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Re-evaluation of the total stopping power of polystyrene for 5.3 MeV electrons

D W O Rogers and B A Faddegon

Ionizing Radiation Standards, Institute for National Measurement Standards, National Research Council Canada, Ottawa, Canada K1A 0R6

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Abstract. The experiment of Feist and Müller to measure the stopping power of polystyrene for 5.3 MeV electrons passing through slabs is re-analysed using a formalism presented here and correction factors obtained from Monte Carlo calculations. The more careful analysis changes the experimentally determined stopping power by 1.7% from 1.913 to 1.882 MeV g^{-1} cm^2 at 5.37 MeV. Appropriate statistical methods show that the uncertainty on the derived stopping power due to the stated uncertainties on the measurements is increased from Feist and Müller's value of 1.2 to 5% if the same data are analysed or 3% if the initial data point is included in the analysis after corrections are made for secondary electrons. Systematic uncertainties in the calculated correction factors lead to an uncertainty in the stopping power of 0.9%. Correction factors are provided for measurements with slabs of thickness up to 1.0 g cm^{-2}. Using two additional measurements for thicker slabs with measurement uncertainties similar to the previous ones, the statistical uncertainty due to the measurement can be reduced to less than 1%. It is shown that in general the average pathlength traversed by electrons passing through a slab of material is not given correctly by the PRESTA pathlength correction nor by the Fermi–Yang correction for thicker slabs. The most reliable way to calculate the average pathlength is to use the Monte Carlo technique.

1. Introduction

Electron stopping powers play a central and critical role in radiation dosimetry. Uncertainties in stopping powers translate almost directly into uncertainty in stopping-power ratios and hence into uncertainty in almost all measurements made with ion chambers, including those of primary standards. The values used in practical dosimetry have been based on theoretical calculations and have been summarized by ICRU Report 37 (1984). Unfortunately, there are few experimental measurements of stopping powers with a precision even approaching the theoretical uncertainties of 2 to 3% in the energy range of interest in radiation dosimetry (i.e. 10 keV to 50 MeV). The most precise measurement reported to date is that of Feist and Müller (1989) who determined the total stopping power of polystyrene for 5.3 MeV electrons by measuring the energy absorbed in a ferrous sulphate solution placed behind polystyrene slabs of various thicknesses and irradiated by an electron beam with a well known energy and total charge. They reported a measurement uncertainty of 1.2% (one standard deviation) and their measured value agreed with the calculated value of ICRU Report 37 (1984) to well within this uncertainty. A measurement with this accuracy represents a significant improvement in our knowledge of electron stopping powers.

As part of an NRC project to measure electron stopping powers (Faddegon *et al* 1991) we have re-analysed the results of Feist and Müller. We have found that changes in the measured absorbed energy of only 0.2% translate into a 3% change in the measured stopping power because the experiment is only measuring a 6% difference in energy loss in the slabs. Seeing such large changes in the fitted result caused by such small corrections led to a re-analysis of all the mechanisms of energy loss and of the uncertainties. By making use of detailed Monte Carlo calculations the data analysis has been refined to give a more accurate estimate of the stopping power. The uncertainty estimates inherent in the fitting of the data were found to be significantly larger than those obtained by Feist and Müller who ignored the measurement uncertainties in their procedure.

After a brief description of Feist and Müller's experiment and the Monte Carlo calculations used here, each of the corrections is discussed in turn as well as an evaluation of the statistical analysis.

2. The measurement of electron stopping powers

2.1. Feist and Müller's experiment

Feist and Müller's measurement of stopping power was based on the Physikalisch-Technische Bundesanstalt (PTB) high-precision Fricke dosimetry system which was used to measure the total energy of electrons after passing through polystyrene slabs ranging in thickness from 0.1 to 0.26 g cm^{-2}. The experimental system is shown in figure 1. By making use of the well established total charge and energy of the mono-energetic accelerator beam, Feist and Müller were able to calibrate the response of their Fricke dosimeter in terms of the energy of the electrons entering it. It was assumed that the calibration did not change as a result of the small changes in the electron spectrum caused by the passage through the slabs.

The thickness of the polystyrene slabs was determined by weight and area measurements and the effective pathlength of the electrons in the slab was calculated taking into account their multiple scattering. To do this Feist and Müller used one half of the Yang (1951) pathlength correction (PLC) based on observations by Hebbard and Wilson (1955) that the Yang correction tended to be high by a factor of about two and a more recent confirmation that the Yang correction was too large (Bielajew and Rogers 1987). Feist and Müller's PLCs ranged from 0.38 to 0.9%.

The uncertainty in the absorbance of the irradiated Fricke solution per unit incident charge from the accelerator beam was given as 0.2% (one standard deviation) which translates directly into the uncertainty in the energy absorbed by the solution per unit incident charge. A precision this high has been achieved at only a few laboratories. The approach used by Feist and Müller to extract the electron stopping powers is described in the following section.

