## The Standard Model

#### **OUTLINE**

#### **Electroweak Unification**

- Chiral fermion states
- Electroweak reactions:  $e^+e^- \rightarrow f \bar{f}$
- PETRA and LEP1 data

#### **Higgs Field**

- Spontaneous symmetry breaking
- LEP2 and LHC

#### Weak Mixing in the Quark Sector

- CKM matrix
- CP violation in the  $B^0$  system

#### Weak Mixing in the Neutrino Sector

- SNO
- Neutrino masses and Oscillations

#### **Conclusion**

## **Elementary Particles**



## **Standard Model**



The SM provides a general description of the physics physics currently accessible with modern particle accelerators. The minimal SM postulates that matter is composed of fundamental spin- $\frac{1}{2}$  quarks and spin- $\frac{1}{2}$  leptons interacting via spin one gauge bosons.

**Electroweak Lagrangian:** 

 $\mathcal{L} = \mathcal{L}(\text{weak CC}) + \mathcal{L}(\text{weak NC}) + \mathcal{L}(\text{em NC})$  $\mathcal{L}(\text{weak CC}) = \frac{g}{\sqrt{2}} \left( J_{\mu}^{-} W^{\mu +} + J_{\mu}^{+} W^{\mu -} \right)$  $\mathcal{L}(\text{weak NC}) = \frac{g}{\cos \theta_{W}} \left( J_{\mu}^{3} - \sin^{2} \theta_{W} J_{\mu}^{\text{em}} \right) Z^{\mu}$  $\mathcal{L}(\text{em NC}) = e J_{\mu}^{\text{em}} A^{\mu}$ 

**QCD:** The gluon couples to the color charge of the quark. The strong potential for short interquark distances  $(r \lesssim R_{\rm hadron} \simeq 1/\Lambda_{\rm QCD} \simeq 1 \text{ fm})$  is:

$$V_{\rm QCD} \simeq -\frac{4\,\alpha_s}{3\,r}\,,$$

where  $\alpha_s$  is the strong coupling constants between quarks and gluons. At large distances (r > 1 fm), a confining term must be added to confine quarks inside hadrons.

## **Chiral Fermion States**

Unification of the E&M and weak interactions: the former has a purely vectorial coupling  $[\gamma_{\mu}]$  while the latter as a V - A character  $[\gamma_{\mu}(1 - \gamma_5)]$ . Let's absorb the  $(1 - \gamma_5)$  in the definition of the spinors:

$$u_L(p) = \frac{(1 - \gamma_5)}{2} u(p)$$
 and  $v_R(p) = \frac{(1 - \gamma_5)}{2} v(p)$ 

$$u_R(p) = \frac{(1+\gamma_5)}{2} u(p)$$
 and  $v_L(p) = \frac{(1+\gamma_5)}{2} v(p)$ 

Here L =left-handed and R =right-handed. Thus:



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## Pure vectorial weak vertex \_\_\_\_

Conseqences:

$$u = \frac{(1 - \gamma_5)}{2}u + \frac{(1 + \gamma_5)}{2}u = u_L + u_R$$

and

$$\bar{u} = \bar{u}\frac{(1+\gamma_5)}{2} + \bar{u}\frac{(1-\gamma_5)}{2} = \bar{u}_L + \bar{u}_R$$

Plus,

$$\bar{u}_L \gamma_\mu u_R = \bar{u}_R \gamma_\mu u_L = 0$$

Thus the electromagntic current can be written as:

$$j^{em}_{\mu} = -\bar{e}\gamma_{\mu}e = -\bar{e}_L\gamma_{\mu}e_L - \bar{e}_R\gamma_{\mu}e_R$$

Define: 
$$j^{\pm}_{\mu} = \bar{\chi}_L \gamma_{\mu} \sigma^{\pm} \chi_L$$
 with  $\sigma^{\pm} = \frac{1}{2} (\sigma^1 \pm i \sigma^2)$ :

$$ec{j}_{\mu} = rac{1}{2} ar{\chi}_L \gamma_\mu \, ec{\sigma} \, \chi_L \;\; ext{and} \;\; J^Y_\mu = 2 j^{em}_\mu - 2 j^3_\mu$$

with

$$\chi_{L}$$

$$\begin{pmatrix} \nu_{e} \\ e \end{pmatrix}_{L} \begin{pmatrix} \nu_{\mu} \\ \mu \end{pmatrix}_{L} \begin{pmatrix} \nu_{\tau} \\ \tau \end{pmatrix}_{L}$$

