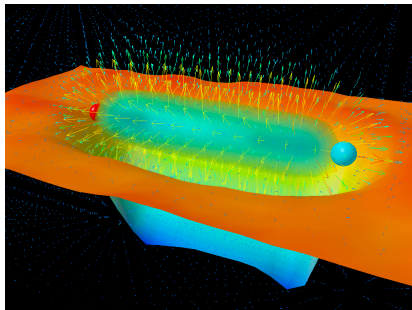


The Lund String Fragmentation Model

Dag Gillberg

Simon Fraser University

Jan 26, 2007



- ➊ Introduction – What is the Lund Model?
- ➋ About QCD – Some features of the strong interaction
- ➌ MRS dynamics – The Massless Relativistic String
- ➍ MRS break-up – The hadronization process
- ➎ Summary

Introduction — What is Lund?



- City Population: 100,000
- Lund University: 43,000 students

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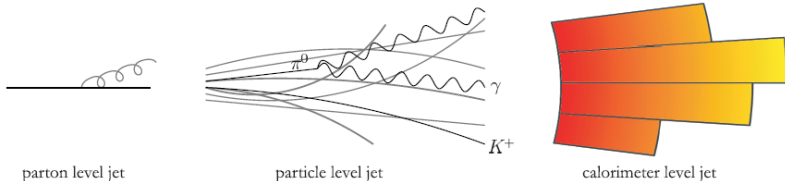


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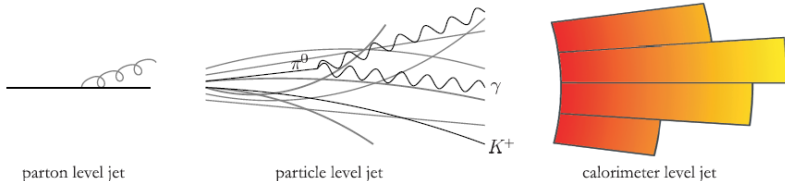


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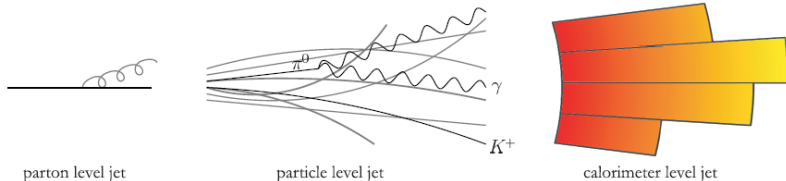


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- **Hadronization:** quark $\rightarrow$ spray of hadrons (**jet**)

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- The Lund Model has been implemented in **Pythia**—today considered the most successful and realistic simulation software for multi-particle production (particle physics)