2.2. A formalism for analysing energy-loss data

Consider a beam of electrons passing through a uniform slab of material of thickness t_0 (in g cm^{-2}). The basic relationship used to determine stopping powers is

$$\left(\frac{S}{\rho}\right)_{\text{tot}} = \frac{\Delta E_{\text{prim}}}{t} \tag{1}$$

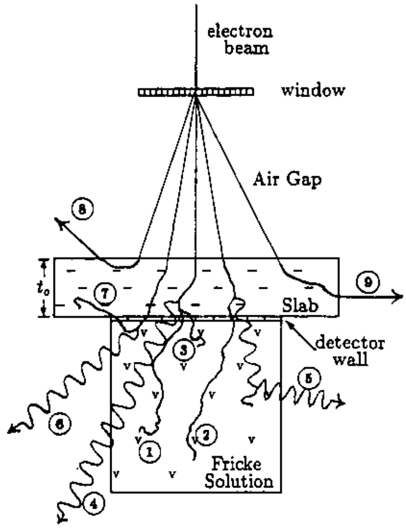

Figure 1. The elements of the Feist and Müller experiment (not to scale). The accelerator beam has an energy of 5.651 MeV and energy width of 25 keV. The accelerator exit window is a 0.005 cm thick piece of mylar (Hostaphan) which is followed by a 9 cm air gap. Polystyrene slabs ranging in thickness from 0.1 to 0.26 g cm^{-2} were placed directly in front of the Fricke detector which was 3 cm in diameter, 3 cm thick and had a 0.005 cm thick polyethylene front wall. Feist and Müller report the average energy of primary electrons incident on the slab as 5.624 MeV. Electrons 1 and 2 lose energy passing through the slab and deposit all of their residual energy in the Fricke solution (this represents 97% of the events). Electron 3 is a secondary electron generated in the slab but which transfers some energy to the Fricke detector. The change in this process with slab thickness is accounted for by the correction $k_{sec}(t)$. Photons 4 and 5 are bremsstrahlung photons created in the slabs, most of which pass through the Fricke detector (4) and some of which deposit some energy there (5). These events are corrected for by the $k_{br}k_{bf}$ factors. Photon 6 represents energy loss from the Fricke solution by transmitted leakage, most of which is from bremsstrahlung generated in the solution. Electron 7 represents backscattered electrons. Both energy losses from the detector are accounted for in the calibration factor K and the changes in these losses with slab thickness are accounted for in $k_\epsilon(t)$. Electron 8 is backscattered from the slabs and electron 9 stops in the slab. The change with thickness of these processes is accounted for by $k_{loss}(t)$.

where $(S/\rho)_{tot}$ is the total electron stopping power (in MeV g^{-1} cm^2), ΔE_{prim} is the average energy lost (in MeV) by a primary electron as it passes through the slab and t, the effective thickness of the slab (in g cm^{-2}), is the average pathlength of the primary electrons as they traverse the slab. If the average energy of primary electrons after traversing a slab of effective thickness t is given by $E_{prim}(t)$, then

$$\Delta E_{prim}(t) = E_0 - E_{prim}(t) \tag{2}$$

where E_0 is the average energy of primary electrons incident on the slab. Hence:

$$\left(\frac{S}{\rho}\right)_{tot} t = E_0 - E_{prim}(t). \tag{3}$$

If $E_{prim}(t)$ is measured for a variety of t values, $(S/\rho)_{tot}$ is given by the slope of $E_0 - E_{prim}(t)$ plotted against t. Stopping powers change with electron energy but since they vary slowly and in a linear fashion with energy and pathlength, the stopping power given by (1) is for the mid-point energy in the slab. In (3), although $(S/\rho)_{tot}$ is a function of t, when fitting the slope to data for several thicknesses as in this experiment, the value obtained is the stopping power at the mid-point energy of the thickest slab included in the fit.

In their experiment, Feist and Müller measured $A(t)$, the absorbance of the irradiated Fricke solution immediately behind a slab of thickness t, per accelerator electron. From this measured value one needs to determine the average energy per transmitted primary electron for use in (3). As a first step, define a calibration factor $k(t)$ such that

$$k(t)A(t) = E_{tr}(t) \tag{4}$$

where $E_{tr}(t)$ is the electron energy transmitted through the slab per transmitted primary electron plus the electron energy absorbed in the Fricke solution by bremsstrahlung photons generated in the slab. The detection efficiency for bremsstrahlung photons is handled separately later. This somewhat complex quantity, E_{tr}, is introduced to allow a rigorous formalism to be developed. The factor $k(t)$ has several components. Consider the case of the electron beam incident directly on the detector. In this case

$$KA(0) = E_{tr}(0) = E_{inc} \tag{5}$$

where E_{inc} is the total electron energy incident on the detector per primary electron incident on the detector and K converts the absorbance per accelerated electron into the electron energy incident on the detector per primary electron incident on the detector. In essence K converts the absorbance to energy units. It also accounts for the energy losses from the detector which amount to a little more than 2% in this case and it accounts for accelerator electrons which are scattered away by the exit window and air (a 0.15% effect in this case). E_{inc} is very slightly (0.02%) *greater* than E_0 because of air and exit window generated secondary electrons which enter the Fricke detector. E_0 is about 0.6% less than the accelerator energy because of energy losses in the exit window and air†. Feist and Müller determined K using (5) with $E_{inc} = E_0$ and then assumed $k(t) = K$.

However, in general there are several other effects which need to be taken into account. As slabs of different thickness are placed on the front of the detector, the detector's efficiency (defined as the energy absorbed in the detector divided by E_{tr}) may change because of the differences in electron energy and angular distributions incident on the detector. This can lead to different bremsstrahlung leakage from the back and sides of the detector and different backscattering from the front. A factor $k_\epsilon(t)$, defined as the detector's efficiency with no slab present divided by its efficiency with a slab thickness t in place, corrects for these changes (note that $k_\epsilon(0) = 1.0$ by definition, and if the detector's efficiency increases, k_ϵ decreases). At the same time as accounting for the detector's efficiency, the overall calibration $k(t)$ converts from absorbance per accelerator electron to an energy per transmitted primary electron (4) and thus must account for any primary electrons that fail to reach the detector. A factor $k_{loss}(t)$, defined as the ratio of primary electrons incident on the slab to primaries transmitted by the slab, accounts for these losses of primary electrons (note

† Feist and Müller give 0.5% for this number, but the difference is inconsequential.

that $k_{\rm loss}(0) = 1.0$ by definition as well and it is assumed that the loss of electrons due to scattering in the air and exit window remains constant and is accounted for in K). To summarize,

$$k(t) = K k_\epsilon(t) k_{\rm loss}(t). \tag{6}$$

In their paper Feist and Müller assumed $k_\epsilon(t) \cdot k_{\rm loss}(t) \equiv 1$. For the range of thicknesses that they considered, this approximation is accurate (shown later).

