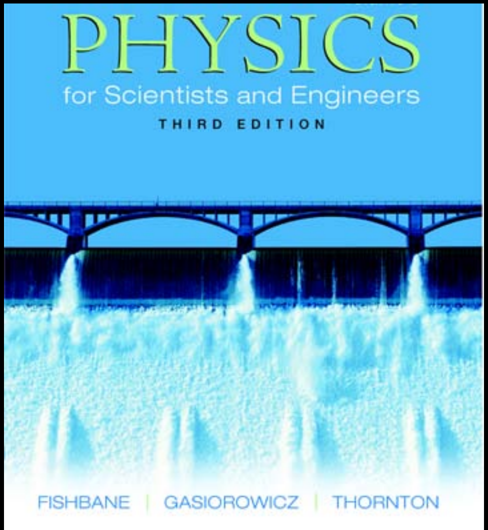


Lecture PowerPoint

Physics for Scientists and Engineers, 3rd edition

Fishbane
Gasiorowicz
Thornton

Chapter 12

Gravitation

Main Points of Chapter 12

- Kepler’s laws
- Universal gravitation
- Potential energy
- Planets and satellites
- Escape speed
- Orbits
- Gravitation and extended objects

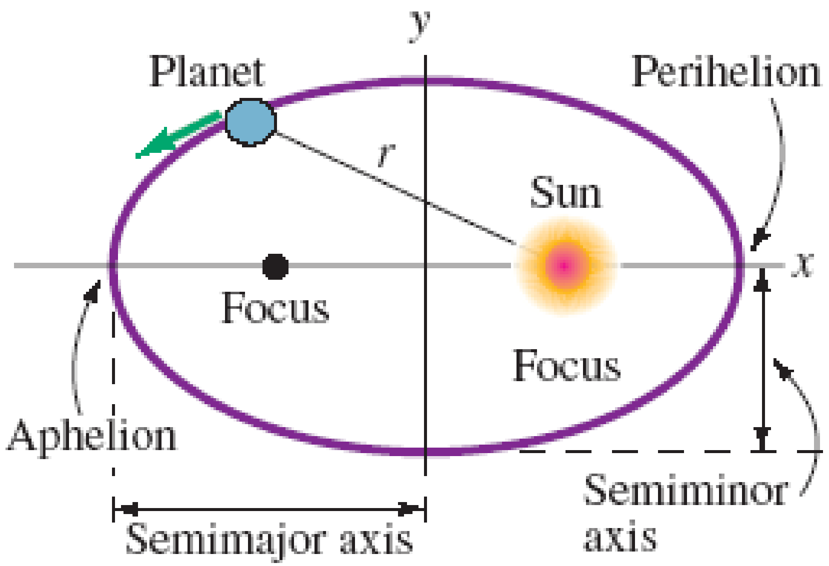
Main Points of Chapter 12

- Tides
- Equality of inertial and gravitational mass
- Einstein’s theory of gravitation
- Equivalence principle

12-1 Early Observations of Planetary Motion

Kepler's laws:

1. **Planets move in planar elliptical paths with the Sun at one focus of the ellipse.**



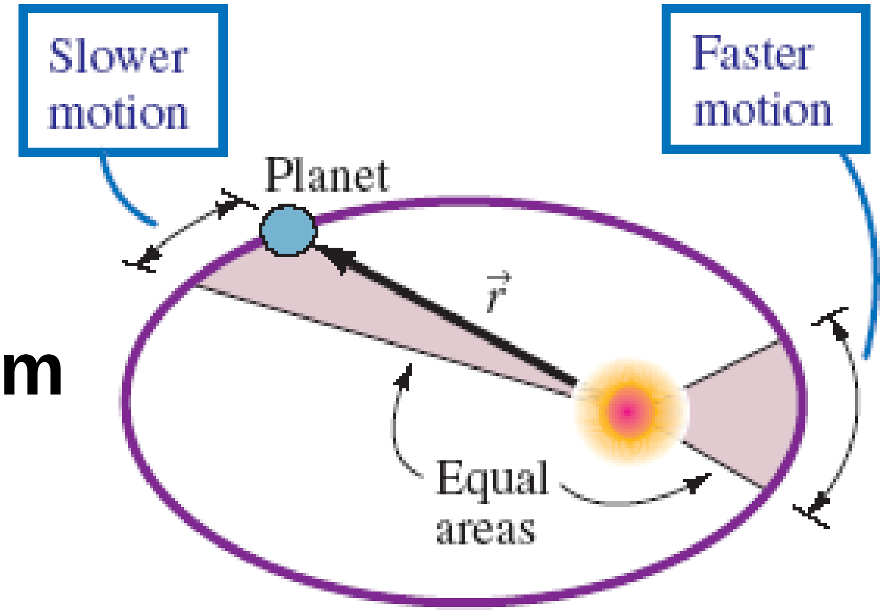
12-1 Early Observations of Planetary Motion

Kepler's laws:

Angular momentum L of a planet is a constant of motion for closed systemt (torque=0)

2. During equal time intervals the radius vector from the Sun to a planet sweeps out equal areas.

dA/dt = L/2m



12-1 Early Observations of Planetary Motion

Kepler's laws:

3. If T is the time that it takes for a planet to make one full revolution around the Sun, and if R is half the major axis of the ellipse (R reduces to the radius of the planet's orbit if that orbit is circular), then:

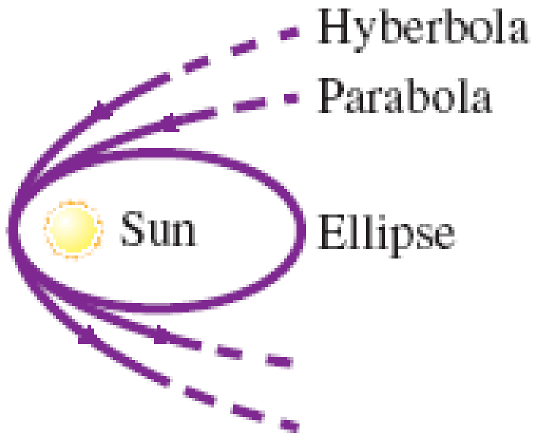
$$\frac{T^2}{R^3} = C \qquad (12\text{-}1)$$

where *C* is a constant *whose value is the same for all planets*.