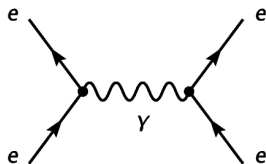
About QCD — QED vs QCD

Two of the four fundamental forces

Electromagnetism

Described by

QED

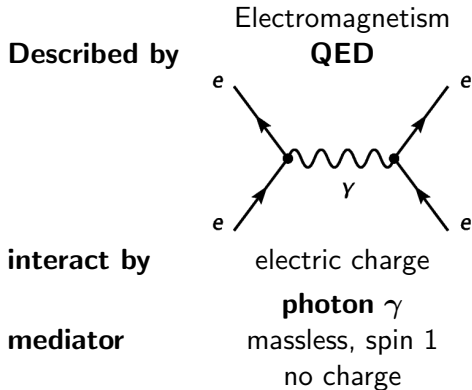


interact by

electric charge

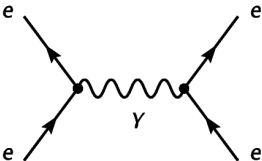
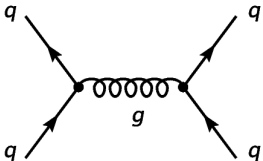
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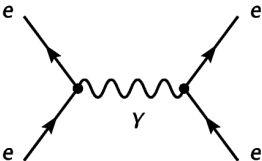
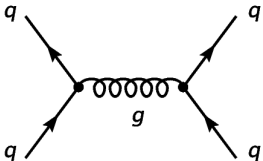
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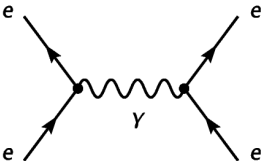
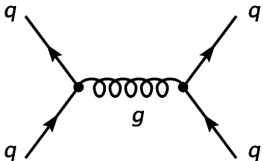
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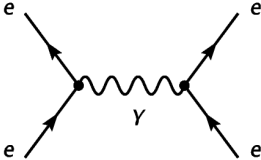
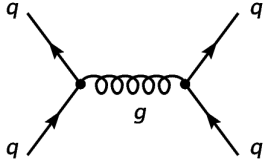
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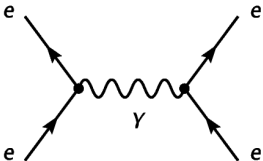
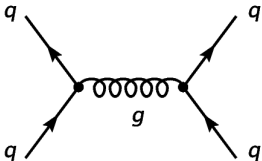
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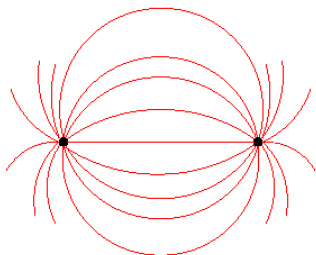
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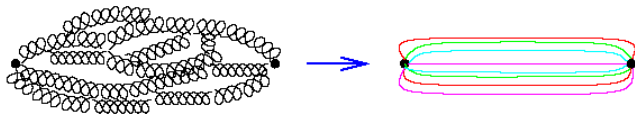
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- QED: Can be calculated very precisely using perturbation theory
- QCD: The gluon self-interaction makes things much harder
Perturbation theory only works at very high E – **asymptotic freedom** –

About QCD — QED vs QCD fields



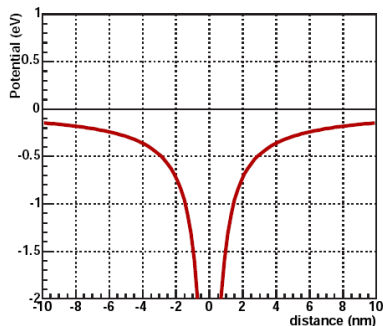
EM: dipole $\Rightarrow$ field $\rightarrow \infty$



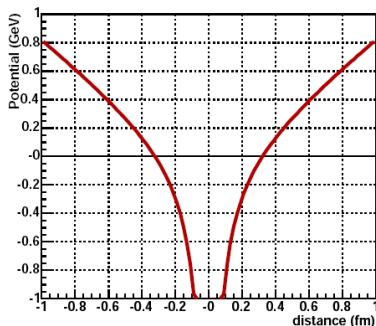
QCD: $q\bar{q}$ colour dipole,
gluon self-interaction $\Rightarrow$ flux tube

About QCD — QED vs QCD potential

QED -- Coulomb Potential (hydrogen atom)



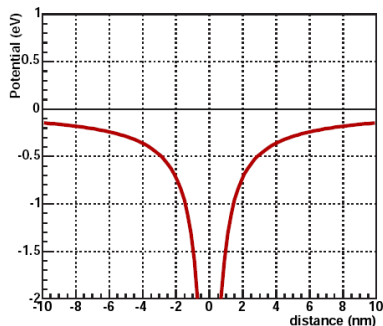
QCD Potential (heavy quarkonia spectra)



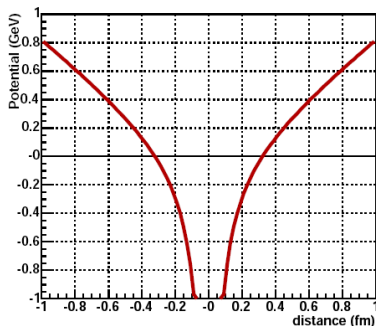
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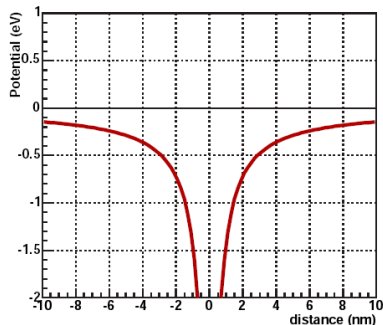
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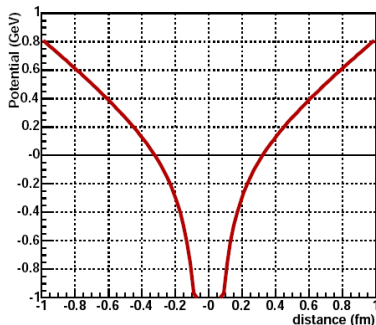
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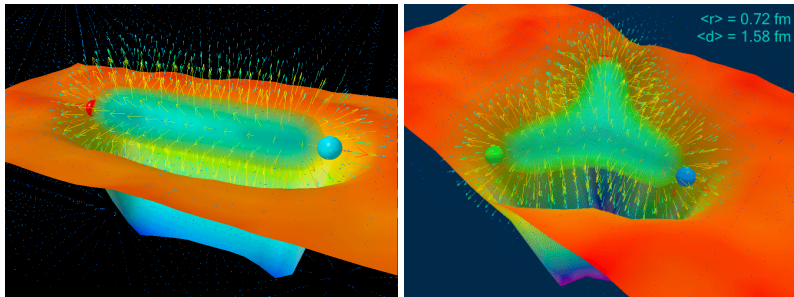