Having established how to determine $E_{\rm tr}(t)$ using (4) and (6), one must determine how to make use of it in (3) to determine the stopping power. From the definition of $E_{\rm tr}$, for particles leaving the back of the slab

$$E_{\rm tr}(t) = E_{\rm prim}(t) + E_{\rm sec}(t) + k_{\rm bf} E_{\rm brem}(t) \tag{7}$$

where $E_{\rm prim}(t)$ is the average energy of transmitted primary electrons (as needed for (3)); $E_{\rm sec}(t)$ is the average energy of transmitted secondary electrons per transmitted primary electron; $E_{\rm brem}(t)$ is the average energy, per transmitted primary electron, of bremsstrahlung photons which were generated in the slab; and $k_{\rm bf}$ is the fraction of the bremsstrahlung energy which is absorbed in the detector. As shown later, the last term is small ($\approx 0.4\%$) and in practice $k_{\rm bf}$ is not a function of t.

Feist and Müller recognized the existence of secondary electron effects but assumed that the thinnest slab was thick enough to create secondary charged particle equilibrium and they assumed that $E_{\rm sec}(t)$ was a constant, i.e. (7) becomes (ignoring the bremsstrahlung terms)

$$E_{\rm tr}(t) = E_{\rm prim}(t) + E_{\rm sec}. \tag{8}$$

Using this equation in conjunction with (3) for $(S/\rho)_{\rm tot}$ and substituting (4) and (6) for $E_{\rm tr}(t)$, and using Feist's and Müller's assumption that $k_\epsilon(t) k_{\rm loss}(t) = 1.0$, leads to

$$A(t) = \frac{E_0 + E_{\rm sec}}{K} - \frac{1}{K}\left(\frac{S}{\rho}\right)_{\rm o} t \tag{9}$$

which holds only for t greater than the buildup thickness and the subscript o on the stopping power indicates it was determined ignoring bremsstrahlung effects. This is the equation used by Feist and Müller to determine $(S/\rho)_{\rm o}$ from the slope of the line fit to $A(t)$ against t with K given by $E_0/A(0)$ ((5) with $E_{\rm inc} = E_0$). Note that the intercept of this line is greater than $A(0) (= E_0/K)$ by $E_{\rm sec}/K$.

However, as shown below, $E_{\rm sec}(t)$ is not a constant and thus (7) for $E_{\rm tr}(t)$ should be rewritten

$$E_{\rm prim}(t) = [E_{\rm tr}(t) - k_{\rm bf} E_{\rm brem}(t)] k_{\rm sec}(t) \tag{10}$$

where

$$k_{\rm sec}(t) = \frac{E_{\rm prim}(t)}{E_{\rm prim}(t) + E_{\rm sec}(t)} \tag{11}$$

is a correction to account for secondary electrons exiting from the back of the slabs along with the primary transmitted electrons.

Consider first the no bremsstrahlung case (i.e. $k_{bf}E_{brem}(t) = 0.0$). Again starting from (3) for $(S/\rho)_{tot}$ and (10) gives

$$\left(\frac{S}{\rho}\right)_o t = E_0 - E_{tr}(t)k_{sec}(t). \tag{12}$$

Substitution of (4) and (6) leads to

$$k_{loss}(t)k_\epsilon(t)k_{sec}(t)A(t) = \frac{E_0}{K} - \frac{1}{K}\left(\frac{S}{\rho}\right)_o t. \tag{13}$$

As before, the stopping power is $-K$ times the slope. But in this case the absorbance data are subjected to a series of correction factors and the absorbance data with no slab in the beam may also be included in the fit. The calibration constant K is given by $E_0/(k_{sec}(t \to 0)A_o)$ where A_o is the fitted absorbance for the no slab case (i.e. $t = 0.0$). This ratio is very close to the value of $E_0/A(0)$.

To this point bremsstrahlung production has been neglected. Since the electrons only lose a small portion of their energy in the slab, the radiative yield does not change significantly and the slabs are thin enough that reabsorption of bremsstrahlung in the slab is negligible. Hence we can write

$$E_{brem}(t) = k_{br}\Delta E_{prim} \tag{14}$$

i.e. the average bremsstrahlung energy from the back of the slab per transmitted primary electron is proportional to the energy lost by the primary electron in the slab. Feist and Müller took

$$k_{br} = \left(\frac{S}{\rho}\right)_{rad} \Big/ \left(\frac{S}{\rho}\right)_{tot} \tag{15}$$

which is confirmed as a good approximation later.