$$\begin{pmatrix} u \\ d' \end{pmatrix}_{L} \begin{pmatrix} c \\ s' \end{pmatrix}_{L} \begin{pmatrix} t \\ b' \end{pmatrix}_{L}$$

$$SU(2)_{L} \otimes U(1)_{Y}$$
Unified electro-weak vertex:  

$$-i \left[ g_{w} \vec{j}_{\mu} \cdot \vec{W}^{\mu} + \frac{g'}{2} J_{\mu}^{Y} B^{\mu} \right]$$
with  $W_{\mu}^{\pm} \equiv \frac{1}{\sqrt{2}} (W_{\mu}^{1} \mp i W_{\mu}^{2})$   
 $\vec{j}_{\mu} \cdot \vec{W}^{\mu} = j_{\mu}^{1} W^{\mu 1} + j_{\mu}^{2} W^{\mu 2} + j_{\mu}^{3} W^{\mu 3}$   
 $\vec{j}_{\mu} \cdot \vec{W}^{\mu} = \frac{1}{\sqrt{2}} j_{\mu}^{+} W^{\mu +} + \frac{1}{\sqrt{2}} j_{\mu}^{-} W^{\mu -} + j_{\mu}^{3} W^{\mu 3}$ 

such that

$$-\frac{ig_w}{\sqrt{2}}j_{\mu}^{\pm} = -\frac{ig_w}{2\sqrt{2}}[\bar{u}\gamma_{\mu}(1-\gamma_5)u]W^{\mu\pm}$$

The neutral underlying  $SU(2)_L \otimes U(1)_Y$  allow the  $W^3$  and the B to mix to the physical states called photon and the Z:

 $A_{\mu} = B_{\mu} \cos \theta_w + W_{\mu}^3 \sin \theta_w$  $Z_{\mu} = -B_{\mu} \sin \theta_w + W_{\mu}^3 \cos \theta_w$ 

and with  $g_w \sin \theta_w = g' \cos \theta_w = g_e$  then

$$-i\left[g_w\,j^3_\mu\cdot W^{\mu3}+rac{g'}{2}J^Y_\mu B^\mu
ight] 
onumber\ -ig_ej^{em}_\mu A^\mu-ig_z(j^3_\mu-\sin^2 heta_wj^{em}_\mu)Z^\mu$$

## Weak Neutral Current

Knowing that  $j_{\mu}^{em} = j_{\mu}^3 + \frac{1}{2}j_{\mu}^Y$  and  $g_Z = \frac{g_e}{\sin \theta_w \cos \theta_w}$ .

 $-ig_z \, (j^3_\mu - \sin^2 heta_w j^{em}_\mu) Z^\mu \qquad [Z^0 \, \, {
m weak \, current}]$ 

where within a particle doublet:

$$j_{\mu}^{em} = \sum_{i=1}^{2} Q_i (\bar{u}_{iL} \gamma_{\mu} u_{iL} + \bar{u}_{iR} \gamma_{\mu} u_{iR})$$

Then the  $Z^0 \rightarrow f \bar{f}$  vertex factor depends on the particular quark and lepton (*i.e.* f) involved:

$$\frac{-ig_Z}{2}\gamma^{\mu}(c_V^f - c_A^f\gamma_5) \qquad [Z^0 \text{ vertex factor}]$$

with

| f         | $c_V$                                   | $c_A$          |
|-----------|---|----------------|
| $ u_\ell$ | $\frac{1}{2}$                           | $\frac{1}{2}$  |
| l         | $-\frac{1}{2} + 2\sin^2\theta_w$        | $-\frac{1}{2}$ |
| q         | $\frac{1}{2} - \frac{4}{3}\sin^2	heta$  | $\frac{1}{2}$  |
| q'        | $-\frac{1}{2}+\frac{2}{3}\sin^2	heta_w$ | $-\frac{1}{2}$ |

## Weinberg Angle

Reaction like the *pure* neutral reaction  $u_{\mu} + e^- \rightarrow \nu_e + \mu^-$  were used to make the first measurement in 1973 of  $c_V^{\ell}$  and  $c_A^{\ell}$  and thus:

 $\sin^2 \theta_w = 0.22 \pm 0.03$ 

At SLC/LEP  $Z^0$  bosons were produced copiously and it allowed a very precise determination of the properties Z. Using all experimental data:

 $\sin^2 \theta_w = 0.23147 \pm 0.00016$ 



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## Mass & Width of the Z Boson



Total Width (final LEP average June 2001)  $\Gamma_{\rm Z} \equiv \Gamma({\rm Z} \rightarrow {X}) = 2.4952 \pm 0.0023 \text{ GeV}$ 