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- linear potential $\rightarrow$ constant force
- slope $\kappa \approx 1$ GeV/fm

About QCD — The QCD field



Meson animation, Baryon animation

These simulations were shown during Prof. Frank Wilczek's 2004 Nobel Prize Lecture – *“for the discovery of asymptotic freedom in the theory of the strong interaction”*

Lattice QCD simulations from 2003 done by Centre for the Subatomic Structure of Matter (CSSM) and
Department of Physics, University of Adelaide, 5005 Australia

MRS dynamics — The Massless Relativistic String

The basic idea/assumptions of the Lund String Model:

- The QCD field between two quarks only exist in a narrow region between the quarks – **the QCD field is modelled as a string** (also called “**flux tube**”)

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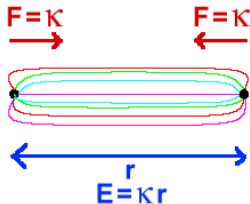
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$$\frac{dE}{dz} = -\kappa, \quad \frac{dp_z}{dt} = -\kappa$$
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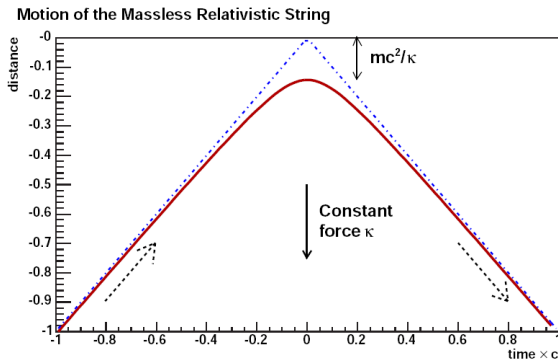
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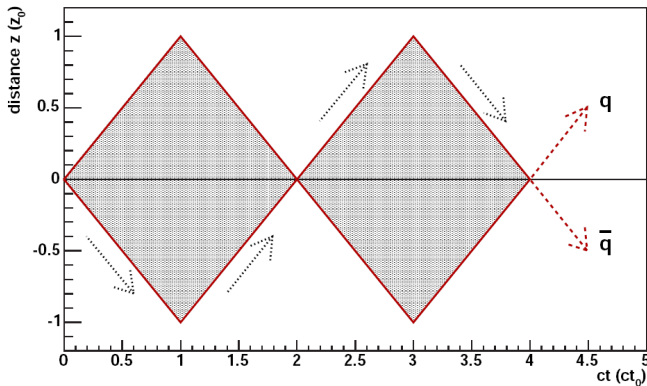
Solving for z gives