Again using (3) for $(S/\rho)_{tot}$ and substituting (10) for $E_{prim}(t)$ and (14) for $E_{brem}(t)$ gives

$$\left(\frac{S}{\rho}\right)_{tot} t = E_0 - E_{tr}(t)k_{sec}(t) + k_{bf}k_{br}t\left(\frac{S}{\rho}\right)_{tot} k_{sec}(t). \tag{16}$$

Here $(S/\rho)_{tot}$ is the stopping power evaluated when bremsstrahlung production is taken into account. The first two terms on the right are just t times $(S/\rho)_o$, the stopping power in (12) evaluated ignoring bremsstrahlung production. Substituting and rearranging gives:

$$\left(\frac{S}{\rho}\right)_{tot} = \left(\frac{S}{\rho}\right)_o \frac{1}{(1 - k_{bf}k_{br})} \tag{18}$$

where $k_{sec}(t)$, which is within 0.4% of unity, has been taken as 1.0 in the already very small bremsstrahlung correction term. This equation allows the correct stopping power to be determined by finding $(S/\rho)_o$ using (12) with no corrections for bremsstrahlung production and then making a correction after the fact. This same technique was used by Feist and Müller. Note that if the detector was not sensitive to bremsstrahlung at all, then $k_{bf} = 0.0$ and $(S/\rho)_o$ is the total stopping power. However, if the detector were 100% efficient at detecting the bremsstrahlung (i.e. $k_{bf} = 1.0$), then under the approximation found in (15), the experimentally measured $(S/\rho)_o$ would be the collision stopping power.

With this formalism in place, the Feist and Müller experiment can be re-analysed using the Monte Carlo calculations described in the following section.

3. Monte Carlo calculations

A series of Monte Carlo calculations have been done to analyse this experiment. The EGS4 code system for simulating electron and photon transport was used (Nelson *et al* 1985, Nelson and Rogers 1989). The electron stopping powers used were those of ICRU Report 37 (Duane *et al* 1989).

The EGS4 user-code MSTEST was used to score the number and energy of photons and of primary and secondary electrons with energies greater than 10 keV which either emerged or were backscattered from slabs of polystyrene with thicknesses corresponding to those used in the experiment. At the same time the total pathlength of each primary electron passing through the slab was scored by adding up the straight line paths of the electrons in each step. Calculations were done with very short electron step lengths (less than 0.5% energy loss by continuous processes per step, i.e. ESTEPE = 0.5%) to ensure that the proper curved path of the electron was simulated in detail. A small correction ($<0.1\%$) is made for the fact that the electron paths are very slightly curved, even in these short steps.

Several hundred thousand electron histories were tracked in each case to obtain a statistical precision on the energy lost traversing a slab *at least* as good as 0.2%. The code MSTEST was used to calculate t, $k_{\rm sec}$ and $k_{\rm loss}$ assuming mono-energetic parallel beams of 5.6 MeV electrons.

The user-code DOSRZ was used to score the energy deposition in the Fricke solution for the various experimental situations. The simulation took into account the exit window of the accelerator (0.005 cm of Hostaphan, i.e. mylar), 9 cm of air, the polystyrene slabs, the 0.005 cm thick polyethylene front wall of the Fricke solution's container and the Fricke solution. The PRESTA algorithm (Bielajew and Rogers 1987) was also used to speed up the calculation since 10^6 incident electron histories were followed in each of several configurations. Electron and photon histories were tracked down to energies of 10 and 1 keV respectively. From these calculations, the factors $k_\epsilon(t)$ and $k_{\rm bf}(t)$ were determined.

4. Effective slab thickness

Table 1 presents four estimates of the ratio t/t_0 where t is the effective thickness or mean pathlength of the primary electrons which pass through the slab, and t_0 is the thickness of the slab:

(i) those from the Monte Carlo code MSTEST;
(ii) those presented by Feist and Müller;
(iii) those predicted by the PLC algorithm which is included in the PRESTA algorithm (Bielajew and Rogers 1987); and
(iv) those given by the Fermi–Yang algorithm used in the default version of EGS4.

In the latter two analytic calculations, the mid-point energy in the slab was used. For thicknesses up to 0.27 g cm^{-2}, the combination of systematic and random uncertainty in the Monte Carlo calculated values is estimated to be $\pm 0.1\%$ which represents an uncertainty of between 5 and 10% on the extra pathlength being calculated.

The results indicate that the values used by Feist and Müller are considerably lower than the correct values from the Monte Carlo calculations although their values are roughly one-half of the Yang correction as they asserted.

Table 1. The ratio t/t_0, of the effective mean pathlength of primary electrons to the slab thickness for 5.6 MeV electrons passing through slabs of polystyrene as calculated: (i) using the Monte Carlo code MSTEST; (ii) by Feist and Müller; (iii) using the PRESTA PLC algorithm evaluated at the mid-point energy in the slab (Bielajew and Rogers 1987) or (iv) using the Fermi–Yang algorithm used in the default version of EGS4, also at the mid-step energy.

t_0	t/t_0			
Thickness g cm^{-2}	Monte Carlo[a]	Feist and Müller	PRESTA	Fermi–Yang
0.1057	1.0074	1.0038	1.0053	1.008
0.1580	1.0114	1.0057	1.0082	1.012
0.2122	1.0157	1.0071	1.0112	1.016
0.2652	1.0200	1.0090	1.0143	1.021
0.53	1.043	—	1.032	1.048
1.06	1.093	—	1.083	1.140

[a] Estimated one standard deviation uncertainty (systematic plus random) of 0.1% for thicknesses used by Feist and Müller.

For the thinner slabs, these results suggest that the Fermi–Yang predictions are more accurate than the PRESTA results. This agreement for the thin slabs may be fortuitous. For the thickest slab the PRESTA PLC is considerably more accurate. The PRESTA PLC starts from the same definitions as the Fermi–Yang PLC but does not introduce a small angle approximation and thus, in principle, includes more accurate physics. The PRESTA PLC has been shown to give more accurate results in Monte Carlo calculations (Bielajew and Rogers 1987). However, PLCs used in Monte Carlo calculations are not applicable in the thin slab case because the Monte Carlo PLC usually addresses the question: 'Given a particular curved pathlength, what is the average straight pathlength of the electron in the original direction?'. In the situation for a thin slab, the straight pathlength is known and the mean curved pathlength is sought. Although the functions involved can be inverted mathematically, the present results show that this is not physically meaningful, especially for the PRESTA PLC in which large angle scattering in a homogeneous medium is taken into account†. The current results demonstrate that the best way to calculate the effective pathlength of electrons passing through a slab is by using Monte Carlo calculations.