12-2 Newton's Inverse-Square Law

Newton wanted to explain Kepler's laws; found that:

- **Force must be central**
- **Inverse-square law**
- **Possible paths must be conic sections:**



12-2 Newton's Inverse-Square Law

Law of universal gravitation includes all requirements:

$$\vec{F} = -\left(\frac{GmM}{r^2}\right)\hat{r} \qquad (12\text{-}4)$$

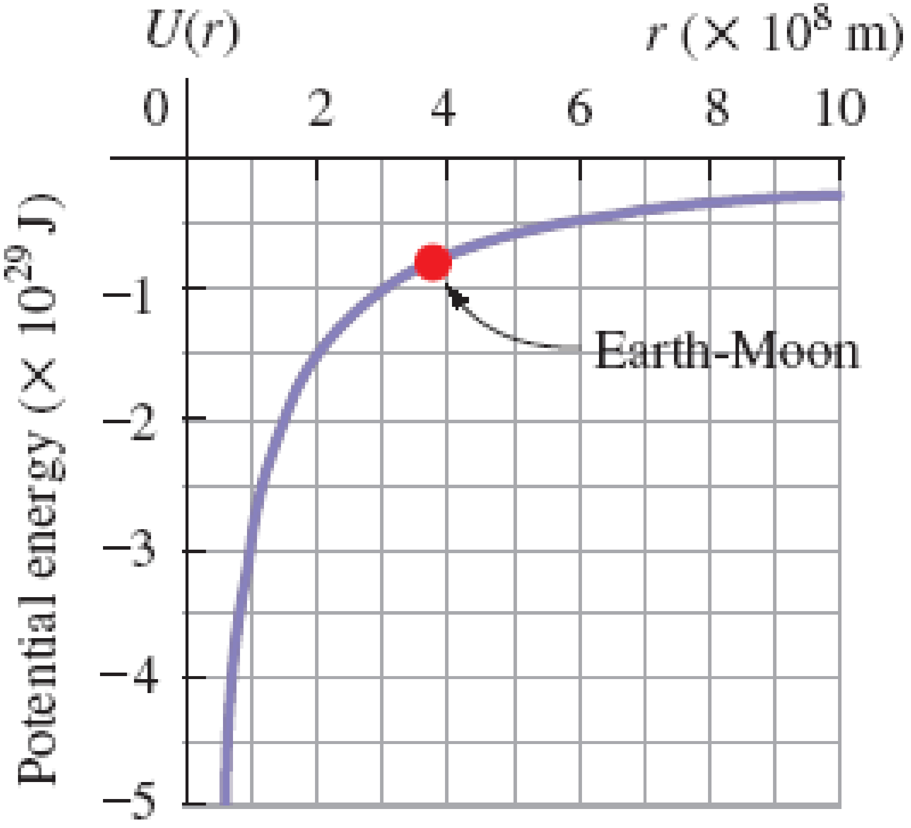
G **is a constant that can be measured using known masses; find:**

$$G = 6.673 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2} \qquad (12\text{-}7)$$

12-2 Newton's Inverse-Square Law

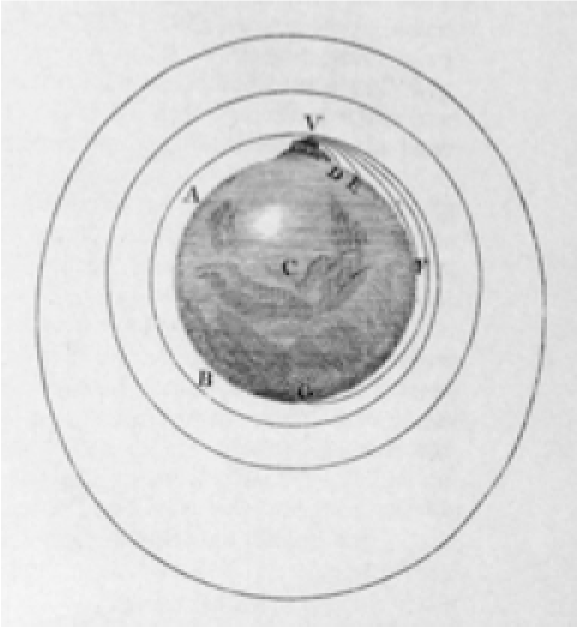
Potential energy can be derived from force:

$$U(r) = -\frac{GmM}{r} \qquad (12\text{-}8)$$



12-3 Planets and Satellites

Newton realized that falling with a sufficiently large initial horizontal velocity is orbiting – that is, the same force that causes the apple to fall from the tree also keeps the Moon in its orbit.

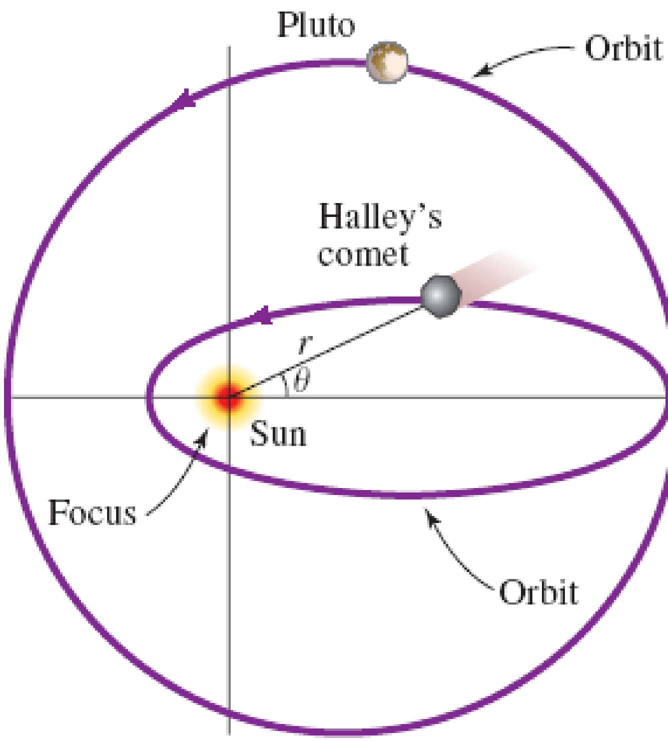


Escape speed is outward speed needed to escape from Earth's gravitational potential well (that is, to make total energy nonnegative):

$$v_{esc} = \sqrt{\frac{2GM_E}{R_E}}$$

12-3 Planets and Satellites

Types of Orbits



Total energy:

