Final

Elementary Particle Physics: PHYS 5602
Professor: Alain Bellerive
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1. **Conservation laws**

   The $\rho^0(770)$ and the $f^0(1274)$ mesons decay via the strong interaction to give $\pi^+\pi^-$ pairs and have spin 1 and spin 2, respectively.

   (a) Show that a meson which decays to $\pi^+\pi^-$ by the strong interaction must have $C = P = (-1)^J$, where J is the spin of the meson. (2 pts)

   (b) Which of the decays $\rho^0 \to \pi^0\gamma$ and $f^0 \to \pi^0\gamma$ is forbidden to occur by an EM interaction? The photon has $J^{PC} = 1^{--}$ and the π^0 has $J^{PC} = 0^{-+}$. (2 pts)

   (c) Which of the decays $\rho^0 \to \pi^0\pi^0$ and $f^0 \to \pi^0\pi^0$ is forbidden by ANY interaction? (2 pts)

2. **$\Upsilon(4S)$ meson production in e^+e^- collisions**

   (a) Explain why the dominant decay mode for the spin-1 $b\bar{b}$ resonance $\Upsilon(4S)$ meson, which has a mass around 1058 MeV, is $\Upsilon(4S) \to B\bar{B}$ and not $\Upsilon(4S) \to$ light hadrons. (2 pts)

   (b) Spin-1 $\Upsilon(4S)$ mesons are produced in S-wave ($L = 0$) electron-positron collisions at $\sqrt{s} \simeq 10.58$ GeV via the electomagnetic process $e^+e^- \to \gamma^* \to \Upsilon(4S)$. What are the parity($P$) and charge-conjugate($C$) quantum numbers of the $\Upsilon(4S)$? (4pts)

3. **Decay of the pion**

   A pion at rest decays into a lepton and a neutrino:

$$\pi^- \to \ell^- + \bar{\nu}_\ell$$

   (a) Draw the Feynman diagram for this decay. (1 pt)

   (b) Why ℓ can only be a muon (μ) or an electron (e)? (1 pt)

   (c) Find the energies of the outgoing particles in term of the various masses. (3 pts)

   (d) Find the magnitudes of the outgoing momenta. (2 pts)

(e) Prove that the decay rate is given by the formula: (2 pts)

$$\Gamma(\pi^- \to \ell^- \bar{\nu}_\ell) = \frac{S\,(m_\pi^2 - m_\ell^2)}{16\pi\hbar m_\pi^3}\,|\,\mathcal{M}\,|^2$$

(f) Use the fact that the antineutrino is always right-handed to explain why $\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu) \gg \Gamma(\pi^- \to e^- \bar{\nu}_e)$. (3 pts)

4. **Decay of the tau lepton and the b quark**

(a) Estimate the ratio (3 pts)

$$\frac{\Gamma(\tau^- \to K^- + \nu_\tau)}{\Gamma(\tau^- \to \pi^- + \nu_\tau)}$$

where the kaon is $K^- = s\bar{u}$ and the pion is $\pi^- = d\bar{u}$.

(b) Find an expression for the lifetime of the bottom quark (τ_b) in term of the tau lifetime (τ_τ), the tau mass (m_τ), the bare mass of the b quark (m_b), and the relevant CKM matrix elements. Use the mass hierarchy $m_b \gg m_s \gg m_d$ and $m_t \gg m_c \gg m_u$ with $m_u \approx m_d$ and $m_c \approx m_s$. (3pts)

5. **Baryon mass**

The Σ^0 and the Λ^0 are both neutral octet baryons with quark content uds and strangeness -1. Explain the difference of mass between the Σ^0 and Λ^0 baryons in term of quark pairings. You do not need to do any lengthy calculations for the size of the mass difference, but indicate all the assumptions used in the context of the static quark model. (5pts)

6. **$|\Delta I| = \frac{1}{2}$ weak selection rule**

(a) Show that the weak quark transition $s \to u W^-$ implies that $\Delta I_3 = -\frac{1}{2}$ and thus that the *favored* isospin weak transition of the strange quark is $|\Delta I| = \frac{1}{2}$ even if isospin is not explicitly conserved in weak interactions. (2 pts)

(b) Show that even if the process $\Xi^- \to n\pi^-$ satisfies most of the conservation laws, it will never be observed experimentally because it violates the $|\Delta I| = \frac{1}{2}$ selection rule. (1 pt)

[see next page for part (c)]

(c) Finally, find with the $|\Delta I| = \frac{1}{2}$ rule the ratio for: (2pts)

$$\frac{\Gamma(\Lambda^0 \to n\pi^0)}{\Gamma(\Lambda^0 \to p\pi^-)}$$

Draw the appropriate weak Feynman diagrams.

Leptons

$$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix} \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix} \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix} \quad \begin{array}{l} Q = -1 \\ Q = \ \ 0 \end{array}$$

Quarks

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix} \quad \begin{array}{l} Q = +\frac{2}{3} \\ Q = -\frac{1}{3} \end{array}$$

Quark Mixing Matrix

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.975 & 0.221 & 0.004 \\ 0.221 & 0.974 & 0.043 \\ 0.004 & 0.043 & 0.999 \end{pmatrix}$$

Quark Model

$$Q = I_3 + Y/2\,,$$

where Q is the charge, I_3 is the 3$^{\mathrm{rd}}$ isospin component, and $Y = B + S + C + \mathcal{B} + T$ is the hypercharge.