The Monte Carlo calculated values for the effective thickness imply that Feist and Müller systematically underestimated the effective pathlength of the electrons in the slabs and thus overestimated the electron stopping power. As shown in figure 2, using the correct pathlengths appears to have a minor effect, but re-fitting the data using the Monte Carlo calculated values of t implies the measured stopping power is reduced by 1.6% because the effective thickness of the three slabs being analysed changes by this amount.

5. Effects of transport of secondary electrons

As the electron beam passes through the slabs of polystyrene it sets in motion some secondary knock-on electrons. The energy absorbed behind these slabs includes the

† We are indebted to our colleague Alex Bielajew for pointing this out.

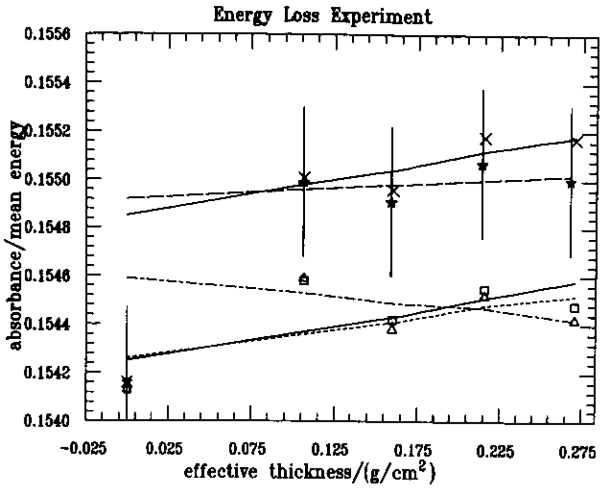

Figure 2. The corrected absorbance per unit accelerator charge divided by a nominal mean energy of primary electrons after passing through a slab of that effective thickness. The nominal mean energy is calculated assuming the electron stopping power is 1.915 MeV g^{-1} cm^2. This plot allows the effects of small variations to be seen. If the *ICRU Report 37* stopping power is correct, the corrected data should fall close to a horizontal line. The stars are the original data of Feist and Müller with the experimental uncertainties of 0.2% shown and their fit is shown by the long broken line. The crosses denote the values using the Monte Carlo calculated effective thickness and the fit is shown as a full line. The values change relative to the Feist and Müller data because of the mean energy implied by the different effective thickness. The fitted values project back above the no slab result (lower left) because of secondary electrons generated in the slab which enter the Fricke solution for all but the first point. The chain line is the fit to the four data points corrected for secondary electron (triangles) whereas the short-broken line is a fit to the same data but including the no slab datum point. The full line represents a fit to the data (boxes) corrected for secondary electrons and $k_{loss}(t)k_\epsilon(t)$. All the data have the same measurement uncertainties as the stars.

energy from these electrons whereas the measurement in the initial beam does not include this component. Thus the initial data point appears low relative to the others (see figure 2). Feist and Müller assumed that E_{sec}, the energy from secondary electrons, was a constant behind the various slabs of polystyrene in order to analyse their data using the equivalent of (9).

Table 2 presents the results of the Monte Carlo calculations where the energy from secondary electrons emerging from each slab has been scored separately and reported as the correction factor, $k_{sec}(t)$ given by (11) as the ratio of the energy of the primary electrons emerging from the slab to the total energy of emerging electrons. It was verified that the choice of low-energy threshold between 10 and 100 keV has no effect on the calculated value of k_{sec}. The statistical uncertainty on the correction factor is 0.01% which represents an uncertainty of 1 to 2% on the value of E_{sec}. A conservative estimate of the systematic uncertainty is somewhat larger, say 0.05%. Contrary to the assumption of Feist and Müller, the table shows that the secondary electron component is not a constant. The region of build-up of secondary electrons could, in principle, be as thick as 1.5 g cm^{-2} since this is the range in polystyrene of the maximum energy knock-on electron that a 5.6 MeV electron can produce.

The values in table 2 were obtained using the code MSTEST with mono-energetic

Table 2. The correction factor $k_{sec}(t)$ given by the Monte Carlo calculated ratio of energy of the primary electrons emerging from the slabs to the total electron energy emerging. The one standard deviation statistical uncertainty in the results is 0.01% and the systematic uncertainty is 0.05%.

t_D (g cm^{-2})	$k_{sec}(t)$ (primary energy/total energy)
0.0	0.9998
0.1057	0.9973
0.1580	0.9963
0.2122	0.9957
0.2652	0.9952
0.53	0.9936
1.06	0.9927

incident pencil beams. Calculations were also done using a code which took into account the accelerator exit window and the intervening air. This leads to a slight correction for secondaries in the open beam case. It was also verified that using the more complete simulation has no effect on the calculated values of k_{sec}.

The correction factor $k_{sec}(t)$ shows that full build-up of the secondary electrons is achieved by the first slab to within 0.2% compared to the fourth slab. However, the total energy loss being analysed is only 5.5% and thus the 0.2% correction leads to a 3% change in the stopping power deduced using (13) and maintaining the Feist and Müller assumption that $k_{loss}k_\epsilon = 1$ as well as not including the $t = 0$ point in the fit.

