Mid-Term

Elementary Particle Physics: Phys-4602
Professor: Alain Bellerive
DEMO 2012

Instructions: Examination for students registered in 4602 (undergraduate level). Attempt all questions. The mid-term exam counts for 25% of the term. The attached sheets have information you may find useful. No additional aids (textbooks, notes, calculators, tables) are permitted nor required.

1. In the Standard Model all matter is composed of quarks and leptons which interact via the exchange of gauge bosons. (5 pts)
   (a) What distinguishes quarks from leptons?
   (b) What are the mediators of the electromagnetic, weak, and strong interactions?

2. If $p \bar{p}$ annihilation at rest proceeds via S-wave states, explain why the reaction $p \bar{p} \rightarrow \pi^0 \pi^0$ cannot be a strong interaction. (5 pts)

3. State which of the following particle reactions are allowed, and if the reaction is not allowed, explain why not (10 pts).
   (i) $\mu^+ \rightarrow e^+ + \gamma$
   (ii) $\Lambda^0 \rightarrow p + e^- + \bar{\nu}_e$
   (iii) $\Sigma^- \rightarrow \Lambda^0 + \pi^-
   (iv) $\pi^- + p \rightarrow \pi^0 + \pi^0 + n$
   (v) $n + p \rightarrow K^+ + \pi^0 + \Lambda^0$
   (vi) $e^- + p \rightarrow e^- + \pi^+ + n$
   (vii) $K^- + p \rightarrow K^0 + \pi^+ + \Xi^-$
   (viii) $e^+ + e^- \rightarrow \mu^+ + \mu^-
   (ix) $e^+ + e^- \rightarrow K^- + K^0 + \pi^+
   (x) $\nu_e + p \rightarrow n + e^+ + \pi^0$

4. The deuteron is a bound state of two nucleus with total spin 1 and positive parity which may only exist in the $^{2S+1}L_J = ^3S_1$ and $^3D_1$ states of the n-p system.
(a) Based on symmetry arguments, show that the isospin of the deuteron is $I = 0$. (5 pts)

(b) Consider the proton scattering on deuteron. At a given centre-of-mass energy, what is the ratio of the cross-sections for the following strong reactions: (5 pts)

(1) $p + d \rightarrow \pi^0 + {}^3\text{He}$

(2) $p + d \rightarrow \pi^+ + {}^3\text{H}$

The nucleus ${}^3\text{He}$ and ${}^3\text{H}$ have the same quantum numbers than the proton and the neutron, respectively.
Leptons

\[
\begin{pmatrix}
e^- \\
\nu_e
\end{pmatrix}
\begin{pmatrix}
\mu^- \\
\nu_\mu
\end{pmatrix}
\begin{pmatrix}
\tau^- \\
\nu_\tau
\end{pmatrix}
\quad Q = -1
\quad Q = 0
\tag{1}
\]

Quarks

\[
\begin{pmatrix}
u \\
\mu \\
\tau
\end{pmatrix}
\begin{pmatrix}
\alpha \\
\beta \\
\gamma
\end{pmatrix}
\quad Q = +\frac{2}{3}
\quad Q = -\frac{1}{3}
\tag{2}
\]

Quark Mixing Matrix

\[
V_{\text{CKM}} = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\begin{pmatrix}
0.975 & 0.221 & 0.004 \\
0.221 & 0.974 & 0.043 \\
0.004 & 0.043 & 0.999
\end{pmatrix}
\tag{3}
\]

Quark Model

\[
Q = I_3 + \frac{Y}{2},
\tag{4}
\]

where \( Q \) is the charge, \( I_3 \) is the 3rd isospin component, and
\( Y = B + S + C + B + T \) is the hypercharge.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Weak</th>
<th>E&amp;M</th>
<th>Strong</th>
</tr>
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<tbody>
<tr>
<td>Energy</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Charge</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Momentum</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Angular Momentum</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Baryon Number</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Lepton Number</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Isospin</td>
<td>no</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>Strangeness</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>

Table 1: Conservation Laws.
Table 2: Quantum numbers for leptons: $L_{\ell}$ is the lepton number, $Q_{\ell}$ is the charge, and $P$ is the parity. For antileptons $L_{\bar{\ell}} = -L_{\ell}$, $Q_{\bar{\ell}} = -Q_{\ell}$, and $P_{\bar{\ell}} = -P_{\ell}$.