#### Hadronic Width

 $\Gamma(Z \rightarrow q\overline{q}) = 1.7442 \pm 0.0020 \text{ GeV}$ 

#### **Leptonic Width**

 $\Gamma(Z \to \ell^+ \ell^-) = 0.083991 \pm 0.000087 \text{ GeV}$ 

#### Such that

 $\Gamma_{\rm Z} = \Gamma({\rm Z} \to {\rm q}{\rm \overline{q}}) + 3\,\Gamma({\rm Z} \to \ell^+\ell^-) + {\rm invisible}$ 

## **Number of Neutrino Types**

Invisible Width (calculated)  $\Gamma(Z \rightarrow \nu_{\ell} \bar{\nu}_{\ell}) = 0.1666 \text{ GeV}$ 

$$\label{eq:Gamma-Lambda} \begin{split} \mathbf{Thus} \\ \Gamma_{\rm Z} &= \Gamma_{\rm q\overline{q}} + 3\,\Gamma_{\ell\bar{\ell}} + N_{\nu}\,\Gamma_{\nu\bar{\nu}} \end{split}$$

 $\label{eq:nonlinear} \begin{array}{l} \mbox{Implies} \\ N_{\nu} = 2.9841 \pm 0.0083 \end{array}$ 



## Higgs Field

Physicists have theorized the existence of the so-called Higgs field, which in theory interacts with other particles to give them **mass**. The Higgs field requires a particle, the Higgs boson. The Higgs boson has not been observed, but physicists are looking for it with great enthusiasm.

Limit on the SM Higgs mass with data collected at LEP2  $\sqrt{s} = 161 - 210$  GeV:

#### $m_H > 113.5$ GeV (95% CL)

#### Spontaneous symmetry breaking:

The form of the Lagrangian that couples the Higgs field and the quarks is constrained by  $SU(2)_L$  gauge invariance:

$$\mathcal{L} = \sum_{j,k} [Y_{jk}(\bar{u}_L^j, \bar{d}_L^j) \begin{pmatrix} \phi^0 \\ -\phi^+ \end{pmatrix}^* u_R^k + Y'_{jk}(\bar{u}_L^j, \bar{d}_L^j) \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} d_R^k]$$

where j and k run over quarks generations, L and R denotes left- and right-handed component, and  $Y_{jk}$  and  $Y'_{jk}$  are the Yukawa couplings. The complex Higgs doublet undergoes spontaneous symmetry breaking:

$$\left(\begin{array}{c} \phi^+ \\ \phi^0 \end{array}\right) \to \frac{1}{\sqrt{2}} \left(\begin{array}{c} 0 \\ v + H(x) \end{array}\right)$$

where v is the Higgs vacuum expectation and H(x) is the Higgs field corresponding to the Higgs boson.

$$\mathcal{L} = \sum_{j,k} [Y_{jk} \bar{u}_L^j u_R^k + Y'_{jk} \bar{d}_L^j d_R^k] \frac{1}{\sqrt{2}} [v + H(x)]$$

The term proportional to v generate the quark mass  $m_{jk} \equiv \frac{-v}{\sqrt{2}} Y_{jk}$ and  $m'_{jk} \equiv \frac{-v}{\sqrt{2}} Y'_{jk}$ .



## LHC: Large Hadron Collider \_\_\_\_

The LHC, in construction at CERN, is a proton-proton collider with  $\sqrt{s} = 14 \text{ TeV}$ . The SPS collider which discovered the W - Z bosons had  $\sqrt{s} = 0.45 \text{ TeV}$  and the Tevatron collider at FermiLab has  $\sqrt{s} = 1.8 \text{ TeV}$ .

Higgs events at the LHC



LHC will take data in 2007 !!!

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## **NLC: Next linear Collider**

The Next Linear Collider (NLC) is proposed as the future generation of accelerator to probe matter. The design of the NLC is a 0.5 TeV  $e^+e^-$  collider to investigate the properties of the W - Z bosons, the top quark, and their couplings; and search for super-symmetric particles (SUSY).