Meson and Baryon Mass Formula

$$M(\mathrm{meson}) = m_1 + m_2 + \mathcal{A}\frac{\vec{S}_1 \cdot \vec{S}_2}{m_1 m_2}$$

$$M(\mathrm{baryon}) = m_1 + m_2 + m_3 + \mathcal{A}' \left[\frac{\vec{S}_1 \cdot \vec{S}_2}{m_1 m_2} + \frac{\vec{S}_1 \cdot \vec{S}_3}{m_1 m_3} + \frac{\vec{S}_2 \cdot \vec{S}_3}{m_2 m_3} \right]$$

Decay Rates and Cross-Sections

$$\Gamma[1 \to 2\,+\,3] \equiv \Gamma[\,1\,(m_1 c, 0) \to 2\,(\frac{E_2}{c}, \vec{p})\,+\,3\,(\frac{E_3}{c}, -\vec{p})\,] = \frac{S\,|\,\vec{p}\,|}{8\pi\hbar\, m_1^2 c}\,|\,\mathcal{M}\,|^2$$

$$\frac{d\,\sigma(1\,+\,2 \to 3\,+\,4)}{d\,\Omega} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{S|\,\mathcal{M}\,|^2}{(E_1 + E_2)^2} \frac{|\,\vec{p}_f\,|}{|\,\vec{p}_i\,|}$$

Quantity	Weak	E&M	Strong
Energy	yes	yes	yes
Charge	yes	yes	yes
Momentum	yes	yes	yes
Angular Momentum	yes	yes	yes
Baryon Number	yes	yes	yes
Lepton Number	yes	yes	yes
Isospin	no	no	yes
G-Parity	no	no	yes
Strangeness	no	yes	yes
P-Parity	no	yes	yes
C-Conjugate	no	yes	yes

Table 1: Conservation Laws.

Particle	L_e	L_μ	L_τ	Q_ℓ	P	Mass (MeV)
e^-	+1	0	0	−1	+1	0.511
μ^-	0	+1	0	−1	+1	105.66
τ^-	0	0	+1	−1	+1	1784
ν_e	+1	0	0	0	+1	0
ν_μ	0	+1	0	0	+1	0
ν_τ	0	0	+1	0	+1	0

Table 2: Quantum numbers for leptons: L_ℓ is the lepton number, Q_ℓ is the charge, and P is the P-parity. For antileptons $L_{\bar{\ell}} = -L_\ell$, $Q_{\bar{\ell}} = -Q_\ell$, and $P_{\bar{\ell}} = -P_\ell$.

Quark	I	I_3	Bare Mass (MeV)	Effective Mass (MeV)
u	$\frac{1}{2}$	$\frac{1}{2}$	1 to 5	310 to 365
d	$\frac{1}{2}$	$-\frac{1}{2}$	3 to 9	310 to 365
s	0	0	75 to 150	480 to 540
c	0	0	∼1100	∼1500
b	0	0	∼4700	∼4900
t	0	0	175000	no top bound states

Table 3: Bare Masses & Effective Masses in hadrons and isospins for quarks.

Particle	Quark	Spin	B	S	I	I_3	P	Mass (MeV)
p	uud	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{1}{2}$	$+1$	938
n	udd	$\frac{1}{2}$	1	0	$\frac{1}{2}$	-$\frac{1}{2}$	$+1$	939
Δ^{++}	uuu	$\frac{3}{2}$	1	0	$\frac{3}{2}$	$\frac{3}{2}$	$+1$	1230
Λ^0	uds	$\frac{1}{2}$	1	-1	0	0	$+1$	1115
Σ^-	dds	$\frac{1}{2}$	1	-1	1	-1	$+1$	1197
Σ^0	uds	$\frac{1}{2}$	1	-1	1	0	$+1$	1192
Σ^+	uus	$\frac{1}{2}$	1	-1	1	$+1$	$+1$	1189
Ξ^-	dss	$\frac{1}{2}$	1	-2	$\frac{1}{2}$	-$\frac{1}{2}$	$+1$	1321
Ξ^0	uss	$\frac{1}{2}$	1	-2	$\frac{1}{2}$	$\frac{1}{2}$	$+1$	1314
Ω^-	sss	$\frac{3}{2}$	1	-3	0	0	$+1$	1672
π^+	$u\bar{d}$	0	0	0	1	$+1$	-1	140
π^0	$u\bar{u}$ or $d\bar{d}$	0	0	0	1	0	-1	135
π^-	$\bar{u}d$	0	0	0	1	-1	-1	140
$\rho^0(770)$	$u\bar{u}$ or $d\bar{d}$	1	0	0	1	0	-1	770
$f^0(1274)$	$u\bar{u}$ or $d\bar{d}$	2	0	0	0	0	$+1$	1274
K^+	$u\bar{s}$	0	0	$+1$	$\frac{1}{2}$	$\frac{1}{2}$	-1	494
K^-	$\bar{u}s$	0	0	-1	$\frac{1}{2}$	-$\frac{1}{2}$	-1	494
$\bar{K}^0$	$\bar{d}s$	0	0	-1	$\frac{1}{2}$	$\frac{1}{2}$	-1	497
K^0	$d\bar{s}$	0	0	$+1$	$\frac{1}{2}$	-$\frac{1}{2}$	-1	497
B^+	$u\bar{b}$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	-1	5279
B^-	$\bar{u}b$	0	0	0	$\frac{1}{2}$	-$\frac{1}{2}$	-1	5279
$\bar{B}^0$	$\bar{d}b$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	-1	5279
B^0	$d\bar{b}$	0	0	0	$\frac{1}{2}$	-$\frac{1}{2}$	-1	5279

Table 4: Quantum numbers for baryons and mesons: B is the baryon number, S is the strangeness, I is the isospin, I_3 is the 3$^{\text{rd}}$ isospin component, P is the P-parity.