If the effects of secondary electrons are properly corrected for by the Monte Carlo calculated factors, then the data measured behind the slabs should extrapolate back to the point measured for the incident beam. The initial point still falls below the extrapolated line (the chain line in figure 2), but it is only 0.3% low compared to being 0.5% low before the secondary electron effects are taken into account. However, if the data point for the incident beam is included in the analysis of (13), the stopping power changes by 3.1% compared to when it is not included. The quality of the fit decreases but is still good (χ^2/degree of freedom = 0.26). Fortunately these various changes tend to cancel each other.

6. Effects of electron losses

As discussed in section 2.2, it is essential, in principle, to correct for the effects of electron losses through the factors k_{loss} and k_ϵ in (13). Table 3 presents values of $k_{loss}(t)$ calculated as the number of primary electrons incident on the slab per primary electron exiting from the back of the slab. The precision of this factor is very good ($< \pm 0.02\%$ except for the thickest slab) since only a very few electrons do not get through the slab. Calculated results which take into account the accelerator exit window and air are indistinguishable from these values of k_{loss}. This correction mostly takes into account the changing backscatter from the slabs. This is seen by considering the factor k'_{loss} (= primary electrons incident on the slab per primary electron either transmitted or reflected) which is within 0.02% of unity except for slabs thicker than used. For the 1.06 g cm^{-2} slab, k'_{loss} shows that about 2% of the electrons actually stop inside the slab.

Table 3. Summary of calculated corrections for electron losses. k_{loss} is the ratio of the number of primary electrons incident on the slab to primary electrons transmitted by the slab. k'_{loss} would correct only for particles which stop inside the slabs (given by the ratio of incident electrons to those primaries transmitted and reflected). k_ϵ shows the changes in the Fricke detector's efficiency relative to the case with no slabs present. $k_{loss} k_\epsilon$ was assumed to be unity by Feist and Müller, a good assumption as shown in the last column. One standard deviation statistical uncertainties in the last digit are shown in brackets.

t_0 g cm^{-2}	k_{loss}	k'_{loss}	k_ϵ	$k_{loss} k_\epsilon$
0.0	1.0000	1.0000	1.0000	1.0000
0.1057	1.0003(1)	1.0000	0.9996(2)	0.9999
0.1580	1.0005(2)	1.0000	0.9997(2)	1.0002
0.2122	1.0007(2)	1.0001	0.9995(2)	1.0002
0.2652	1.0010(2)	1.0002	0.9994(2)	1.0004
0.53	1.0036(2)	1.0012	1.0008(2)	1.0044
1.06	1.0261(10)	1.0209	1.0083(2)	1.0346

Table 3 also presents the correction factor k_ϵ which accounts for changes in the detector efficiency as slabs are added. As the slab thickness increases, the decreasing mean energy of electrons incident on the detector means that the radiative yield of the electrons in the Fricke solution decreases and hence the bremsstrahlung losses from the detector decrease causing the efficiency to increase (by about 0.2% for the 0.26 g cm^{-2} slab). At the same time the backscatter losses increase because of the lower incident energy which causes the efficiency to decrease by about 0.1%. The net effect is that the efficiency varies by only 0.06% for the slab thicknesses analysed by Feist and Müller.

The table shows that what variations there are in k_ϵ and k_{loss} tend to offset each other. As assumed by Feist and Müller, the net effect is that the product $k_{loss}(t)k_\epsilon(t)$ stays within 0.04% of unity (which is the statistical precision of the two calculations).

It is difficult to estimate the systematic uncertainties in k_{loss} and k_ϵ. If it is assumed there is a 10% systematic uncertainty in the calculated losses which are being corrected for, then the uncertainty in k_{loss} is dominated by the statistical precision except for the two thickest slabs where the uncertainty would reach $\pm 0.3\%$. Similarly the uncertainty in k_ϵ is likely dominated by statistical uncertainties since only the ratio of two similar calculations is being sought.

In the no slab case, one can compare the calculated energy loss from the detector of (2.45±0.02)% with the value of 2.7% reported by Feist (1982) (not including the energy loss in the Hostaphan or air).

7. Effects of bremsstrahlung

Using the results of the MSTEST runs, it is found that the the approximation in (15) for the energy of the bremsstrahlung photons produced in the slabs is accurate, the Monte Carlo calculated value of k_{br} being 0.038 ± 0.001 (for all t) rather than the value 0.036 used by Feist and Müller. Monte Carlo calculations with DOSRZ for a 5.6 MeV thin-target bremsstrahlung spectrum (Schiff 1951) incident on the Fricke detector show that absorption of scattered photons in the Fricke solution increases the fraction of the bremsstrahlung energy from the slabs reabsorbed in the Fricke solution, k_{bf}, from the value 0.08 used by Feist and Müller to 0.089 ± 0.001. Thus the correction to the stopping power given by (17) is 1.0034 ± 0.0001 which is comparable to the factor 1.003 used by Feist and Müller.

8. Statistical analysis

Re-analysing the original data using (9) as done by Feist and Müller gives a stopping power value of 1.906_4 MeV g^{-1} cm^2 in good agreement with their value of 1.907. The 68% confidence level internal and external uncertainty estimates on the stopping power are $\pm 4.5\%$ and $\pm 1.5\%$ respectively†. The external uncertainty estimate is much lower than the internal uncertainty estimate because the data fall on a very good straight line, much better that expected based on the random uncertainty on each point (the χ^2 per degree of freedom is 0.06 which has less than a 5% probability of occurring!). Feist and Müller's uncertainty estimate of 1.2% is likely the external error estimate without the student's t distribution factor (1.3 for 2 degrees of freedom) which is needed to get a 68% confidence limit. However, the internal uncertainty estimate of 4.5% represents a lower limit on the uncertainty on the fitting parameters when the uncertainty on each data point is known. This 4.5% value represents the correct uncertainty estimate in this case.

