Co irradiation in the user's laboratory because $N_{{\rm gas},u}$ is a function of the humidity. If we consider formulation $C$ above [Eqs. (31) and (32)], $N^C_{\rm gas}$ is a constant and hence we can derive $N^C_{\rm gas}$ by reference to the calibration setup. Using Eqs. (32) and (34) (now including $P_{\rm wall}$) in the calibration beam (i.e., with $u \equiv c$) we deduce

$$N^C_{\rm gas} = \frac{N_D^{\rm CDN}}{K^c_{\rm ion} P^c_{\rm repl} P^c_{\rm wall} (\bar{L}^c/\rho)^{\rm water}_{\rm air}} \quad ({\rm Gy\ mu}^{-1}). \qquad (39)$$

These values of $N_{{\rm gas},u}$ and $N^C_{\rm gas}$ are formally equivalent to those derived above in terms of exposure calibration factors.

On the assumption that the two calibration factors $N_X$ and $N_D$ are exactly correct, the comments in Secs. II and III imply that the current AAPM protocol will lead to dose estimates which differ by ~0.4% depending on whether the absorbed dose calibration factor is employed (which gives the dose correctly to within ± 0.15%) or the exposure calibration factor is used (in which case the dose is underestimated by 0.4%).

## V. OTHER CORRECTION FACTORS

Until now we have ignored the correction factors (denoted by $K$) which were rather arbitrarily included starting with Eq. (10). The revised AAPM protocol takes into account only $K_{\rm wall} = A^{-1}_{\rm wall}$ and $K_{\rm comp}$. There is no reason why $K_{\rm an}$ and $K_{\rm st}$, the corrections for axial nonuniformity (point of measurement) and stem scatter should not be included. Both are less than unity and both would increase $N_{\rm gas}$. The absorbed dose equations should also include corrections to account for the corresponding effects (if any) in the in-phantom measurements. There is some controversy over the $K_{\rm an}$ factor which is currently taken as unity in both the NRCC and NBS exposure standards[4,12] but is ~0.997 for the BIPM and ~0.993 for the PTB standards.[12] This factor is probably best left at unity if the exposure calibration factor is based on a standard using $K_{\rm an} = 1.00$. $K_{\rm st}$ is 0.996 for the rather large stemmed NRCC exposure chamber[4] and has been shown to be 0.9995 for a particular Farmer chamber.[19] Since these numbers are needed in the protocol for the user's ion chambers, it is clear that some further high-quality data would be useful.

The $K_{\rm el}$ factor corrects for an electrode which is not made of the same material as the chamber wall. This factor is somewhat more problematic. Its value is ≈0.992 for a 1-mm-diam solid aluminum electrode in a graphite chamber as has been shown both theoretically[14] and experimentally.[15] The value 0.992 for a 1-mm-diam electrode is based on a linear model for the magnitude of the effect vs electrode radius. The form of the correction is very similar to the $K_{\rm comp}$ and $P_{\rm wall}$ correction factors which account for composite wall materials, and in a proper formulation may be included in these correction factors once the fraction of the ionization due to the electrode is known in different situations. This linear model is expected to be valid once "full buildup" is achieved for the electrode. Experiment[15] and Monte Carlo calculation[14] confirm this. However, for small radii, this model may overestimate the effect (and, hence, $K_{\rm el}$ may be closer to 1.00) since "full buildup" has not been achieved. Unfortunately, both calculational and measurement uncertainties are too large in this region to provide a direct estimate of $K_{\rm el}$. Its value clearly should be included in Eq. (21) if the goal is to determine the mass of the air in the user's ion chamber. However, if the goal is to determine the dose in a water phantom irradiated by $^{60}$Co, it is probably more accurate to ignore the correction factor in determining $N_{\rm gas}$ because the same physical effect occurs in the phantom and it is not accounted for there—it is therefore more accurate to ignore it in both places. However, if for example, our goal is to measure the dose in a 20-MeV electron beam, the aluminum electrode has a much smaller, probably negligible, effect on the response in the electron beam and hence, $K_{\rm el}$ should be included in the determination of $N_{\rm gas}$. In a proper protocol, it should be accounted for in determining $N_{\rm gas}$ and an additional correction factor introduced in the dose equation to take it into account when needed.