$$z = z_0 - \sqrt{\frac{m^2 c^4}{\kappa^2} + c^2(t_0 - t)^2}$$



MRS dynamics — The yoyo-mode

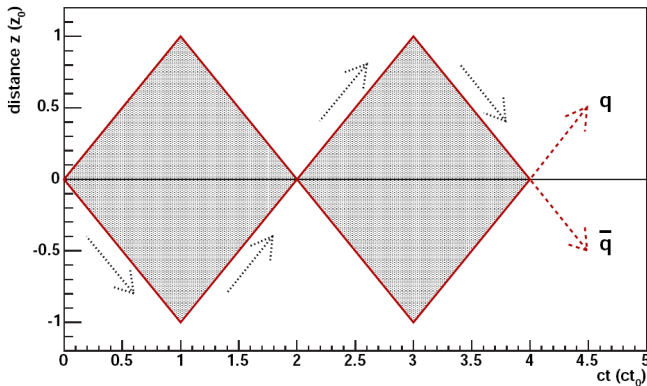
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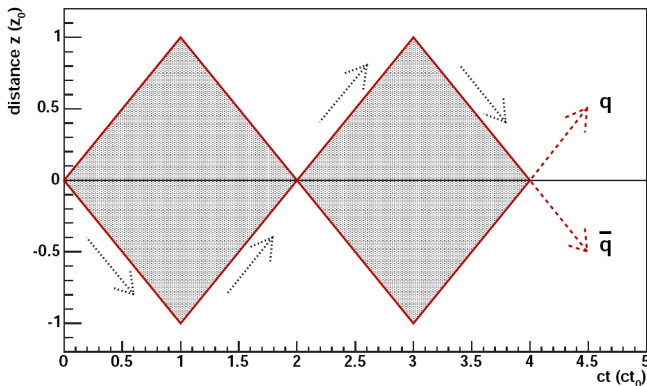
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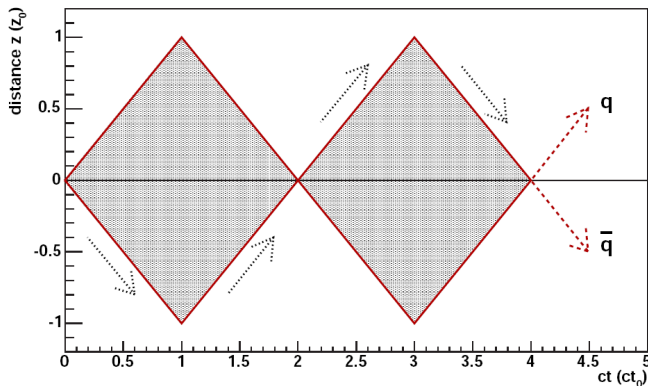
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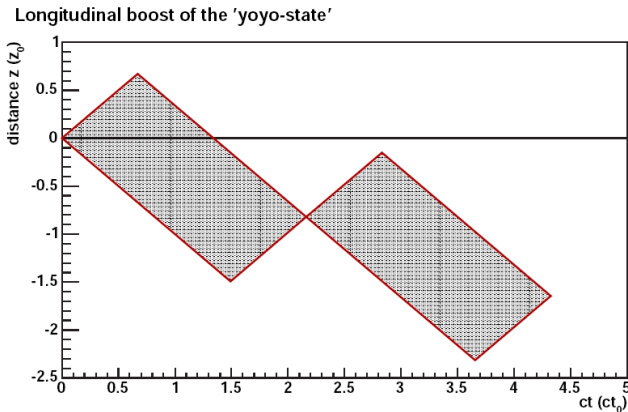
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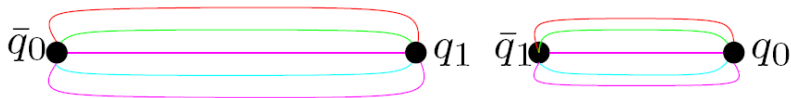
MRS dynamics — boosted yoyo-mode



This is the same system but boosted by $0.38c$ ($\gamma = 0.4$) along the negative z -axis

Area in space time is still directly proportional to the rest mass of the string system (meson) $A = \frac{M^2 c^2}{\kappa^2}$

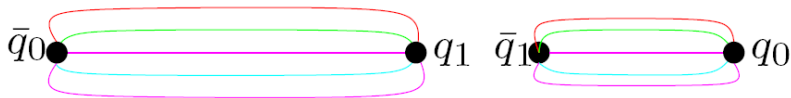
MRS break-up — String decay



Lund string fragmentation:
 $q\bar{q}$ pairs from field energy

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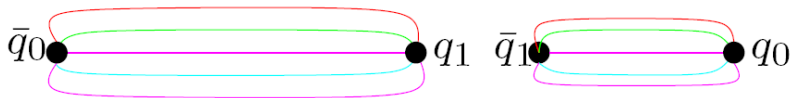
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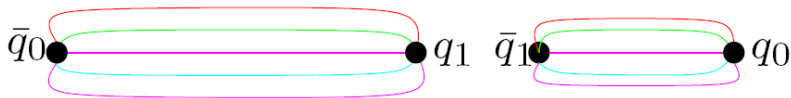
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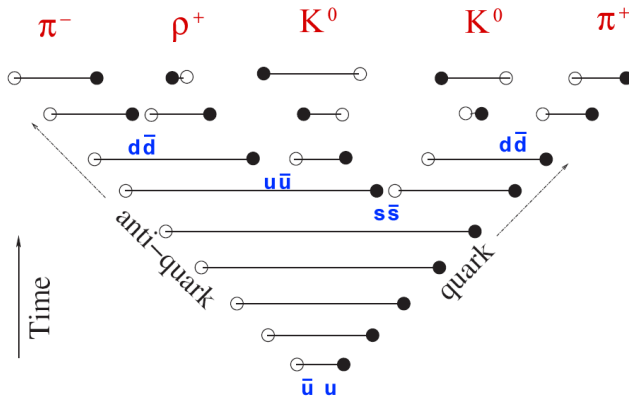
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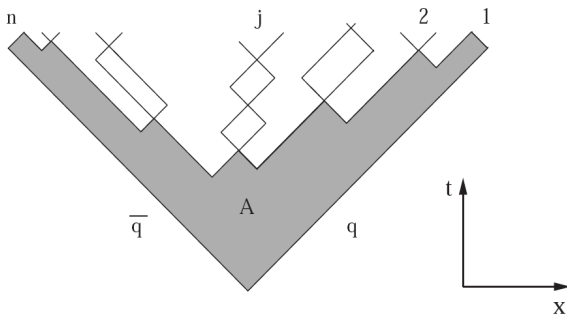
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- **Confinement:** Field disappears between the new $q\bar{q}$