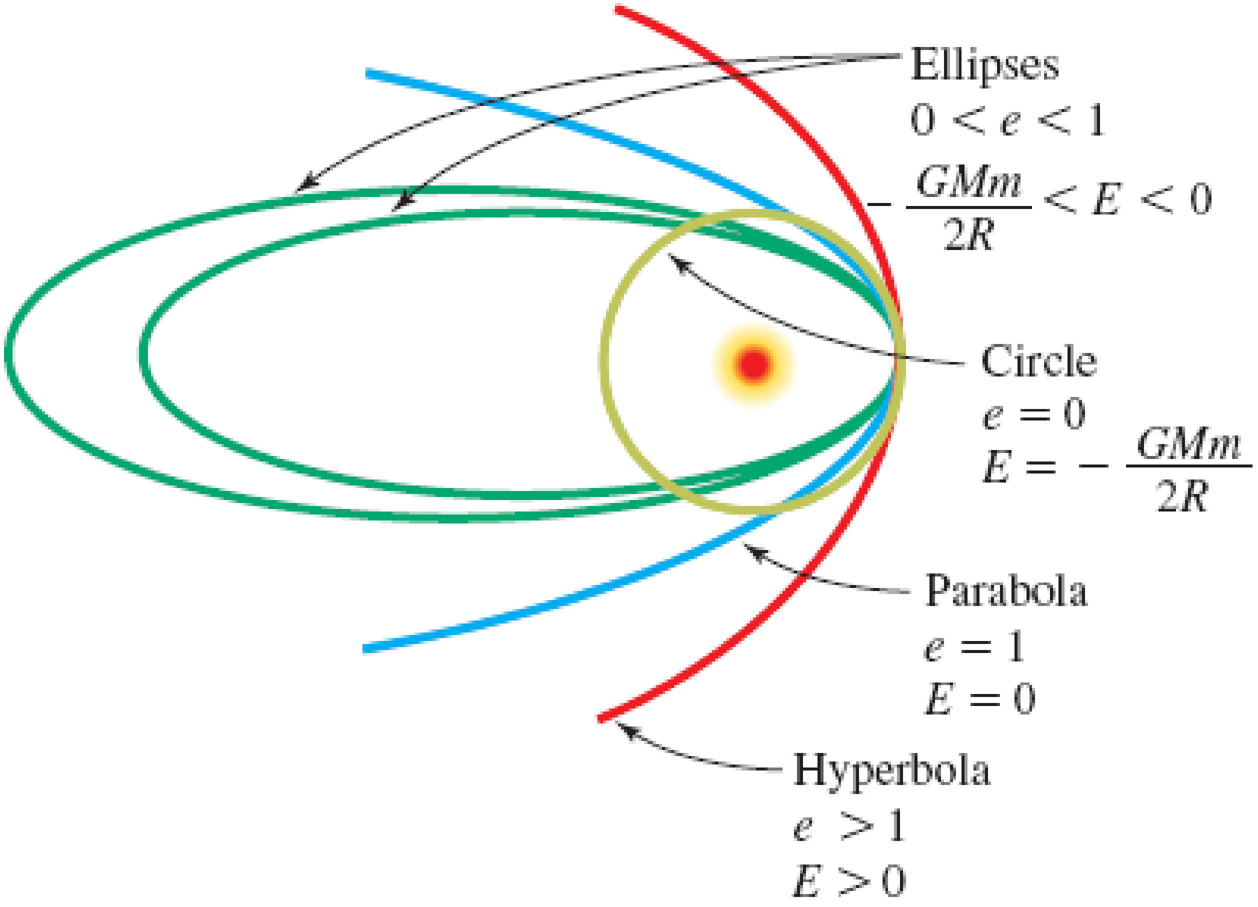
$$E = K + U = \frac{1}{2}mv^2 - \frac{GmM}{r} \tag{12-9}$$

If *E* is negative, orbits are closed – elliptical or circular

If *E* is positive, orbits are open – hyperbola

12-3 Planets and Satellites

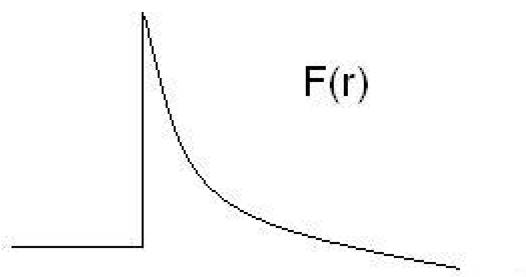
Types of Orbits: circle, ellipses, parabola, hyperbola

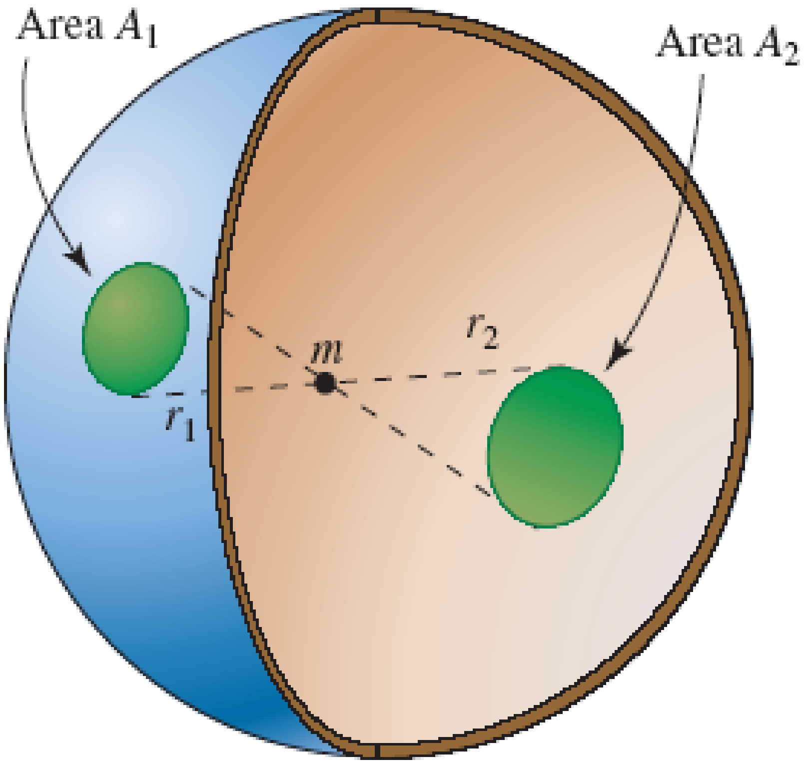


12-4 Gravitation and Extended Objects

The Gravitational Force Due to a Spherically Symmetric Object

Within a hollow shell, the gravitational force is zero – forces from opposite sides cancel