## VI. CONCLUSIONS

A proper treatment of humidity corrections shows that the AAPM protocol underestimates the dose by about 0.4% when using exposure calibrated ion chambers. Care must be taken to distinguish between the NRCC and NBS calibration factors which differ in definition by the humidity correction factor (i.e., $N_X^{\rm US} = K_h N_X^{\rm CDN}$). In reformulating the protocol, we found it more consistent to treat the humidity correction factor as a correction to the instrument reading similar to those for temperature and pressure. We suggest that formulation $C$ [Eqs. (31), (32), and (39)] be adopted as the basis for any future dosimetry protocols. It can easily be rewritten in terms of $N_X^{\rm US}$, and it leads to an exact formulation of the dose specification, with a constant and physically meaningful value of $N_{\rm gas}$ and with all the required physical data being for dry air which are what is available in tables. Although formulation $C$ also holds for chambers filled with gases other than humid air, formulation $B$ is exact and a more natural formulation in this case. Our analysis also demonstrates that formulation $A$, which ignores variations in $K_h$, introduces errors of only $\pm 0.15\%$, which is certainly acceptable for any clinical applications.

This analysis of the humidity correction factor has assumed that the humidity in the ion chamber is the same as that in the laboratory although this assumption may not be accurate for an ion chamber in a water phantom. In the worst case (ion chamber at 100% humidity in a room with 30% humidity), this erroneous assumption leads to a 0.2% underestimate of the dose if we assume $W/e$ behaves smoothly up to 100% humidity [see Fig. (B1)]. This analysis has also ignored the effects of humidity on chamber volume although this can have a significant effect for chambers with nylon and A-150 walls (see Mijnheer[20] and references therein).

The AAPM protocol ignores several other correction factors which may lead to a further underestimate of the dose by about 1% in a bad case (e.g., using a Farmer chamber with an aluminum electrode to measure the dose in a 20-MeV electron beam where the stem and point of measurement corrections are assumed to be similar to those of the NRCC exposure chamber). Further theoretical and experimental work on these effects is clearly justified.

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## APPENDIX A: NOTATION

| | |
|---|---|
| superscript $c$ | Implies quantity refers to conditions which prevailed when the user's chamber was calibrated at the standards laboratory. |
| superscript $u$ | Implies quantity refers to conditions which prevailed when the user's chamber was placed in the user's field. |
| air | Always refers to *dry* air. |
| $A_{\rm ion}$ | The AAPM protocol's notation for $(K_{\rm ion}^c)^{-1}$. |
| $f$ | An electrical calibration factor in C/mu to convert the instrument's output in meter units into coulombs. This factor is not used explicitly in the protocol because it is buried in the exposure calibration factor $N_x$. It is nonetheless an essential element in the development of the protocol. |
| $(K_c)_{\rm material}$ | Collision kerma in the material (J/kg). |
| $K_h$ | Humidity correction factor [see Eq. (12)]. The product of this factor and the charge liberated in the cavity gas is equal to the charge that would be liberated if the cavity were filled with the dry air. |
| $K_{\rm ion}$ | A correction factor which accounts for imperfect collection of charge: $Q_{\rm gas} = M_a K_{\rm ion}$. This factor depends on the beam strength and must be determined for each set of conditions in which the chamber is used (in the AAPM protocol's notation $P_{\rm ion} = K_{\rm ion}^u$ and $A_{\rm ion} = (K_{\rm ion}^c)^{-1}$). |
| $K_{\rm wall}$ | A correction to account for attenuation and scatter in the walls of the ion chamber (it includes $\beta_{\rm wall}$): $K_{\rm wall} = A_{\rm wall}^{-1}$ as defined in the revised AAPM protocol.[7] |
| $\left(\frac{\bar{L}^a}{\rho}\right)_{\rm mat2}^{\rm mat1}$ | The ratio of the spectrum averaged restricted mass stopping power of mat1 to that of mat2, where the electron spectrum is that generated in beam $a$ (either $c$ or $u$). Note, if either material is labeled gas, one *must* specify what gas—e.g., gas,$c$, the gas when the calibration in field $c$ was done. |
| $M$ | The user's instrument reading in meter units (mu). The reading is assumed to be corrected for any leakage and polarity effects, and then normalized to a standard temperature and pressure (22 °C, 101.3 kPa). |
| $M_a$ | The charge in coulombs output by the ion chamber $M_a = Mf = Q_{\rm gas}(K_{\rm ion})^{-1}$. |
| $m_{\rm gas}$ | The mass of gas in kg in the ion chamber (usually at a specified reference temperature and pressure). |
| $N_X$ | Exposure calibration factor provided by the standards laboratory in C kg$^{-1}$ mu$^{-1}$, where mu is whatever units the instrument's meter presents. |
| $P_{\rm ion}$ | The AAPM protocol's notation for $K_{\rm ion}^u$. |
| $Q_{\rm gas}$ | The charge actually liberated in the gas (in C). Incomplete ion collection efficiency will reduce the measured charge, and an absolute electrical calibration of the user's electrometer is necessary to measure $Q_{\rm gas}$ ($Q_{\rm gas} = MfK_{\rm ion}$). |
| $\left(\frac{W}{e}\right)_{\rm gas}$ | Mean energy expended per coulomb of charge liberated in a gas by a slowing electron. It is a function of humidity for moist air (see Appendix B). For dry air, $W/e = 33.97$ J/C.[7,18] |
| $\left(\frac{W}{e}\right)_{\rm gas,u}^{\rm gas,c}$ | The ratio of the value of $W/e$ for the gas in the ion chamber when in the calibration beam to |