MRS break-up — String decay



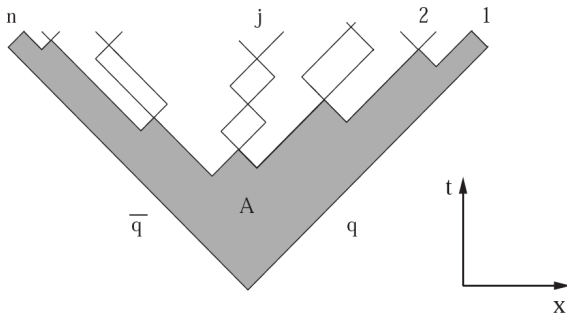
- The Lund String Model uses a stochastic process to simulate the string decay...

String decay



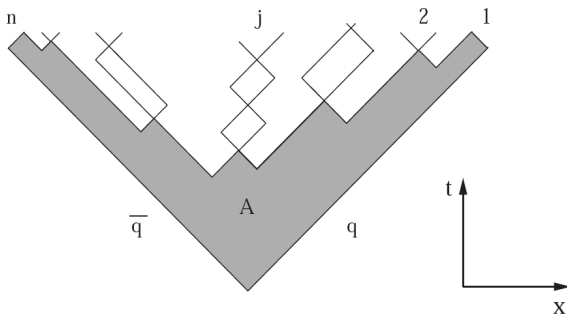
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- Each “yoyo-mode” can further decay and the final yoyo-modes will be on the mass-shells of hadrons
- The final result of a string decay will be a set of hadrons, either moving in the positive or negative z -direction

We have two **jets**

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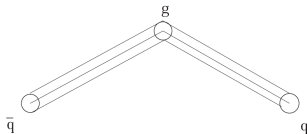
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- Gluons can appear as “kinks” (excitations) on the string
 $\rightarrow$ more complex modes possible
 $\rightarrow$ a gluon can be created with large transverse momentum and result in a third jet

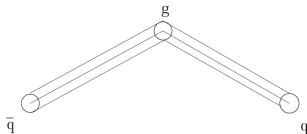


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- As a string breaks, the q and the $\bar{q}$ will tunnel out from the vacuum due to the strength of the field

In this process, the quarks will also get a transverse momentum component $\rightarrow$ the final jets of particles will have a spread

- Gluons can appear as “kinks” (excitations) on the string
 $\rightarrow$ more complex modes possible
 $\rightarrow$ a gluon can be created with large transverse momentum and result in a third jet



- Several models for baryon production – simplest model allows diquark pairs to be created in the string
 $q-\bar{q} \rightarrow q-qq \quad \bar{q}\bar{q}-\bar{q}$

Summary

- A “string” is used to model the QCD field between two endpoint colour charges (quarks)
- Numerous and detailed studies at the LEP collider (at CERN) favours this model over other models for hadronization

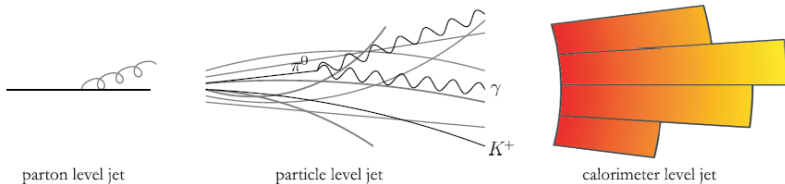


Figure 2.4: A quark radiates a gluon before hadronizing into a jet.

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