Use of this larger uncertainty estimate is more compatible with the 3% change in the measured stopping powers found in section 5 by introducing a relative correction for secondary electrons of 0.2% which is within the stated uncertainty of each data point. The internal uncertainty estimate more accurately reflects the ability of the experiment to measure the stopping power.

Estimates of the systematic uncertainties on all the factors in (13) must be considered to assess the overall uncertainty in the fitted values of the stopping power. These have been given previously as: k_{sec}, 0.05%; k_{loss}, 0.02%; k_ϵ, 0.02%; and $A(t)$, 0.2%. The uncertainty in the PLC was estimated at 0.1% which can be treated as an additional 0.03% uncertainty on A. The overall uncertainty, which is the quadratic sum of these factors is completely dominated by the uncertainty on A and comes to 0.21%. In practice the measurement uncertainties dominate the final uncertainty but if there were no measurement uncertainty, the systematic uncertainties on the correction factors would imply an uncertainty of 0.9% in the stopping power. When including the initial datum point in the fit, the uncertainty in the fitted value of A_0 (which is used to determine K) must also be included in the final analysis, but this is a negligible effect as well. The overall uncertainty when fitting Feist and Müller's five data points and using the present correction factors is 3% (68% confidence level).

9. Summary and conclusions

Table 4 summarizes the values and the internal uncertainties on the electron stopping power obtained by including the various corrections discussed earlier. The 0.34%

† External uncertainty estimates correspond to determining the uncertainty on the average of n points as $s = t(n-1)\sqrt{\sum_i (x_i - \overline{x})^2 / n(n-1)}$ (where $t(n-1)$ is the 68.3% value of the students t distribution for $n - 1$ degrees of freedom) whereas the internal uncertainty estimates correspond to using the known uncertainty on each measured point, ϵ_i, to determine $s = \frac{1}{n}\sqrt{\sum_i \epsilon_i^2}$. For a complete discussion in the case of fitting functions, see Rogers (1975).

increase due to corrections for bremsstrahlung is included in all cases. The χ^2/degree of freedom of the fit increases slightly to no more than 0.1 when fitting four data points and to 0.24 when including the data point with no slab present. Including the fifth data point reduces the fitting uncertainty from 5 to 3%. This reduction occurs because the range of energy loss being fit increases, thereby decreasing the importance of the measurement uncertainty.

Table 4. Re-evaluated total electron stopping powers for 5.37 MeV electrons in polystyrene. The fourth column gives the internal estimate of the 68% confidence limits on the stopping power taking into account only the measurement uncertainty on the absorbance per incident electron of 0.2%. The bremsstrahlung correction is included in all values.

Case	Stopping power (MeV g^{-1} cm^2)	Change from Feist and Müller	68% confidence limit
Feist and Müller	1.913	—	4.6%
Use MC pathlength, t	1.883	−1.6%	4.6%
Use MC t and correct for secondaries	1.942	+1.5%[a]	4.6%
Use MC t, correct for secondaries and include no slab case	1.889	−1.4%	2.7%
Use MC t, correct for secondaries include no slab case and $k_{loss}k_{\epsilon}$	1.881	−1.7%	3.0%[b]

[a] Effect of just correction for secondaries is +3.1%.
[b] Including systematic uncertainties on all correction factors.

This suggests that the best way to increase the accuracy of the experiment is to increase the thickness of the polystyrene slabs. Tables 1 to 3 include the PLCs and corrections for secondary electrons necessary for slabs 0.5 and 1.0 cm thick. In these cases the correction for electron losses, k_{loss} begins to vary significantly from unity because of primaries which completely stop in the slab. Using such correction factors, if a slab thickness of 0.5 g cm^{-2} were used, the uncertainty in the stopping power from the absorbance uncertainties would be reduced to 1.4% and including a 1 g cm^{-2} slab would further reduce the uncertainty to 0.5%. At this level the systematic uncertainties in the correction factors make a significant contribution to the overall uncertainty.

An important result of this work is the realization that correction factors for curved pathlengths of electrons in thin slabs of material cannot be deduced from the PRESTA PLC algorithm because it addresses a slightly different question. At present, the only accurate technique is the use of a complete Monte Carlo simulation.

In summary, analysis of the data of Feist and Müller and making use of Monte Carlo calculated correction factors changes the estimate of the measured stopping power by 1.7% from 1.913 to 1.881 MeV g^{-1} cm^2, corrects the uncertainty analysis and reduces the experimental uncertainty from 5 to 3%. The measured stopping power is 1.8% less than the value in ICRU Report 37 of 1.915 MeV g^{-1} cm^2 power of polystyrene for 5.37 MeV electrons. The experimental uncertainty is dominated by the 0.2% uncertainty on the individual measurements of absorbance per unit charge. With no measurement uncertainty the systematic uncertainty is 0.9%. Without repeating the experiment many times, it is hard to see how this technique could produce a result with an accuracy much better than 3%.