that when in the user's beam. *W*/*e* is assumed independent of the electron spectrum in the two beams but it will depend on the gas in the cavity.

$\left(\frac{\bar{\mu}_{en}}{\rho}\right)^{\text{mat1}}_{\text{mat2}}$ The ratio of the mass energy absorption coefficient averaged over the photon energy fluence spectrum for matl to that of mat2.

## APPENDIX B: EFFECTS OF HUMIDITY ON BASIC QUANTITIES

Considerable effort has gone into studying the effects of humidity on the determination of exposure with primary standards.[1-3,5] Here we will summarize some of these standard results and present a few auxiliary formulas and data which are referred to in the rest of the paper.

The molecular weights of air and water are 28.964 and 18.015 (based on data in Ref. 17). Starting from the ideal gas law which states that the number of molecules of gas in a volume at a given temperature and pressure is fixed, one gets for the ratio of the density of humid air to dry air:

$$\frac{\rho_{\text{humid}}}{\rho_{\text{dry}}} = 1 - \frac{P_{H_2O}}{P}(1-d), \qquad \text{(B1)}$$

where *d*, the ratio of molecular weights is given by

$$d = \frac{M_{H_2O}}{M_{\text{air}}} = \frac{18.015}{28.964} = 0.622,$$

and $P_{H_2O}$ and *P* are the partial pressure of the water vapor and the total pressure, respectively. As we see from Table B1, at 22 °C the density of air varies by 1% going from dry to saturated conditions.

Now consider the stopping power of humid air. In general one determines the collision stopping power for a mixture using the Bragg additivity rule:

$$\left(\frac{S_{\text{col}}}{\rho}\right)_{\text{mixture}} = \sum_i w_i \left(\frac{S_{\text{col}}}{\rho}\right)_i, \qquad \text{(B2)}$$

where $w_i$ is the fraction by weight of constituent *i*. This equation will also hold for restricted collision stopping powers, $(L/\rho)$. The quantity needed is $(L/\rho)_{\text{humid air}}/(L/\rho)_{\text{dry air}}$ which is written $(L/\rho)^{\text{humid}}_{\text{air}}$. Note that here and elsewhere *air* signifies dry air. Using the Bragg rule and the ideal gas law [which gives $w_{H_2O\,\text{vapor}}/w_{\text{air}} = d(P_{H_2O}/P_{\text{air}})$ where *d* has the same meaning as above], one gets

$$\left(\frac{L}{\rho}\right)^{\text{humid}}_{\text{air}} = \frac{1 - \frac{P_{H_2O}}{P}\left[1 - d\left(\frac{L}{\rho}\right)^{H_2O\,\text{vapor}}_{\text{air}}\right]}{\left(1 - \frac{P_{H_2O}}{P}(1-d)\right)}. \qquad \text{(B3)}$$

Since tables of unrestricted collision stopping powers are available,[17] we have used the fact that $(L/\rho)^{H_2O\,\text{vapor}}_{\text{air}} = (S_{\text{col}}/\rho)^{H_2O\,\text{vapor}}_{\text{air}}$ in evaluating Eq. (B3) for Table B1. Note that Eq. (B3) uses the stopping-power ratio for water vapor to air, not water to air. This is an important distinction. The water to air stopping-power ratio changes considerably as a function of electron energy primarily because the density effect affects the water stopping power but not that for air. For the case of water vapor the density effect does not play a role and hence the value of $(L/\rho)^{H_2O\,\text{vapor}}_{\text{air}}$ only changes from 1.137 at 200 keV to 1.126 at 20 MeV. This 1% change has no significant influcence on $(L/\rho)^{\text{humid}}_{\text{air}}$. Furthermore, it means Eq. (B3) can also be applied to spectrum averaged stopping-power ratios.