<table>
<thead>
<tr>
<th>Particle</th>
<th>$L_{e}$</th>
<th>$L_{\mu}$</th>
<th>$L_{\tau}$</th>
<th>$Q_{\ell}$</th>
<th>$P$</th>
<th>Mass (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^{-}$</td>
<td>+1</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>+1</td>
<td>0.511</td>
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<tr>
<td>$\mu^{-}$</td>
<td>0</td>
<td>+1</td>
<td>0</td>
<td>-1</td>
<td>+1</td>
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<tr>
<td>$\tau^{-}$</td>
<td>0</td>
<td>0</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>1784</td>
</tr>
<tr>
<td>$\nu_{e}$</td>
<td>+1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>+1</td>
<td>0</td>
</tr>
<tr>
<td>$\nu_{\mu}$</td>
<td>0</td>
<td>+1</td>
<td>0</td>
<td>0</td>
<td>+1</td>
<td>0</td>
</tr>
<tr>
<td>$\nu_{\tau}$</td>
<td>0</td>
<td>0</td>
<td>+1</td>
<td>0</td>
<td>+1</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3: Quantum numbers for baryons and mesons: $B$ is the baryon number, $S$ is the strangeness, $I$ is the isospin, $I_{3}$ is the $3^{rd}$ isospin component, $P$ is the parity.

<table>
<thead>
<tr>
<th>Particle</th>
<th>Quark Content</th>
<th>$B$</th>
<th>$S$</th>
<th>$I$</th>
<th>$I_{3}$</th>
<th>$P$</th>
<th>Mass (GeV)</th>
</tr>
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<tbody>
<tr>
<td>$p$</td>
<td>$uud$</td>
<td>1</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>+1</td>
<td>0.938</td>
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<tr>
<td>$n$</td>
<td>$udd$</td>
<td>1</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>+1</td>
<td>0.939</td>
</tr>
<tr>
<td>$\Delta^{++}$</td>
<td>$uua$</td>
<td>1</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>+1</td>
<td>1.230</td>
</tr>
<tr>
<td>$\Lambda^{0}$</td>
<td>$uds$</td>
<td>1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>+1</td>
<td>1.115</td>
</tr>
<tr>
<td>$\Sigma^{-}$</td>
<td>$dds$</td>
<td>1</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
<td>+1</td>
<td>1.197</td>
</tr>
<tr>
<td>$\Xi^{-}$</td>
<td>$dss$</td>
<td>1</td>
<td>-2</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>+1</td>
<td>1.321</td>
</tr>
<tr>
<td>$\Omega^{-}$</td>
<td>$sss$</td>
<td>1</td>
<td>-3</td>
<td>0</td>
<td>0</td>
<td>+1</td>
<td>1.672</td>
</tr>
<tr>
<td>$\pi^{+}$</td>
<td>$ud$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>+1</td>
<td>-1</td>
<td>0.140</td>
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<tr>
<td>$\pi^{0}$</td>
<td>$u\bar{u}$ or $d\bar{d}$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>0.135</td>
</tr>
<tr>
<td>$\pi^{-}$</td>
<td>$\bar{u}d$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
<td>0.140</td>
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<tr>
<td>$K^{+}$</td>
<td>$u\bar{s}$</td>
<td>0</td>
<td>+1</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
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<tr>
<td>$K^{-}$</td>
<td>$\bar{u}s$</td>
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<td>-1</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
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<td>0.494</td>
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<tr>
<td>$K_{0}$</td>
<td>$d\bar{s}$</td>
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<td>-1</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>-1</td>
<td>0.497</td>
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<tr>
<td>$K_{0}^{*}$</td>
<td>$d\bar{s}$</td>
<td>0</td>
<td>+1</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>-1</td>
<td>0.497</td>
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