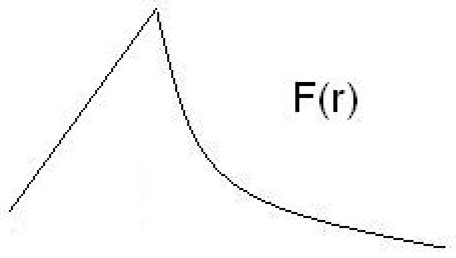


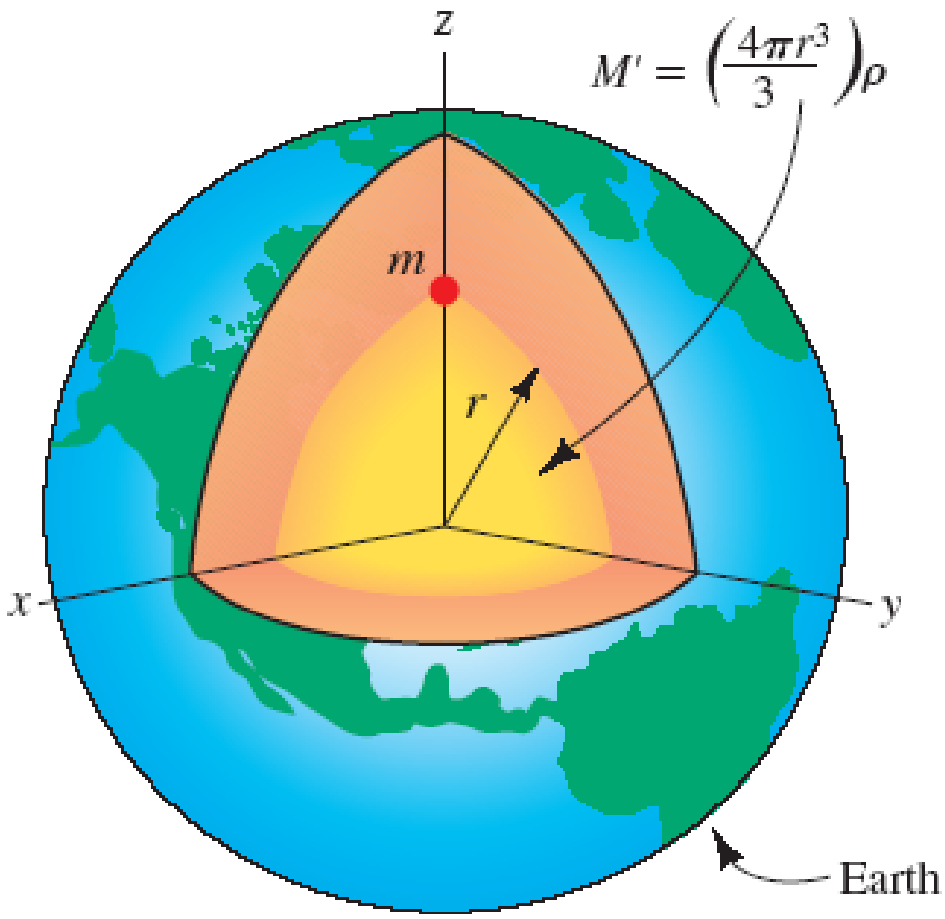


12-4 Gravitation and Extended Objects

The Gravitational Force Due to a Spherically Symmetric Object

Within the object, force at *r* is due to mass inside *r*:





12-4 Gravitation and Extended Objects

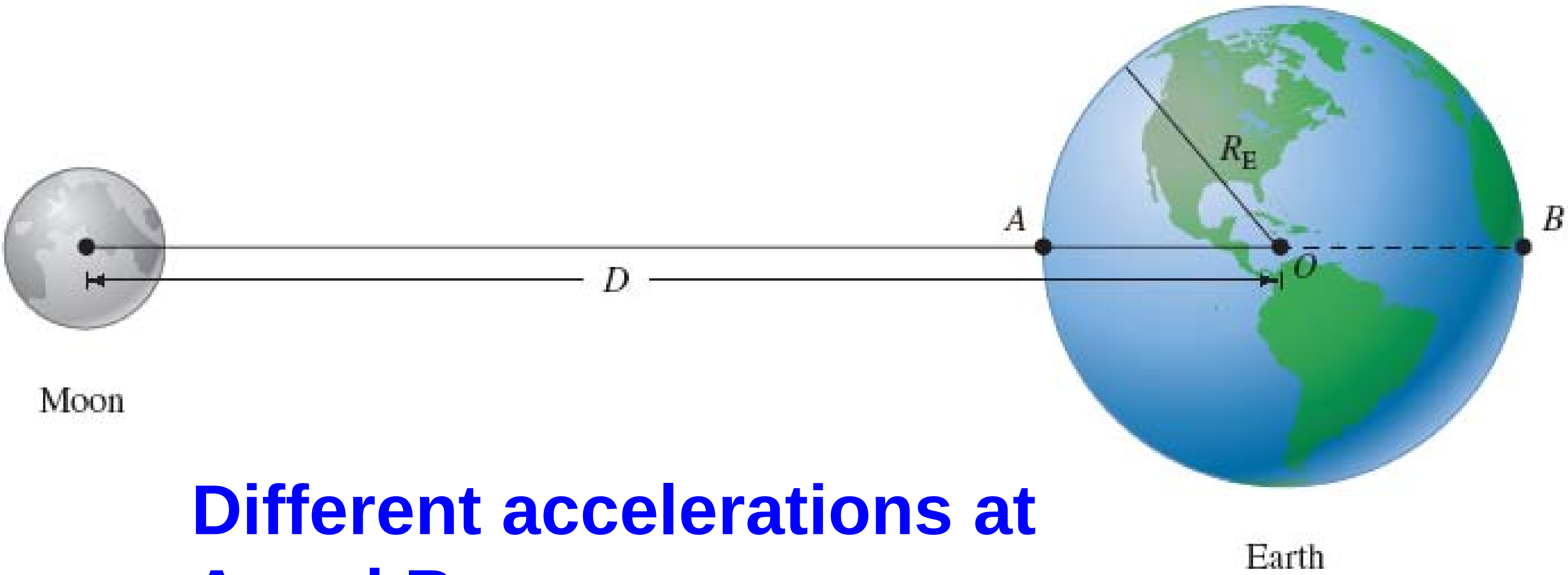
Acceleration of gravity, g, varies with altitude above Earth's surface, due to changing distance from Earth's center:

$$\frac{g(h)}{g(0)} \cong \frac{(GM/R_E^2)[1 - (2h/R_E)]}{(GM/R_E^2)} = 1 - \frac{2h}{R_E} \qquad (12\text{-}19)$$

This is a very small difference, even at the top of Mt. Everest!