#### Superconducting Acceleration Cavity



## CKM Matrix

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$
 [CKM Matrix]

such that

$$\left(\begin{array}{c}d'\\s'\\b'\end{array}\right) = \left(\begin{array}{ccc}V_{ud} & V_{us} & V_{ub}\\V_{cd} & V_{cs} & V_{cb}\\V_{td} & V_{ts} & V_{tb}\end{array}\right) \left(\begin{array}{c}d\\s\\b\end{array}\right)$$

The CKM matrix can be decomposed as:

| $\begin{pmatrix} 1\\ 0 \end{pmatrix}$ | $0 c_{23}$     | $\begin{pmatrix} 0 \\ s_{23} \end{pmatrix}$ | $ \left(\begin{array}{c}c_{13}\\0\\-s_{13}\gamma\end{array}\right) $ | 0<br>1 | ${}^{s}_{13}{}^{\beta}$ |               | $c_{12} - s_{12}$ | ${}^{s}12$ ${}^{c}12$ | $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ |  |
|---------------------------------------|----------------|---|--|--------|-------------------------|---------------|-------------------|-----------------------|--|--|
| 0                                     | $-s_{23}^{20}$ | $c_{23}^{20}$ /                             | $\langle -s_{13}\gamma$  | 0      | $c_{13}$ .              | $\mathcal{F}$ | 0                 | $\overset{12}{0}$     | 1 /                                    |  |

where  $c_{ij} = \cos \theta_{ij}$ ,  $s_{ij} = \sin \theta_{ij}$ , and i, j denote the quark generations. The middle matrix has incorporate the complex phase  $\delta$  such that  $\beta = e^{-i\delta}$  and  $\gamma = e^{i\delta}$  to describe a rotation between quarks that are two generations apart. Multiplying these matrices:

| ( | c <sub>12</sub> c <sub>13</sub>                      | <sup>s</sup> 12 <sup>c</sup> 13                 | $s_{13}e^{-i\delta}$ | ) |
|---|--|---|----------------------|---|
|   | $-s_{12}c_{23} - c_{12}s_{23}s_{13}e_{15}^{i\delta}$ | $c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta}$  | $s_{23}c_{13}$       |   |
|   | $s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta}$       | $-c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta}$ | $c_{23}c_{13}$       | ) |

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## **Wolfenstein Parameterization**

Based on hierarchical we can expand in powers of the Cabibbo angle  $\lambda = s_{12} = 0.22$ , with  $s_{23} = A\lambda^2$  and  $s_{13}e^{-i\delta} = A\lambda^3(\rho - i\eta)$ :

$$V = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

# Complex phase allow CP violation in the framework of Standard Model for a $3 \times 3$ CKM matrix

Or be neglecting the CP phase and the small  $b \rightarrow u$  and  $t \rightarrow d$  transitions:

$$V \simeq \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & 0 \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ 0 & -A\lambda^2 & 1 \end{pmatrix}$$

By neglecting the  $s_{23}$  we have the simple Cabbibo description  $\theta_C \equiv \lambda$ :

$$V \sim \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & 0\\ -\lambda & 1 - \frac{1}{2}\lambda^2 & 0\\ 0 & 0 & 1 \end{pmatrix} \simeq \begin{pmatrix} \cos\theta_C & \sin\theta_C & 0\\ -\sin\theta_C & \cos\theta_C & 0\\ 0 & 0 & 1 \end{pmatrix}$$



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## **B** Factories

Time evolution of the  $B^0$  system since the integration over time gives simply the mass difference and NOT the CP phase: Asymmetric B-factories [BaBar & Belle] operating at the  $\Upsilon(4S)$  with luminosity  $\sim 10^{34} cm^{-2} s^{-1}$ (PETRA in the 1980's had  $\mathcal{L} \sim 10^{31} cm^{-2} s^{-1}$ !!!).

#### Measure the angles of the unitary triangle



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Sudbury

Neutrino

Observatory





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## **Construction Phase**





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#### **Solar Neutrino Event**



#### **Cherenkov Light**

When a particle travels through a medium such that its velocity v is greater than the velocity of light in the medium c/n, radiation is emitted. The radiation is confined to a **cone** around the direction of the incident particle.

#### **SNO** = Heavy Water Cherenkov Detector



#### **Solar Neutrino Problem**

The Solar Neutrino Problem

|  | BP SSM                    | Expt  | Expt/BPSSM   |
|--|---------------------------|---|--|
| Homestake                                  | $9.3^{+1.2}_{-1.4}$ a)    | $2.55 \pm 0.14 \pm 0.14$ a)   | $0.273 \pm 0.021$  |
| Kamiokande<br>Super-Kamiokande<br>Combined | $6.62^{+0.93}_{-1.12}$ b) | $\begin{array}{c} 2.80 \pm 0.19 \pm 0.33 \\ 2.51 \substack{+0.14 \\ -0.13} \pm 0.18 \\ 2.586 \pm 0.195 \\ \mathrm{b} \end{array}$ | $\begin{array}{c} 0.423 \pm 0.058 \\ 0.379 \pm 0.029 \\ 0.391 \pm 0.029 \end{array}$ |
| SAGE<br>GALLEX<br>Combined                 | $137^{+8}_{-7}$ a)        | $69 \pm 10^{+5}_{-7}$<br>$69.7^{+3.9}_{-4.5}$<br>$69.5 \pm 6.7$<br>a)   | $0.504 \pm 0.089$<br>$0.509 \pm 0.059$<br>$0.507 \pm 0.049$                          |