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Résumé

Réévaluation du pouvoir de ralentissement total d'électrons de 5.3 MeV dans le polystyrène

Les auteurs ont réanalysé l'expérience de Feist et Müller destinée à la mesure du pouvoir de ralentissement d'électrons de 5.3 MeV traversant des lames de polystyrène, utilisant un formalisme présenté dans ce travail et des facteurs de correction obtenus par des calculs à l'aide de la méthode de Monte Carlo. L'analyse la plus approfondie a conduit à un changement de 1.7% dans la valeur du pouvoir de ralentissement déterminée expérimentalement, celle-ci passant de 1.913 à 1.882 MeV cm^2 g^{-1} à 5.37 MeV. Des méthodes statistiques approfondies montrent que l'incertitude sur le pouvoir de ralentissement qui a été ainsi déterminé, liée aux incertitudes annoncées pour les mesures, est augmentée par rapport aux valeurs de Feist et Müller, de 1.2% à 5% si les mêmes données sont analysées, ou 3% si la valeur initiale des données est incluse dans l'analyse après une correction pour les électrons secondaires. Les incertitudes systémitiques des facteurs de correction calculés conduisent à une incertitude sur le pouvoir de ralentissement de 0.9%. Les auteurs fournissent des facteurs de correction pour des mesures effectuées avec des lames dont l'épaisseur atteint 1.0 g cm^{-2}. Utilisant deux mesures supplémentaires pour les lames plus épaisses, avec des incertitudes de mesure comparables aux précédentes, l'incertitude statisque dûe à la mesure peut être réduite à moins de 1%. On montre que, en général, le parcours moyen des électrons à la traversée d'une lame n'est pas donné correctement par les corrections de parcours de Fermi–Yang ou PRESTA, qui en fait se rapportent à un autre effet. Actuellement, la seule solution précise est de calculer le parcours moyen à l'aide de la méthode de Monte Carlo.

Zusammenfassung

Neuberwertung des Gesamtbremsvermögens von 5.3 MeV Elektronen in Styropor

Das Experiment von Feist und Müller zur Messung des Bremsvermögens von 5.3 MeV Elektronen beim Durchgang durch eine Styroporplatte wurde mit Hilfe des hier vorgestellten Formalismus, sowie mit Korrekturfaktoren aus Monte Carlo Rechnungen überprüft. Aufgrund der genauen Analyse ändert sich das experimentell bestimmte Bremsvermögen um 1.7% von 1.913 auf 1.882 MeV g^{-1} cm^2 bei 5.37 MeV. Geeignete statistische Methoden zeigen, daß sich die Ungenauigkeit des abgeleiteten Bremsvermögens aufgrund der Meßfehler von 1.2% nach Feist und Müller, auf 5% erhöht, wenn die gleichen Daten analysiert werden oder um 3%, wenn der Anfangspunkt in die Analyse miteinbezogen wird, nachdem auf Sekudärelektronen korrigiert wurde. Systematische Fehler der berechneten Korrekturfaktoren führen zu einer Ungenauigkeit im Bremsvermögen von 0.9%. Korrekturfaktoren werden angegeben für Messungen an Platten bis zu 1.0 g cm^{-2}. Mit Hilfe zweier zusätzlicher Messungen für dickere Platten mit ähnlichen Meßfehlern, kann der statistische Fehler aufgrund der Messung auf weniger als 1% reduziert werden. Es wird gezeigt, daß im allgemeinen die durchschnittliche Weglänge, die von den Elektronen beim Durchgang durch eine Materieschicht mit Hilfe der Fermi–Yang oder PRESTA Weglängenkorrektur angegeben wird, nicht korrekt ist. Diese Korrektur betrifft außerdem eine andere Art der Fragestellung. Eine genaue Berechnung der durchschnittlichen Weglängen ist deshalb zur Zeit nur mit Monte Carlo Verfahren möglich.

References

Bielajew A F and Rogers D W O 1987 PRESTA: The parameter reduced electron-step transport algorithm for electron Monte Carlo transport *Nucl. Instrum. Methods* B **18** 165–81

Duane S, Bielajew A F and Rogers D W O 1989 Use of ICRU-37/NBS collision stopping powers in the EGS4 system *NRCC Report* PIRS-0173 March 1989

Faddegon B A, Ross C and Rogers D W O 1991 Measurement of collision stopping power of graphite, aluminium and copper for 10–50 MeV electrons *Phys. Med. Biol.* submitted

Feist H 1982 Determination of the absorbed dose to water for high-energy photons and electrons by total absorption of electrons in ferrous sulphate solution *Phys. Med. Biol.* **27** 1435–47

Feist H and Müller U 1989 Measurement of the total stopping power of 5.3 MeV electrons in polystyrene by means of electron beam absorption in ferrous sulphate solution *Phys. Med. Biol.* **34** 1863–9

Hebbard D F and Wilson R P 1955 The effect of multiple scattering on electron energy loss distribution *Aust. J. Phys.* **8** 90

ICRU 1984 Radiation dosimetry: electron beams with energies between 1 and 50 MeV *ICRU Report* 35

ICRU 1984 Stopping powers for electrons and positrons *ICRU Report* 37

Nelson W R, Hirayama H and Rogers D W O 1985 The EGS4 Code System *Stanford Linear Accelerator Center Report* SLAC-265

Nelson W R and Rogers D W O 1989 Structure and operation of the EGS4 code system *Monte Carlo Transport of Electrons and Photons Below 50 MeV* ed T M Jenkins, W R Nelson, A Rindi, A E Nahum and D W O Rogers (New York: Plenum) pp 287–306

Rogers D W O 1975 Analytic and graphical methods for assigning errors to parameters in non-linear least squares fitting *Nucl. Instrum. Methods* **127** 253–60

Rogers D W O 1984 Low energy electron transport with EGS *Nucl. Instrum. Methods* A **227** 535–48

Schiff L I 1951 Energy-angle distribution for thin target bremsstrahlung *Phys. Rev.* **83** 252–3

Yang C N 1951 Actual path length of electrons in foils *Phys. Rev.* **84** 599–600