12-4 Gravitation and Extended Objects

Tidal Forces



Different accelerations at A and B:

$$a_A = \frac{GM_M}{D^2} + 2\frac{GM_M R_E}{D^3}$$

$$a_B = \frac{GM_M}{D^2} - 2\frac{GM_M R_E}{D^3}$$

Chapter 13

Oscillatory Motion

Main Points of Chapter 13

- Kinematics and properties of simple harmonic motion
- Relationship among position, velocity, and acceleration
- Connection to circular motion
- Springs
- Energy
- Pendulums, simple and physical
- Damped and driven harmonic motion

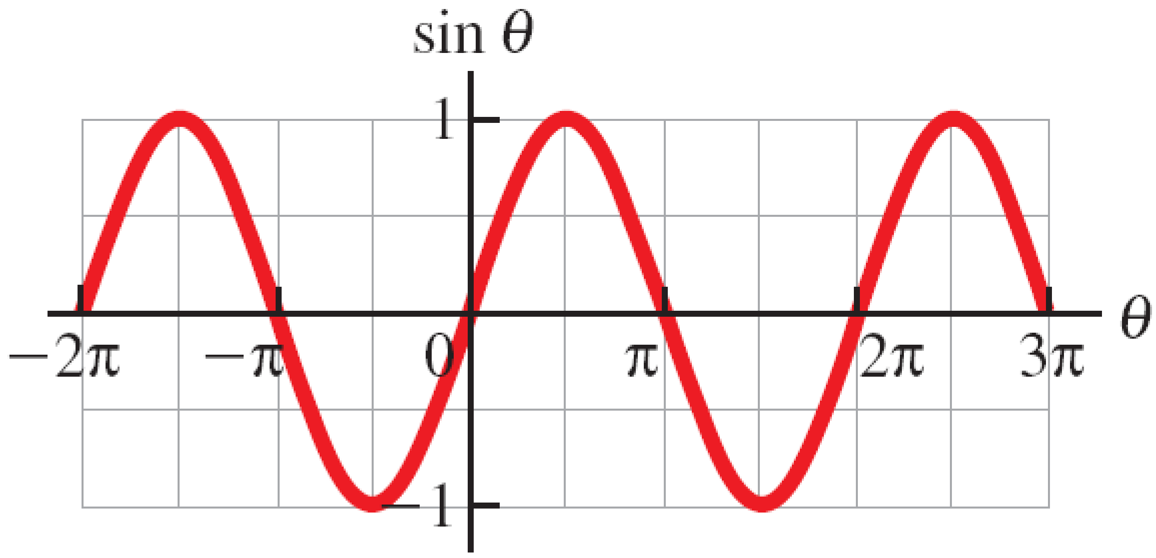
13-1 The Kinematics of Simple Harmonic Motion

Motion is sinusoidal:

$$x(t) = A\sin(\omega t + \delta) \qquad \text{(13-1a)}$$

Here, *ω* is the angular frequency, and *δ* is the phase angle (which sets the position at *t* = 0)

For *δ* = 0:

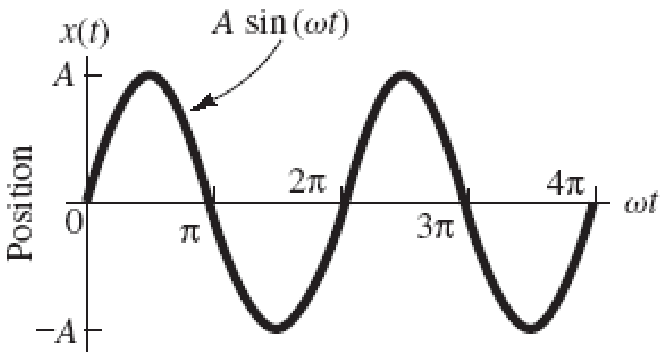


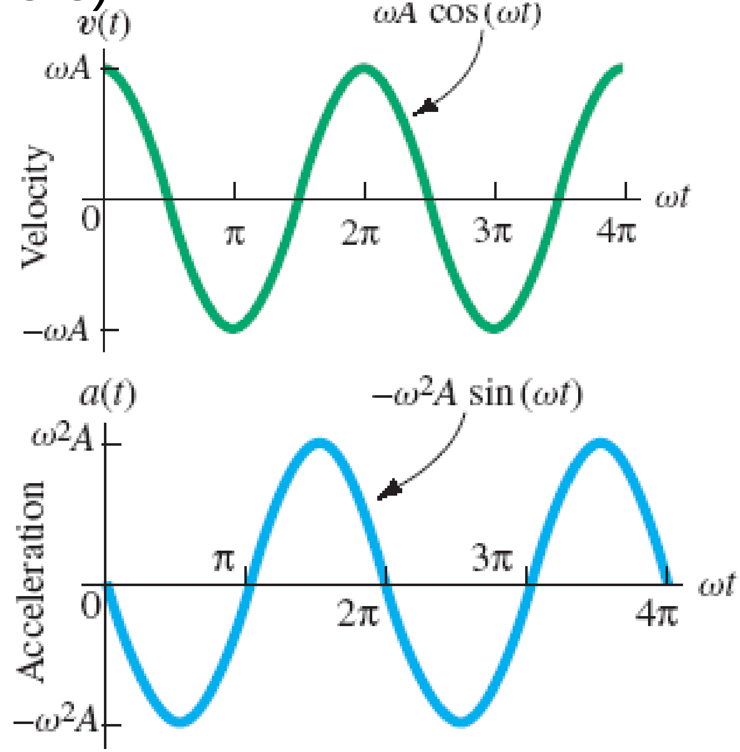
13-1 The Kinematics of Simple Harmonic Motion