Units a) SNU  $(10^{-36}/s/tgt atom)$ b)  $10^{6}/cm^{2}/s$ 



From Hata and Langacker, preprint 1997.

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#### **Neutrino Oscillations**

#### **Neutrino Oscillations**

For simplicity consider only two neutrino flavours,  $\nu_e$ ,  $\nu_{\mu}$ 

• Suppose the flavour eigenstates,  $|\nu_e\rangle$ ,  $|\nu_{\mu}\rangle$  are not the mass eigenstates,  $|\nu_1\rangle$ ,  $|\nu_2\rangle$  Then the flavour eigenstates can be represented as a superposition of the mass eigenstates,

$$\left(\begin{array}{c}\nu_e\\\nu_\mu\end{array}\right) = \left(\begin{array}{c}\cos\theta&\sin\theta\\-\sin\theta&\cos\theta\end{array}\right) \left(\begin{array}{c}\nu_1\\\nu_2\end{array}\right)$$

where  $\theta$  is the mixing angle between the mass states.



The time evolution of the flavour states becomes,

$$\begin{aligned} |\nu_e\rangle_t &= \cos\theta e^{-iE_1t} |\nu_1\rangle + \sin\theta e^{-iE_2t} |\nu_2\rangle \\ |\nu_\mu\rangle_t &= -\sin\theta e^{-iE_1t} |\nu_1\rangle + \cos\theta e^{-iE_2t} |\nu_2\rangle \end{aligned}$$

Writing the time evolution in terms of the mass matrix gives,

$$i\frac{d}{dt}\left(\begin{array}{c}\nu_{e}\\\nu_{\mu}\end{array}\right) = \frac{1}{2}\left(\begin{array}{c}-\frac{\Delta m^{2}}{2E}\cos 2\theta & \frac{\Delta m^{2}}{2E}\sin^{2}\theta\\\frac{\Delta m^{2}}{2E}\sin^{2}\theta & \frac{\Delta m^{2}}{2E}\cos 2\theta\end{array}\right)\left(\begin{array}{c}\nu_{e}\\\nu_{\mu}\end{array}\right)$$

where E is the energy of the electron neutrino in MeV and

$$\Delta m^2 \equiv |m_2^2 - m_1^2|$$

• The survival probability of an electron neutrino after travelling a distance, L, is

$$P_e = 1 - \sin^2 2\theta \sin^2 \left[ \pm \frac{1.27\Delta m^2 L}{E} \right]$$

• Furthermore, you can get an enhancement of flavour conversion in the sun due to the Mikheyev Smirnov Wofenstein (MSW) Effect

#### **Deuterium Reactions**

#### **Detecting Neutrinos with Deuterium**

#### **Charged Current**





**Electron Scattering** 



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## **SNO: First Results**



$$\begin{split} \Phi_{CC}^{\rm SNO}: & \text{Sensitive to } \nu_e \text{ only!} \\ \Phi_{ES}^{\rm SK}: & \text{Sensitive to } \nu_e, \ \nu_\mu, \text{ and } \nu_\tau \\ & \text{Here } \Phi_{ES}^{\rm SK} = \Phi(\nu_e) + 0.154 \Phi(\nu_{\mu\tau}) \end{split}$$

 $\Phi(\nu_{\mu\tau}) \neq 0$  at the 3.3 standard deviation.  $\rightarrow$  First evidence of solar neutrino oscillation !!!

**Next:** measure CC/NC will provide an unambiguous statement on whether neutrinos oscillate on their way to the earth from the core of the sun.

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## Summary \_\_\_\_

The open questions of particle physics:

- Weak flavor mixing in the quark and neutrino sectors.
- Search of the Higgs boson and new physics beyond the SM.

Elementary Particles 75-462 & 562:

- Constituents of matter
- Fundamental forces
- Conservation Laws
- Invariance Principles and Symmetries
- Relativistic Kinematics
- Quark Model
- QED and QCD
- Feynman Rules
- Electroweak interactions
- Open questions!

URL: http://www.physics.carleton.ca/~alainb/