Consider now the ratio of linear stopping powers. By definition:

$$(\bar{L})^{\text{humid}}_{\text{air}} = \frac{\rho_{\text{humid}}}{\rho_{\text{air}}}\left(\frac{L}{\rho}\right)^{\text{humid}}_{\text{air}}. \qquad \text{(B4)}$$

From this definition, and using Eqs. (B1) and (B3) one gets

$$(\bar{L})^{\text{humid}}_{\text{air}} = 1 - \frac{P_{H_2O}}{P}\left[1 - d\left(\frac{L}{\rho}\right)^{H_2O\,\text{vapor}}_{\text{air}}\right]. \qquad \text{(B5)}$$

This is the same equation developed by Niatel[1,2] and Henry.[3]

TABLE B1. Variation of several physical parameters as a function of the fractional partial pressure of water vapor in the air.

| $\frac{P_{H_2O}}{P}$ | Relative humidity at 22 °C, 1 atm | $\left(\frac{W}{e}\right)^{\text{humid}}_{\text{air}}$ (note a) | $\frac{\rho_{\text{humid}}}{\rho_{\text{air}}}$ | $\left(\frac{L}{\rho}\right)^{\text{humid}}_{\text{air}}$ for $(L/\rho)^{H_2O\,\text{vapor}}_{\text{air}} = 1.136$[b] | $(\bar{L})^{\text{air}}_{\text{humid}}$ | $K_h = \left(\frac{W}{e}\right)^{\text{humid}}_{\text{air}}(\bar{L})^{\text{air}}_{\text{humid}}$ |
|---|---|---|---|---|---|---|
| 0.0 | 0% | 1.0[c] | 1.00 | 1.00 | 1.00 | 1.00 |
| 0.0026 | 10% | 0.9966 | 0.9990 | 1.0002 | 1.0008 | 0.9974 |
| 0.0052 | 20% | 0.9955 | 0.9980 | 1.0004<br>(1.0004) | 1.0016 | 0.9971 |
| 0.0078 | 30% | 0.9945 | 0.9971 | 1.0007 | 1.0022 | 0.9967 |
| 0.0104 | 40% | 0.9938 | 0.9961 | 1.0009 | 1.0030 | 0.9968 |
| 0.0130 | 50% | 0.9933 | 0.9951 | 1.0011<br>(1.0010) | 1.0038 | 0.9971 |
| 0.0157 | 60% | 0.9928 | 0.9941 | 1.0013 | 1.0046 | 0.9974 |
| 0.0183 | 70% | 0.9924 | 0.9931 | 1.0016 | 1.0053 | 0.9977 |
| 0.0209 | 80% | 0.9921 | 0.9921 | 1.0018 | 1.0062 | 0.9982 |
| 0.0235 | 90% | 0.9916 | 0.9911 | 1.0020<br>(1.0018) | 1.0070 | 0.9985 |
| 0.0261 | 100% | ... | 0.9901 | 1.0022 | 1.0078 | ... |

[a] References 1 and 5.

[b] Corresponds to $E_e = 300$ keV. Values for 20 MeV $[(L/\rho)^{H_2O\,\text{vapor}}_{\text{air}} = 1.126]$ are given in parentheses.

[c] Current best estimate is $(W/e)_{\text{air}} = 33.97$ J/C (Ref. 18).

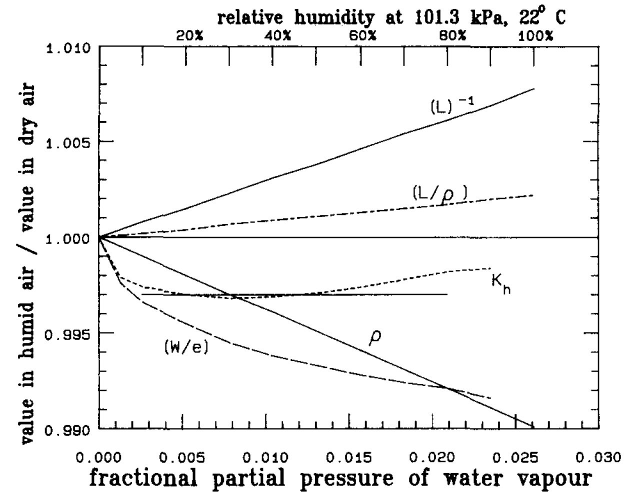

FIG. B1. Variation of various physical quantities and the humidity correction factor as a function of the partial pressure of water vapor in air.

The factor $(\bar{L})^{\text{air}}_{\text{humid}}$ is included in Table B1 which also includes $(W/e)^{\text{humid}}_{\text{air}}$ as measured by Niatel[1] and presented in ICRU 31.[5] The standard humidity correction factor which is used throughout the paper [Eq. (12)], is also presented in Table B1. Its constancy over the range of 10% to 70% humidity is very useful.

Figure B1 shows graphically the values in Table B1.

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