Can take derivatives to find velocity and acceleration:

$$v(t) = \omega A \cos(\omega t + \delta) \quad (13\text{-}7)$$

$$a(t) = -\omega^2 x(t) \quad (13\text{-}8)$$





13-1 The Kinematics of Simple Harmonic Motion

Properties:

- **amplitude A**
- **angular frequency ω**
- **phase angle δ**

Derived quantities:

Period: $$T = \frac{2\pi}{\omega}$$ (13-2)

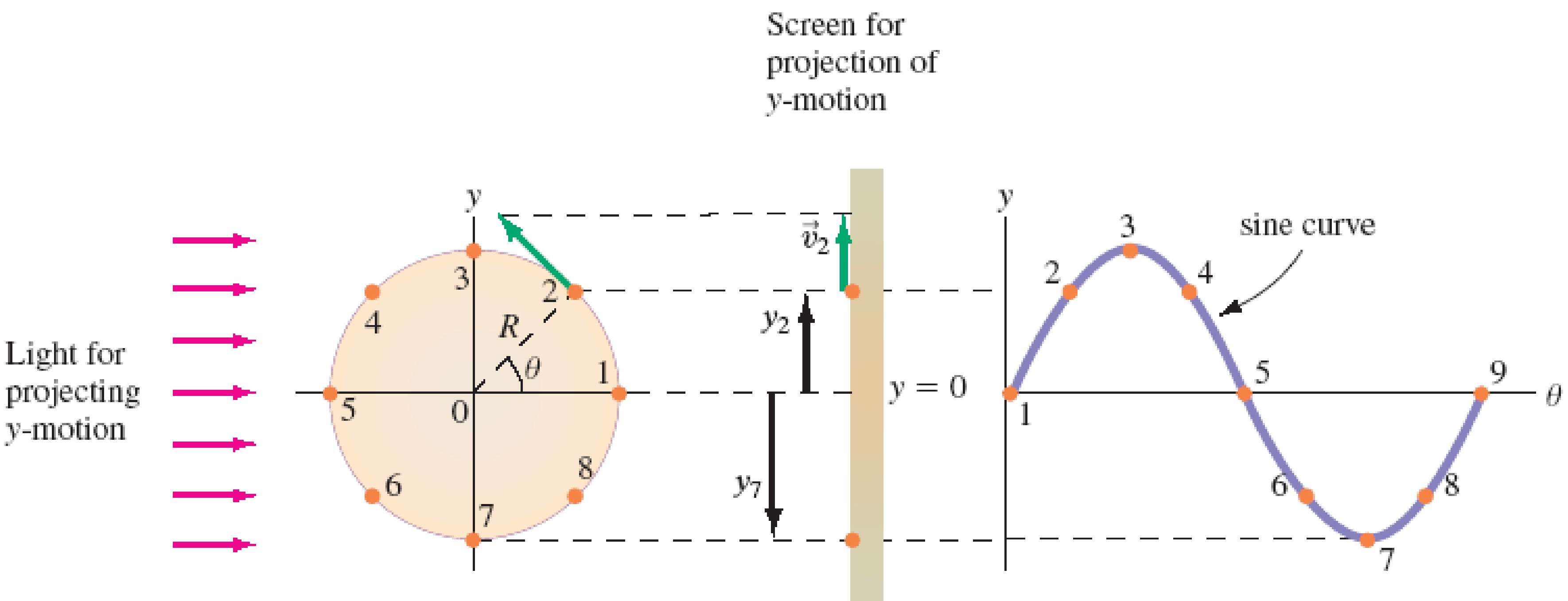
Frequency: $$f = \frac{1}{T} = \frac{\omega}{2\pi}$$ (13-3,5)

13-2 A Connection to Circular Motion

Projection of object in uniform circular motion onto a single axis shows that each component of the motion is simple harmonic:

$$x = R\cos\theta = R\cos(\omega t + \delta) \qquad (13\text{-}10)$$

$$y = R\sin\theta = R\sin(\omega t + \delta) \qquad (13\text{-}11)$$



13-3 Springs and Simple Harmonic Motion

Spring force depends on displacement:

$$F = -kx \qquad (13\text{-}13)$$

Here, *k* is spring constant, different for every spring.

Combining with Newton's second law gives:

$$a = -\frac{k}{m}x \qquad (13\text{-}14)$$

13-3 Springs and Simple Harmonic Motion

As in simple harmonic motion, acceleration is proportional to the negative of the displacement, and has a similar solution, with

$$\omega = \sqrt{\frac{k}{m}} \quad (13\text{-}16)$$

13-4 Energy and Simple Harmonic Motion

Potential energy of mass on a spring:

$$U(x) = \frac{1}{2}kx^2 \qquad (13\text{-}18)$$

As usual,

$$K = \frac{1}{2}mv^2 \qquad (13\text{-}19)$$

Substituting for *x* and *v*:

$$U = \frac{1}{2}kA^2 \sin^2 \theta \qquad (13\text{-}20)$$

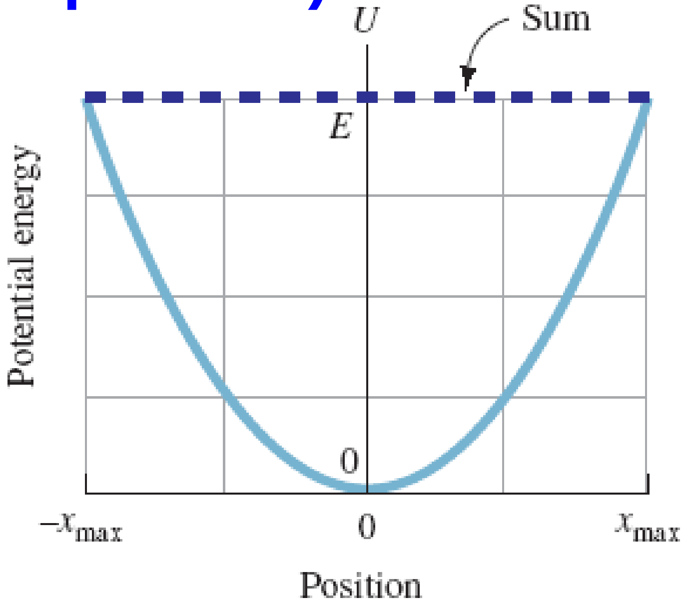
$$K = \frac{1}{2}kA^2 \cos^2 \theta \qquad (13\text{-}21)$$

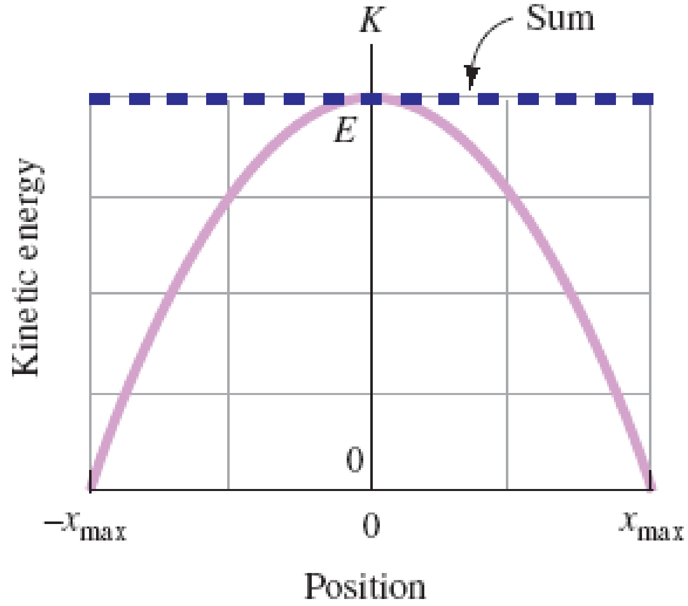
13-4 Energy and Simple Harmonic Motion

As $\sin^2\theta + \cos^2\theta = 1$, the sum of the kinetic and potential energies is constant:

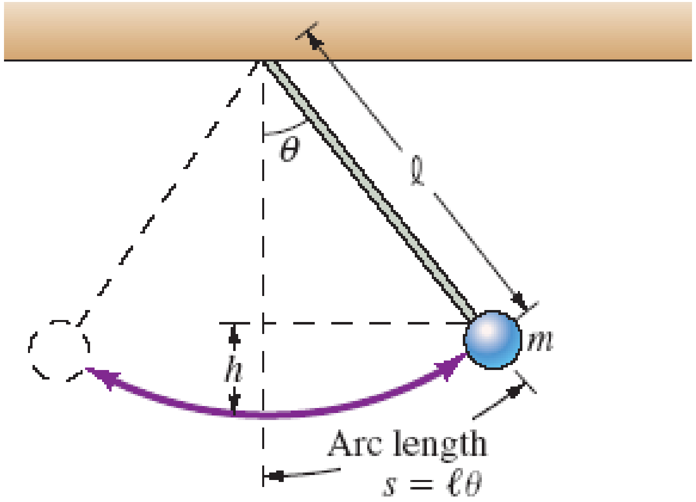
$$E = \frac{1}{2}kA^2 \qquad (13\text{-}23)$$

The total energy varies from being all potential (at extremes of motion) to all kinetic (when spring is neither stretched nor compressed):





13-5 The Simple Pendulum



Position of mass along arc:

$$s = \ell\theta \qquad (13\text{-}26)$$

Velocity along the arc:

$$v = \frac{ds}{dt} = \ell\frac{d\theta}{dt} \qquad (13\text{-}27)$$

Tangential acceleration:

$$a = \frac{dv}{dt} = \ell\frac{d^2\theta}{dt^2} \qquad (13\text{-}28)$$

13-5 The Simple Pendulum

The tangential force comes from gravity (tension is always centripetal for a pendulum):

$$F_t = -mg \sin \theta \qquad (13\text{-}29)$$

Substituting,

$$\ell \frac{d^2\theta}{dt^2} = -g \sin \theta \qquad (13\text{-}30)$$

This is almost a harmonic-oscillator equation, but the right-hand side has sin *θ* instead of *θ*.

13-5 The Simple Pendulum

Fortunately, if *θ* is small, sin *θ* ≈ *θ*:

$$\ell \frac{d^2\theta}{dt^2} = -g\theta \qquad (13\text{-}33)$$

$$\theta = \theta_0 \sin(\omega t + \delta) \quad \text{with} \quad \omega = \sqrt{\frac{g}{\ell}} \qquad (13\text{-}35)$$

Energy of a simple pendulum:

$$K(\theta) = \frac{1}{2}mv^2 = \frac{1}{2}m\ell^2\left(\frac{d\theta}{dt}\right)^2 \qquad (13\text{-}37)$$

$$U(\theta) \cong mg\ell\left(1 - 1 + \frac{1}{2}\theta^2\right) = \frac{1}{2}mg\ell\theta^2 \qquad (13\text{-}39)$$

13-6 More About Pendulums

The Physical Pendulum

Any object, if suspended and then displaced so the gravitational force does no run through the center of mass, can oscillate due to the torque.

$$\tau = I\alpha \qquad \text{(13-40)} \qquad \alpha = d^2\theta/dt^2$$

Also, $\tau = rF \sin\theta, = rMg \sin\theta$ (13-41)

And therefore $Mgr \sin\theta = -I\dfrac{d^2\theta}{dt^2}$ (13-42)

13-6 More About Pendulums

As before, sin *θ* can be replaced by *θ* if *θ* is small, and the motion is simple harmonic with frequency:

$$\omega = \sqrt{\frac{Mgr}{I}} \qquad (13\text{-}43a)$$

13-7 Damped Harmonic Motion

Look at drag force that is proportional to velocity; *b* is the damping coefficient:

$$\vec{F}_d = -b\vec{v} = -b\frac{d\vec{x}}{dt} \qquad (13\text{-}44)$$

Then the equation of motion is:

$$-kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2} \qquad (13\text{-}45)$$

Trial solution – α and ω´ need to be found:

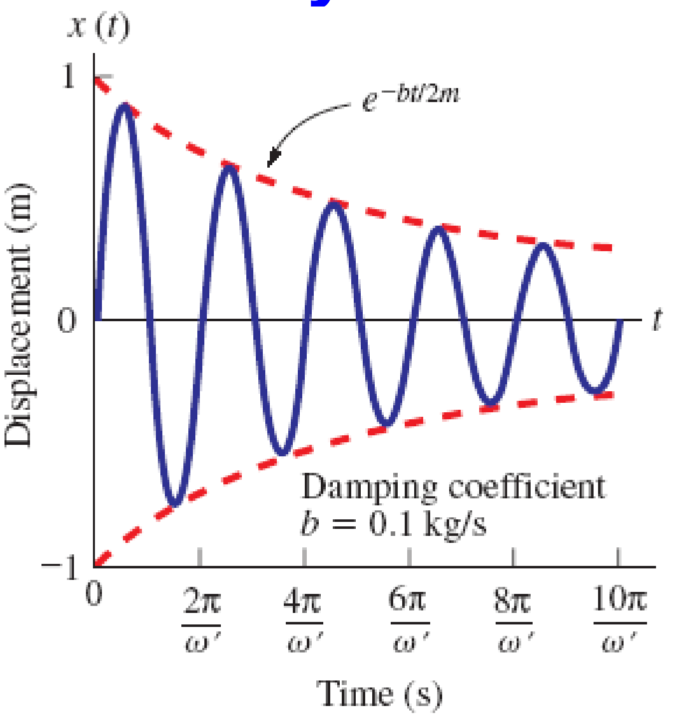
$$x = Ae^{-\alpha t}\sin(\omega' t + \delta) \qquad (13\text{-}46)$$

13-7 Damped Harmonic Motion

Solving, $$\alpha = \frac{b}{2m}$$ (13-47)

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} = \sqrt{\omega_0^2 - \frac{b^2}{4m^2}}$$ (13-48)

For light damping, motion is oscillatory within an exponential envelope:



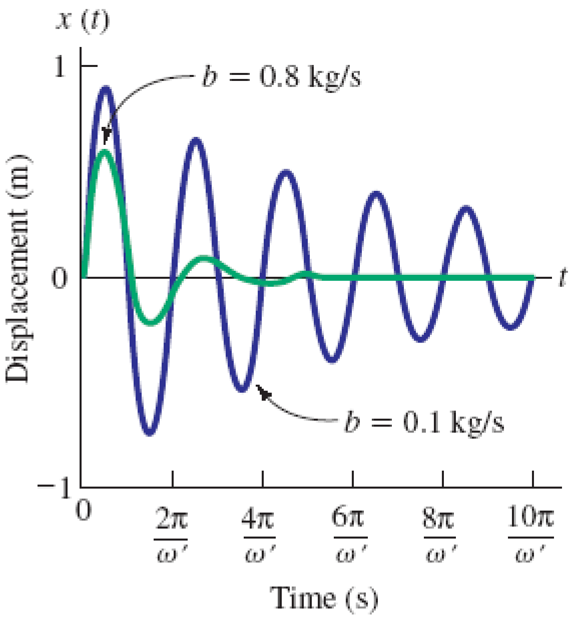
Exponential envelope has time constant:

$$\tau \equiv \frac{m}{b}$$ (13-50)

Can also define quality factor Q: $$Q \equiv \omega_0 \tau$$ (13-51)

13-7 Damped Harmonic Motion

For heavier damping, but still underdamped, we have the green curve:



When $\omega' = 0$,

$$b^2 = 4mk$$

This is critical damping, and the value of b for which this occurs is b_c:

When $b > b_c$, overdamped

When $b < b_c$, underdamped

13-7 Damped Harmonic Motion

Exponential envelope has time constant:

$$\tau \equiv \frac{m}{b} \qquad (13\text{-}50)$$

Can also define quality factor Q:

$$Q \equiv \omega_0 \tau \qquad (13\text{-}51)$$

13-8 Driven Harmonic Motion

Now, have a sinusoidal driving force, which may or may not be at the natural frequency of the system.

Equation of motion becomes:

$$-kx - b\frac{dx}{dt} + F_0 \sin(\omega t) = m\frac{d^2x}{dt^2} \qquad (13\text{-}52)$$

Test solution:

$$x = A\sin(\omega t + \delta) \qquad (13\text{-}53)$$

13-8 Driven Harmonic Motion

Solving for the amplitude:

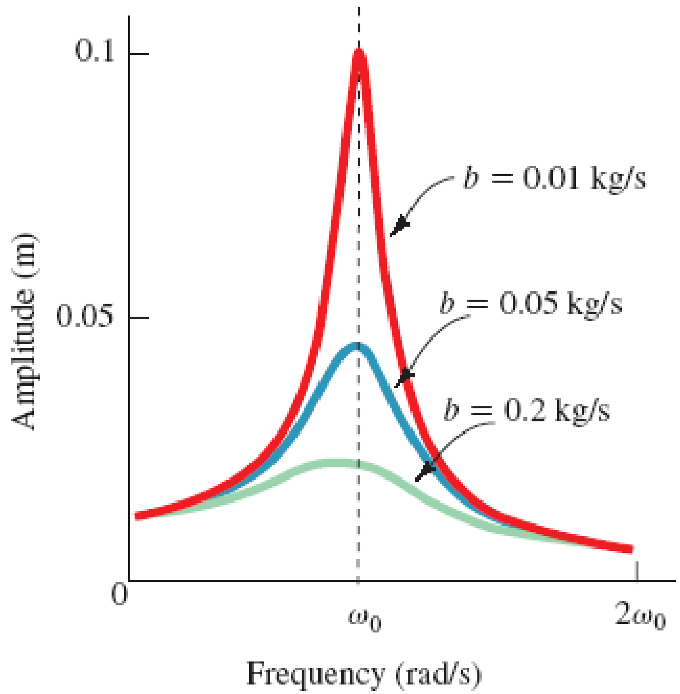
$$A = \frac{F_0}{\sqrt{m^2(\omega^2 - \omega_0^2)^2 + b^2\omega^2}} \quad (13\text{-}54)$$

- **Amplitude is maximum when $\omega = \omega_0$**
- **Must be some damping, or amplitude would become infinite**

13-8 Driven Harmonic Motion

Position of peak amplitude shifts as *b* increases:

$$\omega_{\max}^2 = \omega_0^2 - \frac{1}{2}\left(\frac{b^2}{m^2}\right) \quad (13\text{-}55)$$



13-8 Driven Harmonic Motion

Also, peak becomes broader as *b* increases:

$$\Delta\omega \cong \frac{2b}{m} \quad (13\text{-}56)$$